NWT Apprenticeship Support Materials

Math

Reading Comprehension

Science

* Module 1 – Foundations
* Module 2 – Patterns and Relations
* Module 3 – Variables and Equations
* Module 4 – Measuring Time, Shapes and Space
* Module 5 – Special Topics

P A R T N E R S

SNAP LAKE DIAMOND PROJECT
De Beers

Aurora College

Northwest Territories Education, Culture and Employment

Skills Compétences Canada
Northwest Territories Nunavut

Genesis Group Ltd.

Canada

Alberta Advanced Education
Apprenticeship and Industry Training

First Printing: February 2003
Second Printing: March 2004

Acknowledgements:
The Apprenticeship Support Materials Project has been a true partnership. The challenge has been to develop a set of study materials, which covers every competency in the "Entrance Level Competencies for Apprenticeship Programs" developed by Alberta Advanced Education, Apprenticeship and Industry Training. These study materials although written for Northerners using northern examples, issues and problems, are appropriate for any individual entering the trades. The goal is to equip Northerners for the Trades Entrance Exam as well as to support them through the apprenticeship process.

The following partner organizations have all contributed to the development of these materials:

De Beers Canada Mining Inc. – Snap Lake Diamond Project
Government of the Northwest Territories – Education, Culture and Employment
Government of Canada – Indian and Northern Affairs Canada
Government of Canada – Human Resources Development Canada
Government of Canada – Canadian Rural Partnership
Aurora College
Skills Canada NWT/NU
The Genesis Group Ltd.

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The Partnership Steering Committee wishes to acknowledge his dedication to the goals of northern adult education.

ISBN : 0-7708-0117-4

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Introduction[1]

Entrance Level Competencies In Mathematics (Levels 1, 2, 3, 4, 5)

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Module 3 – Variables And Equations

Module 4 – Measuring Time, Shapes, And Spaces

Organization of Topics

The emphasis in trades is on using mathematics to solve practical problems quickly and correctly. Each topic in this curriculum guide includes:

1. Background and theory
2. Examples with explanations
3. Practice Exam Questions with answers and explanations

This apprenticeship support material outlines competencies, but does not provide detailed lessons, as in a textbook, college classroom, or GED Study Guide. If you need more instruction on a particular competency, you may find these and other textbook resources helpful. If you want to build up speed as well as accuracy in a competency you will find these additional resources helpful as a source of additional practice questions. Need to know information for the trades entrance exam is singled out for your attention by the use of text boxes and bold type.

1 A detailed introduction with sections on self-assessment and study tips is provided with Math – Module 1 – Foundations.

Introduction

Examples are the focus

In this curriculum guide, examples with explanations are the primary tool used for review. Background for each competency is also given with a brief overview of what you need to know. Before any examples are given, the main ideas in each topic are explained and "need- to- know information" is summarized in rules and definitions.

Please Note:

When you work on an example, cover the text below with a piece of paper so that you don't see answers and explanations prematurely.

You may want to skip the background given on a topic and go right to the examples to see how well you do. You can always go back to the theory if you find you need it.

In addition, a set of practice exam questions for Module 3 – Variables and Equations accompanies this learning guide to enable you to assess yourself, decide what you need to study, and practice for the exam.

Unit 1

Understanding Equations

Topic 1 – Patterns and Equations

Background

In the trades, equations are used to describe patterns and solve problems. Formulas and recipes can be explained in words or expressed by equations. Equations set one expression equal to another. For example, an equation can be used to answer this question: "What number of workers needs to be added to the 30 people who show up for work if a crew of 80 is required?"

Choose a letter to represent the number we are seeking and then write an equation that expresses the relationship between what we want to find out and what we know. We can show the relationship between what we want to know and what we are given by this equation:

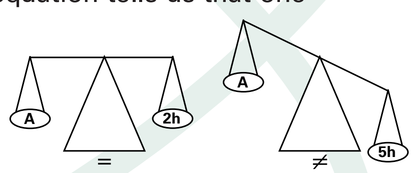
$x + 30 = 80$ (some number added to 30 will equal 80)
$x = 80 - 30$ (subtract 30 from both sides)
$x = 50$
Check: $50 + 30 = 80$

We let x represent the number of workers that would need to be added to the 30 who show up for work, but any letter could be chosen to play this role. We conclude that 50 workers will need to be added to get a crew of 80 if only 30 show up for work. You probably worked this out in your head. Algebra is useful when relationships are too complex to do in your head.

You can use a question mark (?) to represent the number you seek just as well as a letter. Whatever symbol is chosen, it will represent what we want to find, or the "unknown" in the equation. The unknown is also called a variable. This equation was solved by a one step process, sometimes additional steps are needed to get a variable by itself in an equation so that we can see what it is equal to.

Equations

Algebraic expressions appear in equations. An equation tells us that one expression is equal to another. **An equation produces true statements when values are substituted for the variables in the equation**. Each side of the equal sign in an equation can contain more than one term. A term can be a number, or the product of a number and one or more variables represented by letters.



Like a scale, equations compare expressions that produce the same values when they are equal

Unit 1 – Understanding Equations

Topic 1 – Patterns and Equations

Math – Module 2 – Patterns and Relations introduced expressions that contain numbers and variables. These are known as **algebraic expressions**. For example, 2h is an expression. It tells us to multiply a number by two. The value we choose for h can vary or change, but any choice we make will be multiplied by two in order to satisfy the expression 2h. In this expression, h is the **variable** and 2 is a **constant**.

If h = 2, then 2h = 4, if h = $\frac{1}{2}$, then 2h = 1, and so on. **A number multiplied by a variable is also called a coefficient.** 2 is the coefficient of h in the expression 2h. Our choice of a value for h determines what the expression 2h will be equal to.

If we are given an equation using an expression with h, we can solve for h by isolating the variable, h, and finding what it equals. For example, if 2h = 66,

Equations are useful

People working in trades create equations to solve problems and describe relationships that are useful. When the variables in an equation describe specific quantities, they are called **formulas**.

1) $R = \frac{V}{I}$ Electrical resistance equals voltage divided by current (If we know the value of any two variables, we can solve for the third variable).

2) $\frac{L}{16} = N$ The length of a wall divided by 16 inches equals the number of studs required in some construction. (If we know the value of L we can solve for N, and vice versa).

3) $D_1 = D_2$ The two diagonals of a floor must be equal for the corners to be at right angles.

then h = 66 ÷ 2, which makes h = 33.

For example, "distance travelled equals miles per hour (the speed, or rate) multiplied by time". This relationship can be described by the formula d = vt, where d = distance, v = velocity (speed in a direction), and t = time.

Formulas use equations: for example the area of a square = side^2, and this can be written as the equation $A = s^2$. Sometimes a subscript is used to help identify the meaning of a variable in an equation. A_{square} could be used to specify area of a square, and $A_{triangle}$, the area of a triangle and so on[2].

[2] All formulas use equations, but not all equations are formulas. Variables in an equation must refer to specific quantities to be called a formula.

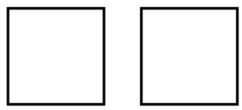
Unit 1 – Understanding Equations

Topic 1 – Patterns and Equations

Use pictures and concrete objects to model equations

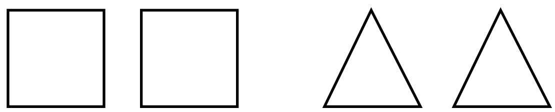
Simple expressions can be shown with diagrams or objects such as tiles or coloured cards. Use one object to represent the variables, and another to represent constants.

1) For example use a square to represent the variable h and the expression 2h can be shown as:



Here any number can be substituted for each square, and their sum will equal the value of the expression 2h. Addition is the operation meant by placing the objects next to each other.

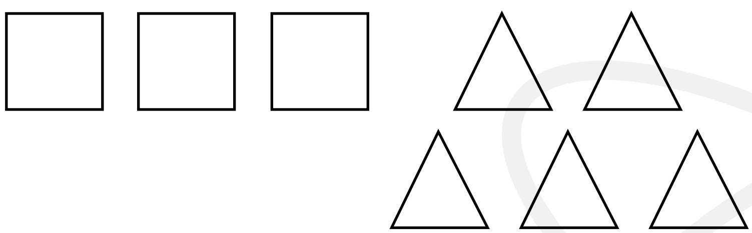
2) Use a triangle to represent one, and 2h + 2 could be shown as:



Here h is a variable and 2 is a constant. Once h is chosen, the value of the expression will be h + h + 2. If h = 5, then 2h + 2 = 12.

Colours can also be used to identify opposites. For example, a black triangle could represent 1, and a white triangle –1. Use a code of this kind to help you understand the expressions used in equations.

3) Example: Use a square for the variable, and a triangle for the constant to represent the expression 3y + 5.



Unit 1 – Understanding Equations

UNIT 1

Topic 1 – Patterns and Equations

Expressions use letters to give instructions

An algebraic expression contains letters to represent quantities. For example, the expression $2x^2$ tells us to square a number and multiply the result by 2. If we put 6 in place of x, we get 72 as the value of the expression when x = 6 because 2 x 6 x 6 = 72. When x = 6, $2x^2 = 72$ is a true statement. However, if we choose to let x = 4, then we would get 32 as the value of the expression. $2x^2 = 32$ is a true statement when x = 4. The solutions for an equation can all be expressed as "if… then…" statements. "If x = 4, then $2x^2 = 32$".

For any input number in an equation, we get an output number that results from doing what the expression tells us to do with the input number. All of the correct output numbers are solutions for the equation and are sometimes called the **solution set** for the equation.

Letters in algebraic expressions are called variables, as opposed to constants, because they can be replaced by any number unless we are told otherwise.[3] An algebraic expression will produce a numerical value when a number is substituted for a letter. An algebraic expression tells us what to do with a number that we put in the place of a variable (i.e. a letter) in the expression.

Evaluating expressions

x, x^3, 2x, 2x – 1 are all algebraic expressions. Any number can be selected to play the role of x in these expressions. The expression tells us what to do with any number that is chosen. We could have chosen other letters to represent what is variable in these expressions. For example, we could replace the letter x with r, or k, and nothing would change. What we do with the variable doesn't change when we change the letter that represents the variable quantity in an expression.

If we let x = 5, then x = 5, $x^2 = 25$, 2x = 10, 2x – 1 = 9

If we let x = 10, then x = 10, $x^2 = 100$, 2x = 20, and 2x – 1 = 19

> **Measurements are often substituted for variables**
>
> Our choice of a value for variable will often depend on what we are using an equation for. In the case of a formula, we often measure the value we assign to one or more of the variables. For example, in the formula d = vt, we may measure d = 3 miles, and t = 2 minutes, and then find our velocity in this situation by solving for v = d ÷ t, substituting to get v = 3 ÷ 2 = 1.5 miles per minute.

If we let x = $\frac{1}{2}$, then, $x^2 = \frac{1}{4}$, 2x = 1, 2x – 1 = 0

3 Sometimes a letter is used as shorthand for a constant value. For example, c is used to represent the speed of light, 186,000 mps in physical formulas.

Unit 1 – Understanding Equations

Topic 1 – Patterns and Equations

Example: An equation that predicts height based on age

Suppose we wanted to know how to predict a child's height when they grow up. After a series of measurements, we might discover that (in general) people in Yellowknife double the height that they reach at age 4 when they grow up. If h is allowed to represent a person's height at age 4, then the expression 2h will represent their height when they grow up. An equation can show this relationship by setting 2h equal to a person's adult height. If A = adult height, then A = 2h. In this equation h is a variable because children will have different heights at age four, however, once we know the height of a child we can predict that they will likely double this height when they grow up.

We can use this equation to predict that a child who is 3 feet tall at age four will likely be 6 feet tall at age 21. We can also use the same equation to prove that a person who is 5 feet 6 inches tall at age 21, was probably 2 feet nine inches tall at age four. We get this result by rearranging A = 2h to get h = A/2 by dividing both sides of the equation by 2. **An equation can be solved for any letter appearing in it.**

For greater clarity:

Equations can be rearranged by doing the same thing to both sides.

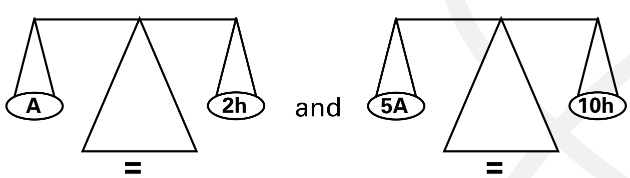
(Equals can be substituted for equals). The relationship of equality will not change as a result of performing the same operation on both sides.

A = 2h describes the same relationship as

5A = 10h (multiply both sides of A = 2h by 5) or,

A + 3 = 2h + 3 (add 3 to both sides)

These are equivalent equations and there is no limit to the number of equivalent equations we can create. **Equivalent equations produce the same solutions – that is why they equal each other.** As you will see in Topic 2 below, the simplest form of an equation will have all possible operations completed. A = 2h is the simplest form of 5a = 10h, and A + 3 = 2h + 3.



Unit 1 – Understanding Equations

Topic 1 – Practice Exam Questions

Question 1

How many terms are in this expression $x^2 + 4x - 3$?

a) 1

b) 2

c) 3

d) 4

Answer: c

Explanation
A term is defined as a number or a number multiplied by one or more variables. Terms are separated by + and – in algebraic expressions. The three terms in this expression are x^2, $4x$, and -3

Question 2

If D is chosen to represent the distance in kilometres from Yellowknife to Ekati, which expression describes the point that is one kilometre more than the halfway point?

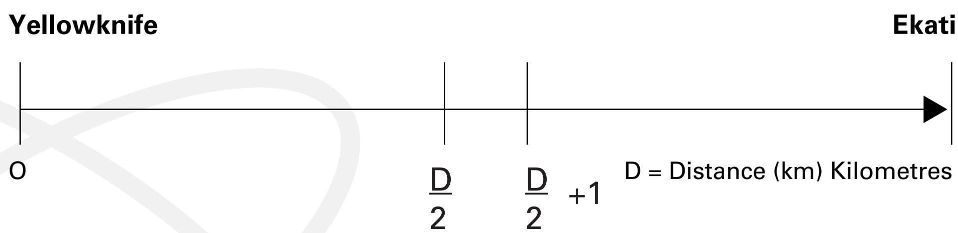
a) $2D + 1$

b) $\frac{D}{2} + 1$

c) $\frac{1}{2}D$

d) $\frac{D}{2} - 1$

Answer: b



Explanation
A sketch will help you see that the halfway point is the midpoint on a line from Yellowknife to Ekati.

We find this point by dividing the distance, D, in half. $D \div 2$ will give us the distance halfway. Now add 1 km to this term to get the expression we need $^{D}/_{2} + 1$. describes the point that is 1 kilometre more than the halfway point. Because this is a sketch, and not a scale drawing, we cannot label this point on the sketch exactly, but we do know it is to the right of the halfway point.

Unit 1 – Understanding Equations

Topic 1 – Practice Exam Questions

Question 3

A carpenter thinks that the width of a $1\frac{1}{2}$-inch thick rafter should measure 1 inch for every 16 inches of distance being spanned. If w is chosen to represent width, and S is chosen to represent the distance to be spanned by a rafter, which equation describes how this carpenter will figure out the width of a rafter?

a) w = 16S

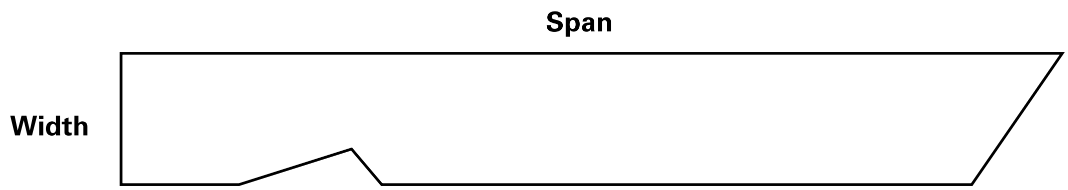
b) w = S/16

c) S = 16w

d) S +1 = w/16

Answer: b

Explanation



A sketch can help you to see the relationship described in this question.

Draw a rectangle to show a basic rafter and label the width w, and the length S. The question asks how the carpenter will calculate the width for a given span. We need an equation that shows what S is equal to in terms of w. Pick a number for either variable, w or S, and use the relationship that is described in the problem. This is a useful method for solving problems: create an example using simple numbers.

Suppose the rafter spans 10 feet, or 120 inches. Then S = 120. To find w, we divide by 16 because we are told that for every 16 inches of span we need 1 inch of rafter width. The answer will be 7.5 inches. The equation, or formula, we used to get this answer is $w = {}^{S}/_{16}$, using inches as our units. Notice that choice c expresses the same formula solved for S. Choice c tells how to find the span when we know the width. Span and width will always have the same relationship to each other. You can change choice b into choice c by multiplying both sides of the equation in choice b by 16. These two equations will have the same set of solutions for w and s. **However, the question asks for an equation to figure out the width. This makes b the correct answer to the question.**

Topic 1 – Practice Exam Questions

Question 4

Which of the following equations is equivalent to 5x – 25 = 0?

a) 10x + 25 = 25

b) 5x + 25 = 25

c) 5x – 15 = 10

d) x – 5 = 5

Answer: c

Explanation

Choice c is equivalent to 5x – 25 = 0 by adding 10 to both sides.

Question 5

What is the value of 5b – 32 when b = 60?

a) 262

b) 140

c) 268

d) 362

Answer: c

Explanation
To find the answer substitute 60 for b in the expression. Multiply first by 5 to get 300, and then subtract 32 to get 268.

Question 6

Peter works at two job sites. In the last 30 days he worked on one jobsite 4 more days than at the other jobsite. How many days did he work at each of the job sites in the last 30 days?

a) 13 and 17

b) 11 and 19

c) 28 and 2

d) 14 and 16

Answer: a

Explanation
In this question we need to find the parts of a total that equals 30. We are told that the days worked at one site are 4 more than at the other site. You might see the answer by thinking about the choices and realizing that each pair of numbers adds to 30, and that only choice a has a pair of numbers that add to 30 and differ from each other by four. Or, you can use an equation to find the unknowns involved.

This relationship can be expressed by x days at one jobsite, and $x + 4$ days at the other jobsite: we don't know x yet, but we do know that whatever it turns out to be, the days worked at the other jobsite will be 4 greater. Now we can set $x + (x + 4) = 30$ to describe our problem. This equation is solved by combining like terms to get $2x + 4 = 30$. Now isolate the variable by subtracting 4 from each side to get $2x = 26$. Finally divide both sides by 2 to get $x = 13$, and then $x + 4 = 17$.

Unit 1 – Understanding Equations

Topic 2 – Solving Inequalities

Background

Equations describe equality between expressions, and inequalities describe relationships that involve one expression being larger or smaller, as well as equal to, another. Equations are really a special case, or subset of inequalities. The same rules that solve for an unknown in an equation can be used to solve for an unknown in an inequality statement.

An inequality is a statement that says two expressions are not equal to each other. Like an equation, an inequality will have a collection of numbers that make the inequality true when they are substituted for the variables in the inequality. The solutions for an inequality can also be expressed by "if… then…" statements. For example, "If $A = 3$ then the inequality $A < 5$ is true".

What you need to know about inequalities

The letters in inequalities are variables. Statements of inequality become either true or false when values are substituted for the letters. Inequalities are evaluated by substituting values for the letters and testing the result to see if it satisfies the relationship described by the inequality. Many values will make an inequality true.

$A < B$ is true only when A is less than B, for example if: $A = 3$ and $B = 5$

$A > B$ is true only when A is bigger than B, for example if: $A = 5$ and $B = 3$

$A \leq B$ is true only when A is either less than or equal to B, for example if: $A = 4$ and $B = 4$ or when $A = 3$ and $B = 5$

$A \geq B$ is true only when A is either greater than or equal to B, for example if: $A = 4$ and $B = 4$, or when $A = 5$ and $B = 3$

When a value is given for one term in an inequality, solve for the other term by describing all the values that make the statement true:

Example:

Solve for A in the inequality $A < 5$

All numbers less than, but not including 5, will make this inequality true.

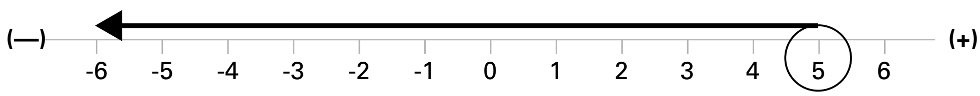
Topic 2 – Solving Inequalities

Use a number line to graph the solutions to an inequality

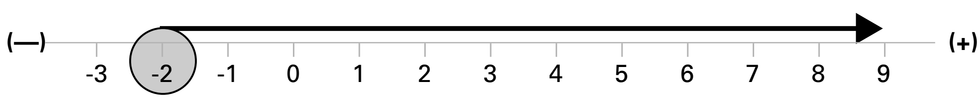
Example

$A < 5$

The solution will be all of the numbers left of five. An unfilled circle represents 5 as not being included in the collection of numbers that solve the inequality. 5 is not less than five, and therefore is left out.



$y \geq -2$



The solution will be all of the numbers greater than and including –2. The filled in circle represents –2 as a value included in the solution.

Inequalities are solved by isolating the variable. We do this by choosing operations, (add, subtract, multiply, divide) which get the variable letter by itself on one side of the inequality. The relationship of inequality will not change when the same thing is done to both sides – this is the same fact we used to work on equations.

Example

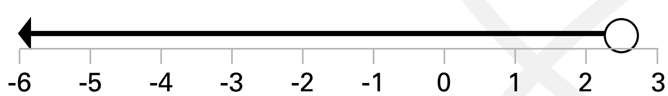
Solve the inequality $3x + 7 < x + 12$

Subtract x from both sides to get $2x + 7 < 12$

Subtract 7 from both sides to get $2x < 5$

Divide both sides by 2 to get $x < 2.5$

This inequality will be solved by any number that is less than, but not including, 2.5. All of the numbers to the left of 2.5, not including 2.5, will solve the inequality. You can graph this solution on a number line:



Topic 2 – Practice Exam Questions

Question 1

Which inequality describes all the numbers that are greater than 8?

a) $x < 8$

b) $x > 8$

c) $x \leq 8$

d) $x \geq 8$

Answer: b

Explanation
Choice b says x is greater than 8. This means that the inequality will be true for any number greater than 8. For example, 10 is larger than 8 and $10 > 8$ is a true statement.

Question 2

Solve the inequality $2x + 3 < x + 1$

a) $x < 1$

b) $x < 2$

c) $x < -2$

d) $x > 1$

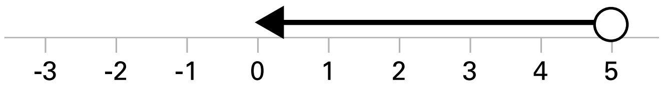
Answer: c

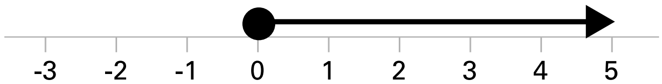
Explanation
Solve this inequality by isolating the variable, x. First subtract x from both sides to get $x + 3 < 1$, now subtract 3 from both sides to get $x < -2$.

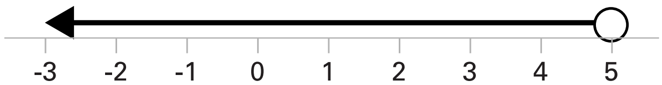
Topic 2 – Practice Exam Questions

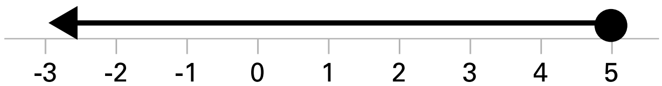
Question 3

Which graph represents the inequality $x \leq 5$?

a)


b)


c)


d)


Answer: d

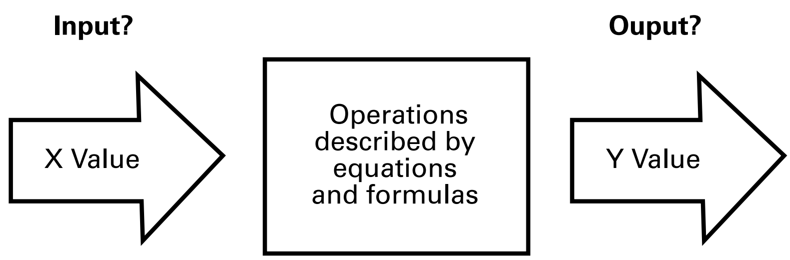
Explanation
Only choice d shows that 5 is included in the solution set for all numbers equal or less than five. Choices a and c are wrong because they do not include 5, and b is wrong because it describes all numbers greater than or equal to zero.

Unit 1 – Understanding Equations

Topic 3 – Solving Linear Equations

Background

Linear equations allow us to solve problems involving straight-line relationships that are regular. For every input number in a table there will be an output number that is determined by an equation. You may want to review Math – Module 2 – Patterns and Relations, finding the nth value in a series, for this topic. Equations describe operations on input numbers that produce output numbers. In a problem with equations, we need to know if we are looking for an input number, an output number, or a description of the operations that produce an output from an input.



Examples:

1) We are given an equation and an input number and have to find the output number.

   If the equation is y = 3x, and an input number is x = 3, then the output will be 9, because y = 9 when x = 3.

2) We are given an output number and an equation and have to find the input number.

   If y = 50 in the equation y = 10x - 2, then 50 = 10x - 2 (substitute 50 for y). Next, add 2 to both sides to get 52 = 10x, and x = 5.2 when both sides are divided by ten.

3) We are given several input-output pairs in a table and asked for the equation that relates them. In this example the input numbers could represent the number of people on a framing crew and the output could be the number of walls completed in a day.

Input (x)	Output (y)
3	18
4	24
5	30

What happens to each value for x in this table? Answer: it is multiplied by 6 to give the output (y). An equation for this relationship will show how the y value is the result of operations on x, in this case multiplying by 6. y = 6x is the equation that works. Notice that y and x are variables, and 6 is a constant.

Unit 1 – Understanding Equations

Topic 3 – Solving Linear Equations

What you need to know about equations:

An equation states that one expression is equal to another. In addition to using numbers on each side of the equal sign, we can use letters to stand for any numbers we might choose. These letters are called **variables** because they can vary (change) according to the values we assign to them. For example, the equation $P = k^2$ makes P = 9 when we set k = 3. P becomes 25 when we set k = 5. The values we choose for k will depend on the problem we are working on. For example, k could refer to the number of tiles on a floor, the number of trees in a forest, or the number of tons of gravel needed for a roadbed.

1) **A term is a number or, a number and one or more variables that are multiplied together.**

   14, 14x, 14xy, -7k, x^2

2) **An algebraic expression can have more than one term.**

   14x is an expression with one term

   $14 + 14x^2$ is an expression with two terms

   $2c + 4d – 2c^2$ is an expression with three terms and two variables, c and d.

   5k + 7 is an expression with one variable, k, two terms, and one constant, 7.

3) **An equation sets one expression equal to another.**

   14 + 3x = 25 (this equation has three terms and one variable, x. No variables are raised to a power; this is a first degree or **linear equation**. Its graph will be a straight line.)

4) **Replacing a variable with a number is also called:**

   Plugging in a value for the variable

   Letting the variable equal the number we choose

   Substituting a value for a variable

   Replacing a variable with a constant

   Unless we are told otherwise, any number (integers, decimals, fractions) can be used as values for a variable in an algebraic expression

5) **An equation can be solved for any variable appearing in it.**

   Area of a square = side x side and the length of a side = square root of area:

   $A = s^2$ and $s = \sqrt{A}$

   Salary = hours x wage per hour, and wage per hour = salary ÷ hours

   S = h x w and w = S ÷ h

Unit 1 – Understanding Equations

Topic 3 – Solving Linear Equations

Three methods are described below that can help you solve and check problems like these that use linear equations.

Linear equations will have a straight-line graph.

All equations that have the form ax + b, where a and b are any number and x is a variable, are linear, or first-degree, equations. Y = 6x is a linear equation because b = 0. y = 6x – 2 is linear and the role of a is played by 6, and the role of b is played by –2.

Example

A recipe tells a chef to take an ingredient, weigh it in pounds, divide it in half, and then add 3 pounds. What equation will describe this formula?

Solution

If we let R equal the output of this process, and I equal the starting amount of the ingredient, then R = (I ÷ 2) +3, or R = $\frac{1}{2}$ I + 3. These are equivalent forms of the same equation. You can see that the equation for R has the form of y = ax + b. I plays the role of the variable, a is played by $\frac{1}{2}$, and b is played by 3. We can conclude that this equation will produce a graph that is a straight line.

Some pure algebra

The solution set for an equation forms a set of ordered pairs, (x, y). The x value of the pair is the input number for the equation; the y value is the corresponding output value. These are the values that appear in a table, and that also name the points in a graph of the equation.

Example:

In the equation y = 2x, the following ordered pairs are in the solution set: (1, 2), (2, 4), ($\frac{1}{2}$, 1), but (4, 2), and (6, 5) are not solutions. **Any ordered pair can be checked by substituting to see if it is in the solution set.**
Check (1, 2): 2 = 2 x 1 = 2, (2, 4): 4 = 2 x 2, but (6,5): 5 = 2 x 6

R = 1/2 I + 3 is a formula. **A formula is an equation with two unknowns.**

If we know the value of R we can find I and vice versa.

Topic 3 – Solving Linear Equations

Tables and Graphs

In the last example we were asked for an equation that describes what happens when a number is first divided by two (halved) and then has three added to it.[4] A table can help us to understand the formula given in this problem. We can also use the table to make a graph of the equation. Notice that we can choose a small selection of input values that interest us, but that the graph will produce a line that gives an output value for every possible input value.

In this example, each output number will be the result of taking an input number, dividing it by two, and then adding three. **The graph of this information will produce a straight line that connects all of the points that satisfy the equation.** On a graph the horizontal x axis is used to show the input numbers, and the vertical y axis is used to show the corresponding output values.

$$R = \tfrac{1}{2} I + 3$$

Input numbers (x axis) (Starting pounds of ingredient = I)	**Output numbers (y axis)** (Amount of ingredient to use = R)
5	5 ÷ 2 + 3 = 5.5
7	7 ÷ 2 + 3 = 6.5
10	10 ÷ 2 + 3 = 8
33	33 ÷ 2 + 3 = 19.5
50	50 ÷ 2 + 3 = 28

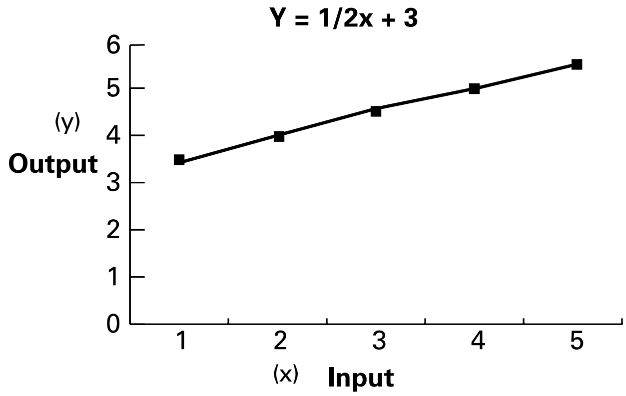
A graph of this information will use the x axis to show the input numbers and the y axis to show the output numbers. Each pair of numbers names a point on the graph (coordinate system). The role of R is played by y and the role of I is played by x in the equation. This equation has the form of a linear equation and its graph will be a straight line.

By choosing small values for x, beginning with zero, we can discover the line that connects all the points that solve this equation. Notice that the graph for $y = \tfrac{1}{2}x + 3$ is the same as the graph for $R = \tfrac{1}{2}I + 3$. **The pattern is the same and any choice of letters will reveal it.** Because graphs are based on an x axis and a y axis, it is customary to change linear equations using other variable letters into the equivalent x and y form.

Here is the graph of $R = \tfrac{1}{2}x + 3$ expressed in terms of points in an x, y coordinate system using input values for (x) of 1, 2, 3, 4, and the corresponding output (y) values of 3.5, 4, 4.5, 5, and 5.5. If the line is extended it will go through the points described in the earlier table for this formula as well.[4]

4 Remember that dividing by two is equivalent to multiplying by $\tfrac{1}{2}$.

Topic 3 – Solving Linear Equations



Notice that additional pairs of numbers that solve the equation can be found by looking at the graph. For example, when x = 8, y = 7 because the point (8,7) is on the graph and 7 = 8 ÷ 2 + 3. Any point on the graph will solve the equation and all solutions to the equation will be on the graph.[5] The values given earlier in the table for this equation will be on the graph. For example the point (50, 28) is a solution and will be on the line because 28 = 50 ÷ 2 + 3.

You can also see that many solutions are on the line, but not easily read by looking at the scale given for x and y. For example, you cannot use this graph to find exactly what y is when x = 2.25. This is why a graph can help understand an equation but not necessarily find the solutions we seek.

Solving and checking linear equations

You need to know:

1. How to rearrange an equation to show what each of its variables equals.
2. How to find a solution for the equation when we know the value of its variables.
3. How to build a table based on an equation and the information given in a problem.
4. How to make a graph of an equation.

5 You may want to review the discussion of interpolation and extrapolation in Unit 2, Topic 6 in Module Two: Patterns and Relations.

Unit 1 – Understanding Equations

Topic 3 – Solving Linear Equations

Solve for one unknown at a time

An equation can have several variables. When we solve for one unknown we identify the variable that we want to find a value for. The solution of an equation involves simplifying terms and isolating the variable on one side of the equation to see what it is equal to. To get the variable on one side, we will use the fact that we can add or subtract the same thing to both sides without changing the equality relationship. This is also true of multiplying or dividing both sides by the same quantity or expression. The distributive property of multiplication over addition and subtraction will also help solve equations. Combine like terms to simplify.

For example, if you are given d = vt, you may be asked to "solve for v", or "find v". This is done by rearranging the equation to show what v equals in terms of d and t, or what velocity will be when we know distance and time. Isolate v on one side of the equal sign by dividing both sides by t and then you can see that v = d/t. You may also be given d = v/t and asked what v is when d = 6 miles and t = 2 hours. Substitute these values and find that v = 6/2 = 3 miles per hour.

Three methods for answering questions about linear equations are described next.

Examples

The following question is based on the last example using the formula $R = \frac{1}{2} I + 3$ and its table and graph.

1) If R = 100 what will be the value of I?

   a) 194

   b) 53

   c) 153

   d) 203

Answer: a

Explanation
We are given R and have to find I when R = 100.

Method one: Substitute and rearrange

This is usually the quickest method, and it is recommended as the best way to solve linear equations when a value is given for a variable.

Topic 3 – Solving Linear Equations

After substituting 100 for R, we evaluate the equation:

$100 = \frac{1}{2} I + 3$

Now rearrange to "get I by itself" so we can see what I equals with the values "plugged in".

$97 = \frac{1}{2} I$ (Subtract 3 from both sides)

$2 \times 97 = I$ (Multiply both sides by 2, which is the same as dividing both sides by $\frac{1}{2}$)

$I = 194$

Check:

Now substitute 194 for I in the original equation we evaluated and see if it produces a true statement:

$100 = \frac{1}{2} \times 194 + 3$

$100 = 97 + 3$

$100 = 100$

Method two: Use a table

The output number is R and we have to find the input number. Look at the table given earlier and discover the pattern that gives the correct output for any input. Every output is the result of taking an input, dividing it by two, and then adding three. We need to ask: "what number divided by two with three added to the result will equal 100?" You can use trial and error mentally or on paper. We see from the table that when I = 50, R = 28. If we choose larger values for inputs we can search for a number that will give 100 as the output:

Input numbers (x axis) (Starting pounds of ingredient = I)	**Output numbers (y axis)** (Amount of ingredient to use = R)
50	50 ÷ 2 + 3 = 28
55	55 ÷ 2 + 3 = 30.5
100	100 ÷ 2 + 3 = 53
200	200 ÷ 2 + 3 = 103

200 is an input number that produces an output of 103, which is close to our target of R = 100, but it is too large by 3. We can also see from this table that the input we are looking for will be an even number because any odd number divided by two will not produce a whole number for R, and 194 is an even whole number. We can be guided by these observations and try smaller even numbered inputs that are close to, but smaller than, 200:

198	199 ÷ 2 + 3 = 102
196	196 ÷ 2 + 3 = 103
194	194 ÷ 2 + 3 = 100

194 is the correct value for I when R = 100. You can check this answer in the same way as for method one above. Put algebraically, (194, 100) names the point in the solution set that solves our problem.

Method three: Look at the graph

We are given the value for R, which appears on the vertical y axis of the graph. By looking down from R = 100 we should be able to find the input number of 194.

This method only works if we have a graph that has a scale that includes the values we want to investigate. Inspecting a graph to find a solution for an equation only works when the numbers in the line are given precisely.

More practice solving for variables in formulas

This optional section provides further explanation and additional examples that will help you rearrange and solve equations on the exam.

Equations with two or more unknowns are also called formulas. People working in the trades use equations with two variables when they calculate the area of a circle or triangle, and when they figure out discount and mark-up.[6] Recall that the problems using these formulas provided you with values for one or more of the variables so that you could solve for the remaining unknowns.

> **A formula is an equation with more than one variable.** You can express a formula in terms of a solution for each variable.
>
> Example: area of a rectangle equals length (l) x width (w)
>
> A = l x w, w = A/l, l = A/w
>
> These are three ways to express the same relationship between area, length, and width.

The key to solving a formula for one of its variables is to isolate the variable you are interested in on one side of the equal sign. This will allow that variable to be defined in terms of the other numbers and variables in the equation.

Examples

1. The formula for the area of a triangle is A = 1/2bh, or one half the base times the height. There are three variables in this equation: the area, the base length, and the height length. If we know any two we can find the value of the remaining one. If we plug in values for any two we can calculate the value of the third by using the formula. If we do not know any of the values that will be substituted for the variables in the formula, we can still rearrange the formula to show what each variable equals in terms of the other variables.

   $A = 1/2\ bh$

   $h = 2\,\frac{A}{b}$ (The result of dividing both sides by $\frac{1}{2}$ b)

   $b = 2\,\frac{A}{h}$ (The result of dividing both sides by $\frac{1}{2}$ h)

These are equivalent formulas. If we seek the value for the height we use h = 2 A/b, if we seek the value of the base we use b = 2 A/h, and if we seek the value of the area we use A = $\frac{1}{2}$ bh.

6 See Math – Module 1, Unit 4, Topic 7 – Problems Involving Discount and Topic 8 – Problems Involving Markup. Also see Math – Module 4 – Measuring Time, Shapes and Spaces, for area formulas.

Topic 3 – Solving Linear Equations

Sample problem:

Solve A = ½ bh when you know that A = 16 square feet and b = 4 feet.

We are given values for two of the three variables. We need to find h.
You can substitute the values in the equation as given, or you can rearrange it to solve for h.

$h = 2\frac{A}{b}$

$h = 2 \times \frac{16}{4} = 8$ feet

Check:

A = ½ bh

16 = ½ x 4 x 8

16 = 16

2) Find r when k = 32 in the formula 5r + 2k = 25

We can substitute 32 for k in the formula and then solve for r. After we make the substitution we are solving an equation in one unknown and the rules from Unit 1, Topic 1 – Patterns and Equations will apply.

5r + 2k = 25

5r + 2 x 32 = 25

5r + 64 = 25

5r = 25 – 64 (subtract 64 from both sides)

5r = -39

r = -39/5 (divide both sides by 5)

r = -7 4/5 or -7.80

Check:

5r + 2k = 25

(5)(-7.80) + (2)(32) = 25

-39 + 64 = 25

25 = 25

Topic 3 – Solving Linear Equations

3) Solve the equation 4x + 3y = 38 for y

We have to get y by itself on one side of the equal sign so that we can see what it is equal to. Begin by subtracting 4x from both sides

$3y = 38 - 4x$

Now divide both sides by 3

$$y = \frac{38 - 4x}{3}$$

$y = 38/3 - 4x/3$ (rule for dividing algebraic expressions)

$y = 12.66 - 1.33x$ (we could also have left everything in fraction form)

y is now expressed as a function of x. If we know the value of x we can calculate the value of y using this formula.

Unit 1 – Understanding Equations

Topic 4 – Practice Exam Questions

Question 1

Which pair of numbers would be found in a table showing the solutions for $y = x \div 4$

a) 1,2

b) 8,2

c) 10,5

d) 12,4

Answer: b

Explanation
A table shows the input values, or x coordinate values on the left, and the output or y coordinate values on the right. We need to test each pair of numbers to see if it satisfies the equation. Do this mentally by asking if the y value equals the x value divided by 4. Choice a fails because 2 is not equal to $1 \div 4$. Choice b is able to satisfy the equation because $2 = 8 \div 4$, and an input of 8 with an output of 2 would be in a table for this equation. 8, 2 would also be a point on the graph of $y = x \div 4$. Choices c and d fail to satisfy the equation.

Question 2

Which of these equations will not have a straight-line graph?

a) $P = kt$

b) $2x – 1 = y$

c) $x^2 = 32 + b$

d) $m = 32 + m/2$

Answer: c

A straight line is the graph of any equation with the form $ax + b$, where a and b are any number and x is a variable. Choice c has a variable, x, that is squared. It has the form $ax^2 + b$ and its graph will not be a straight line.

Topic 4 – Practice Exam Questions

Question 3

What value of k will make r = 5 in the equation r = 25k?

a) 5

b) 25

c) 1

d) 1/5

Answer: d

Any of the three methods discussed above can be used to answer this question, but method one: rearrange and substitute is recommended. The question asks us to find an output, k, when we are given an input, r. We need to rearrange the equation to solve for k because we are looking for a value for k when we are given a value for r. Because r = 25k, k = r/25 (divide both sides by 25 to get k alone). Now substitute 5 for r and see that k = 5/25 = 1/5. (Choice b)

You could also do mental trial and error to plug in numbers that help you use the pattern in the equation. For example, try k = 1 to get r = 25, K = 2 to get r = 50. From this experiment you can see that you are getting further away from the target of r = 5. You can see that k will be less than one. This method takes varying amounts of time. The rearrange and substitute method is quick and efficient – an important factor in solving problems in the trades.

Question 4

A technician knows that V = IR (voltage = current x resistance). He is able to measure a resistance of 10 ohms in a circuit and wants to find the voltage needed to drive a current of 25 amps. Which equation will give him the solution?

a) $V = I \times 25 + 10$

b) $V = \frac{I}{25} + 10$

c) $I = \frac{V}{25} + 10$

d) $V = 10 \times 25$

Answer: d

This equation does not need to be re-arranged because the question asks for the amount of voltage (V) needed to drive 25 amps through a circuit with a 10 ohm resistance. We can substitute 25 for I and 10 for R to get V = 10 x 25 = 250 voltage.

Unit 1 – Understanding Equations

Topic 4 – Practice Exam Questions

Question 5

What is the value of y when x equals 23 in the equation $y = 2x - x$?

a) 44

b) 46

c) 23

d) 66

Answer: c

Simplify to get $y = x$ because $2x - x = x$. Substitute 23 for the variable x in the equation to get $y = 23$. You can check this answer by letting $y = 23$ and verifying that $23 = 46 - 23$.

Question 6

If $V = IR$, what is the solution for R?

a) $V = R/I$

b) $R = V/I$

c) $R = VI$

d) $R = IV$

Answer: b

We need to rearrange this equation to solve for R. We get R by itself by dividing both sides by I. $R = V/I$. Now R is expressed in terms of V and I.

Question 7

Which point is on the graph of $y = \frac{2x}{3}$?

a) 1, 2/3
b) 2, 3/2
c) 1, 3/2
d) 1/2, 32

Answer: a

Each point gives an input number, the x value, and an output number, the y value. We can test each pair of numbers given in the choices by substituting for x in the equation, and seeing what y output we get. Choice a is a point on the graph because when $x = 1$, $y = 2/3$.

Topic 4 – Practice Exam Questions

Question 8

Bill discovers a linear relationship between the size of his savings account and the amount he deposits each week. What can we conclude about the amount of the deposits he makes each week?

a) They range between $1.00 and $100.00
b) He doesn't contribute more than $100 each week
c) His contributions are 20% of his salary
d) His contributions are always for the same amount

Answer: d

A linear relationship is one that will produce a straight-line graph. If we graph contributions that are different each week, we would not get a straight line – the line would be jagged. Sketch a graph with the y axis showing the total in the account, and the x axis showing the amount contributed on week 1, 2, 3, and so on. We know that the increase each week is linear, this is only possible if every contribution is for the same amount.

Question 9

Solve $x + 3x = 28$

a) $x = 28 - 3x = 28$

b) $x = 4x + 28$

c) $5x = 24$

d) $x = 7$

Answer: d

This equation says that some number plus three times itself equals 28. Or, put the other way around, 28 is equal to three times some number plus that number. We can read an equation from the left or from the right side of the equal sign. The answer is 7. You can find this answer by testing some numbers to find that 7 works, or by simplifying:

Divide both sides by 4 to find $x = \frac{28}{4} = 7$

Check: put 7 in place of x and solve: $7 + (3 \times 7) = 28$.
(Do multiplication inside brackets first)

Any numbers that can be substituted for the letters (also known as variables) that makes the equation a true statement is a solution for the equation. Seven is the solution for the equation in the last example. There was one variable, also called an unknown, in this equation. **The variable x appears twice in the equation, but it will have the same value both times.** An equation in more than one unknown will have additional letters.

Unit 1 – Understanding Equations

Topic 4 – Practice Exam Questions

Question 10

Solve 4 (y +12) = 36

a) y = 39

b) y = 48

c) y = -3

d) y = -36

Answer: c

We need to rearrange the terms in this equation to get the variable, y, by itself. This will show us what y is equal to, and that solves the equation. Begin by distributing (i.e. multiplying) 4 over y + 12

Now subtract 48 from both sides to get 4y by itself

4y + 48 – 48 = 36 – 48

= 4y = -12

Now divide both sides by 4 to get y by itself. These steps do not alter the equality relationship because we are doing the same thing to the terms on each side of the equal sign.

4y/4 = -12/4

y = -3

Check: put -3 in place of y in the equation and see if the sides are equal

4(y + 12) = 36

4(-3 + 12) = 36

4(9) = 36

36 = 36

Topic 4 – Practice Exam Questions

Question 11

Solve 0.12x + 0.40 (x + 34) = 50

a) x = 70

b) x = 35

c) x = .75

d) x = .48

Answer: a

In this equation the coefficients are decimal numbers. We can solve for x by distributing 0.40, combining terms and isolating the variable on one side.

$0.12x + 0.40\,(x + 34) = 50$

$0.12x + 0.40x + (0.40)(34) = 50$ (distributive rule)

$0.52x + 13.6 = 50$ (combine like terms)

$0.52x = 50 - 13.6$ (subtract 13.6 from both sides)

$x = 36.4/0.52$ (divide both sides by 0.42)

$x = 70$

Check: plug into the equation and test for equality

$0.12\,(70) + 0.4\,(70+34) = 50$

$8.4 + 41.6 = 50$

$50 = 50$

Unit 1 – Understanding Equations

Topic 4 – Practice Exam Questions

Question 12

Solve (1/4)x + 3/2 = 2x

a) x = 5/2

b) x = 6/7

c) x = 3/4

d) x = 1 3/4

Answer: b

This equation has one unknown, x, and three terms. One term is a number with no coefficient, 3/2. The other two terms have a whole number coefficient of 2 in 2x, and a fraction coefficient of 1/4 in (1/4)x. **Proceed as before by combining like terms, and isolating the variable.** The rules for combining fractions will apply.

We need to get the like terms of (1/4)x + 3/2 = 2x on one side. In order to get (1/4)x and 2x on one side, we choose one of these terms and subtract it from both sides to preserve the equality relationship. Let's try subtracting 2x from both sides of the equation, then: (1/4)x – 2x + 3/2 = 0.

Now we can combine like terms to simplify, or if you prefer, first eliminate 3/2 from the left by subtracting this term from both sides. Let's combine like terms first and deal with 3/2 after doing that.

(1/4)x – 2x = 7/4x (we get a mixed number by combining the coefficients, 1/4 + (-8/4) = -7/4

Now after combining like terms we have:
(-7/4)x + 3/2 = 0

Isolate the term with x in it by subtracting 3/2 from both sides.
(-7/4)x = -3/2

Solve for x by dividing both sides by -7/4 now
x = -3/2 divided by -7/4

Now use the rules for dividing fractions to invert the divisor, -7/4, and multiply times -3/2. The answer is (-3/2) (-4/7) = 12/14 = 6/7

Check:

Let x = 6/7 and solve (1/4)x + 3/2 = 2x

(1/4) + (6/7) + 3/2 = (2) (6/7) (use brackets to keep terms separated in multiplication) and you will get:
6/28 + 3/2 = 12/7

Use 14 as a common denominator and add terms
3/14 + 21/14 = 24/14
24/14 = 24/14

Unit 2

Working with Equations

Topic 1 – Simplifying Algebraic Expressions

Background

Equations contain algebraic expressions that can be simplified in order to find solutions more easily. Expressions can be added, subtracted, multiplied and divided. Operations on equations and the expressions in equations build on what you already know about operations in arithmetic.

The first step in solving an equation is to simplify both sides as much as possible. Each side of an equation will have an expression with one or more terms in it. An algebraic expression is a collection of terms connected by the four basic operations of adding, subtracting, multiplying and dividing. An algebraic expression can be simplified by completing the operations that are given, and by combining similar terms so that there are fewer of them. Fewer terms makes the expression simpler.

Like terms can be combined.

Like terms are numbers in the expression, and terms with the same variables and numbers multiplied together. Like terms can be either positive or negative.

- $y = x + 3 - 7$ becomes $y = x - 4$ (numbers are combined to simplify)
- $y = 3x - 2x$ becomes $y = x$ (like terms are combined to simplify)

Like terms: two or more terms with the same variable part are similar and can be combined

$4x + 5x = 9x$, $2y - y = y$, $10xy + 12xy = 22xy$

Checking your work
By simplifying we create an equation that can be tested by plugging in a value for the variables on both sides of the equal sign.

Test $4x + 5x = 9x$ by substituting $x = 1$, then $4 + 5 = 9$, a true statement.

Unit 2 – Working with Equations

Topic 1 – Simplifying Algebraic Expressions

Remember: a variable by itself is one of something.

x = 1x, or 1 times x

y = 1 times y, or 1y

A variable next to a number or variable is multiplied

3x = y means 3 times x = y,

4xy = 4 means 4 times x times y = 4

Unlike terms cannot be simplified by combining them

12y – 7x + 3

Is in simplest form already.
x cannot combine with y

Remember the order of operations:

Multiply before adding or subtracting

Remove parentheses before combining similar terms

Review the order of operations covered in Math – Module 1, Unit 3, Topic 4 – The Order of Operations, and review how to distribute **multiplication over addition and subtraction**.

Example:

4(5 + x) = 4 x 5 + 4x = 20 + 4x

4(5 – x) = 20 – 4x

You can simplify an expression by doing the reverse and remove a factor that can be distributed by multiplication. This is called "**factoring**". Notice that simplifying is a process that keeps things equal to each other. A simpler expression is also an equivalent expression.

20 – 4x = 4(5 – x) (4 is "factored out" to give a simpler expression)

$2x^2 + 16 = 2(x^2 + 8)$ (2 is factored out to give a simpler expression)

Example:

Simplify this expression: 4 + 7x + 18y – 7x + 22xy – 2 + x

This expression has six terms. First see if any numbers stand alone because they are not multiplied by a variable. Combine them first.

We have 4 and -2, combining we get 2. (4 – 2 = 2)

Topic 1 – Simplifying Algebraic Expressions

Next look for terms with the same variable, either alone or with a number as a factor. We have 7x, -7x, and x. The coefficient is different for each, but the variable is the same, so we can combine them. They are equal to x. You can see this either by adding them or by using the distributive property (factoring) to isolate the variable, x, and then combining the number parts of the expression.

1. Adding: $7x + (-7x) + x = x$
2. Factoring:

   $7x - 7x + x = x(7 - 7) + x = x(0) + x = x$ or
   $7x\,(1 - 1) + x = 0 + x = x$

Our simplified expression is $2 + x + 18y + 22xy$, we have simplified from six terms to four terms. There are no like terms left to combine. We could use the distributive property to express $18y + 22xy = y(18 + 22x)$, but we can't simplify further than this because 18 cannot be combined with 22x.

Notice also that the order of the terms can be rearranged with no change in the meaning of the expression. For example:

$2 + x + 18y + 22xy = x + 18y + 22xy + x + 2$

Unit 2 – Working with Equations

Topic 1 – Practice Exam Questions

Question 1

Simplify each expression by combining like terms, 77x + 11x.

a) 77 x 2

b) 18 x 2x

c) 18x

d) 77x

Answer: c

Explanation
This expression has two like terms. We can combine them by adding into 18x. By combining the terms we have created this equation: 7x + 11x = 18x.

Check: 7x + 11x = 18x Pick a few numbers and substitute for x, if you choose x = 2, then 7x = 14 and 11x = 22. The sum of 14 + 22 = 36. 18 x 2 also equals 36.

Question 2

4y + 10y – 6y

a) 46y

b) 8y

c) 14y

d) 16y

Answer: b

Explanation
This expression has three like terms. Combine them in any order to simplify or use the distributive property and combine the number parts of the terms;

4y + 10y – 6y = 14y – 6y = 8y or, using the distributive property;

y(4 + 10 – 6) = y8 or 8y (it is customary to put the numerical coefficient to the left of the variable in a term).

Check: select a value for y and check the equation we have created:

4y + 10y – 6y = 8y

plug in y = 2

8 + 20 – 12 = 16

28 – 12 = 16

16 = 16

Topic 1 – Practice Exam Questions

Question 3

7a – 9a

a) 2a

b) -63a

c) -2a

d) -16a

Answer: c

Explanation

This expression has two like terms. 7a – 9a = -2a. Or, we can factor out the variable a and get a(7 – 9) = 7a – 9a. Now combine 7 and -9 to get -2 inside the brackets. This produces a(-2) or -2a. The answer is once again -2a.

If you think with a number line in mind, this expression asks us start at 7 and move 9 digits to the left. We will land on -2, the answer. Alternatively, we can start on -9 and move 7 in the positive direction to the right. We will land on -2 this way as well. The order doesn't change the answer when we combine signed numbers.

Check:

7a – 9a = -2a

Plug in 3 (or any other convenient number) and test the equation.

21 – 27 = -2 x 3

6 = 6

Question 4

6b + 8x – 3b

a) 9b + 8x

b) 48b x -3

c) 3b + 8x

d) 3b – 8x

Answer: c

Explanation

This expression has three terms but only two of them are like terms. Combine the like terms:

6b – 3b = 3b

Now combine the result with the remaining unlike terms 3b + 8x. We have simplified the expression from three terms to two terms by combining like terms.

Check:

6b + 8x – 3b = 3b + 8x

Plug in b = 2 (or any convenient number) and test the equation

12 + 8x – 6 = 6 + 8x

6 + 8x = 6 + 8x

We could also have checked by choosing a value for x and seeing that a true statement results.

Let x = 2, then

6b + 8x – 3b = 3b + 8x

3b +16 = 3b + 16 (this is also a true statement)

Question 5

Simplify 8(3y – 8) – 3

a) 24y – 24

b) 24y – 67

c) 64 – 9y

d) 24 – 64y

Answer: b

Explanation
You might think this is already in simplest terms, but often when parentheses are in an expression, we can remove them by using the distributive rule and then simplify the result.

8(3y – 8) – 3 = 8 x 3y – 64 – 3 = 24y – 67

The answer is 24y – 67

Question 6

Simplify 6x + 12 – x – 5

a) 7x – 7

b) 5x + 17

c) 5x + 7

d) 6x – 12x

Answer: c

Explanation

This expression has four terms. Combine like numerical terms first:
12 – 5 = 7

Now combine like terms with the same variables in them:
6x – x = 5x

Combine the results: 5x + 7 or 7 + 5x

Topic 1 – Practice Exam Questions

Question 7

Simplify 7(y – 3) – (4y + 8)

a) 28y – 11

b) -3y + 5

c) 5 + 21y

d) 3y – 29

Answer: d

Explanation

We need to remove the parentheses first. Notice that
-(4y + 8) = -1(4y + 8) and allows us to distribute -1 to get -4y – 8. Here is the solution by distributing and then combining like terms:

7(y – 3) – (4y + 8) = 7y – 21 – 4y – 8

Now combine like terms:
7y – 21 – 4y – 8 = 3y – 29

Remember

-(a + b) = -1(a + b) = -a – b
(– times + equals –)

-(a – b) = -1(a – b) = -a + b
(– times – equals +)

Unit 2 – Working with Equations

Topic 2.A – Adding Expressions

Background

Everything you know about simplifying expressions is based on combining and factoring. Combining involves both adding and subtracting, and factoring involves multiplying and dividing. Working with equations requires the ability to add, subtract, multiply and divide algebraic expressions.

Topic 2.A – Adding Algebraic Expressions

Adding algebraic expressions means combining like terms as discussed in the last section. Group similar terms together, combine them, and then combine all of the results. Simplify the result using the distributive rule if possible to get the simplest form in the answer.

> Adding and subtracting are both ways to combine numbers

We can add the coefficients of similar terms.

A number times a variable is called a **coefficient.**

$x^2 + 3x^2 + 4x^2 = 8x^2$
$14y + 2y = 16y$

We cannot add the coefficients of unlike terms

$x^2 + y^2$ does not equal $2xy^2$
$12x + 2y$ does not equal $14xy$

Plug in some numbers and check that this is so.

Unit 2 – Working with Equations

Topic 2.A – Adding Expressions

Examples

1. Add $(2x^2 - 3x + 5)$ and $(4x^2 + 9x - 6)$

   First group like terms using parentheses (brackets): put the x squared items together, the x items together, and the numbers that are not the coefficients of any variables together.

   $(2x^2 + 4x^2) + (-3x + 9x) + (5 - 6)$ (keep the sign for each term inside its grouping)

   Next, Combine the like terms inside each grouping

   $(6x^2) + (6x) + (-1)$

   Now Simplify

   $6x^2 + 6x - 1$ or,

   $= 6(x^2 + x) - 1$ (use distributive rule to factor 6 out)

   $= 6x(x + 1) - 1$ (use distributive rule to factor x out)

   These are all equivalent expressions. **You can check to see that**

   $(6x^2) + (6x) + (-1) = 6x(x + 1) - 1$

   Let $x = 1$, then,

   $6 + 6 - 1 = 6(2) - 1$

   $11 = 11$

2. Add $2xy - 2x + 8$ and $2y + 2x - 2$

   Rearrange by grouping like terms together in brackets
   $(2x + 2x) + (8 - 2) + 2xy$

   Combine like terms
   $(4x) + (6) + 2xy$

   We cannot combine 2xy with the other terms because it has different variable letters. 2xy is unlike 6 and unlike 4x.

Unit 2 – Working with Equations

Topic 2.B – Subtracting Expressions

Background

In algebra we can subtract without knowing which term or expression is larger. Subtraction can be understood as a way of combining items, some of which may have negative values.[7]

> **a – b = a + (-b)**
>
> We can combine variables and numbers by realizing that some will have a negative sign to indicate that we are "taking them away".
>
> Arrange subtraction problems so the expression being subtracted is grouped in a bracket with a minus sign in front of it.

Examples:

1. **Subtract x^2 from $3x^2$**

   Put the problem in this order:

   $3x^2 – x^2 = 2x^2$. The expression being taken away is put to the right. We have similar terms, so the coefficients (3 and -1) can be combined to give 2. Notice that you can set up any subtraction problem by multiplying the expression being subtracted (i.e. "taken away"), by -1. In this example x^2 becomes $-x^2$. Once this is done, the problem becomes one of combining like terms in order to simplify the expression.[8]

2. **Subtract $3x^2 + 4x – 3$ from $x^2 – 2x – 3$**

   Put the problem in the order that shows what is being subtracted from what, in this case:

   $(x^2 – 2x – 3) – (3x^2 + 4x – 3)$

   Simplify by distributing the minus sign. Recall that $-(a) = -1a$.

   In this problem $-(3x^2 + 4x – 3) = -1(3x^2) + (-1)(4x) + (-1)(-3)$ this becomes:

   $-3x^2 – 4x + 3$

7 Review Math – Module 1, Unit 2, Topic 1.C – Signed Numbers.

8 If you prefer, you can write this problem vertically, grouping like terms below each other. However, the horizontal method is recommended for work with equations.

Topic 2.B – Subtracting Expressions

Now complete the original subtraction problem as a problem in simplifying an algebraic expression by combining like terms.

Rearrange to group similar terms and combine

$x^2 - 2x - 3 - 3x^2 - 4x + 3 =$

$x^2 - 3x^2 - 2x - 4x - 3 + 3 =$

$-2x^2 - 6x + 0$

or factor out 2x to get the equivalent expression $-2x(x + 3)$.

You can check your work by substituting a value for x to see if this answer is equivalent to the original expression. Let $x = 1$:

$(x^2 - 2x - 3) - (3x^2 + 4x - 3) = -2x(x + 3)$

$(1 - 2 - 3) - (3 + 4 - 3) = -2(4)$

$(-4) - (4) = -8$

$-8 = -8$

Topic 2.C – Multiplying Expressions

To multiply algebraic expressions start by multiplying the coefficients of the terms being multiplied and then follow the rules for multiplying powers and roots.[10] The results can then be simplified as before by grouping like terms and combining where possible.[11]

Examples:

1. **Multiply $(-2x^2)(3x^3)$**

   This expression has two terms. The coefficients are -2 and 3. Their product is -6. The variables are both x. Like variables are like bases, we multiply by adding the exponents of like bases. The answer is $(-6)(x^2\ x^3) = -6x^5$

   Check: try plugging in x = 2

   (-8)(24) = (-6)(32) = -192 (this is a true statement)

2. **$(-2k^2m)(3k^3m^2)$**

   The coefficients are -2 and 3. The bases are k and m. The product of the coefficients is -6. Now add the exponents for k to get the product of k^2 x k^3 (multiplying exponents rule) and likewise for m. Now you can see that:

   $(-2k^2m)(3k^3m^2) = -6k^5m^3$

   Check by letting k = 1: (or by choosing a value for m, or for both m and k. Any of these choices will allow you to see if the answer is an equivalent expression for the original product of expressions given in a problem)

   $(-2m)(3m^2) = -6m^3$

   $-6m^3 = -6m^3$

3. **Multiply $2x^2(3x^2 + 5x + 7)$**

   This expression has four terms. One term is multiplied by the other three. Use the distributive rule to multiply $2x^2$ times each term in the other expression.

   $= 2x^2(3x^2) + 2x^2(5x) + 2x^2(7)$

   (By writing it out this way you keep track better)

   $= 6x^4 + 10x^3 + 14x^2$

   Check: let x = 1, then,

   2(3 + 5 + 7) = 6 + 10 + 14

   2(15) = 30

   30 = 30

10 See Math – Module 1, Unit 3, Topic 6 – Bases, Exponents and Square Roots.

11 The topic of factoring algebraic expressions is covered in the level five math curriculum.

Topic 2.C – Multiplying Expressions

4. Multiply $(4x - 2)(3x + 4)$

The distributive rule tells us to multiply each term in the first expression with each term in the second. We then add like terms:

$(4x)(3x) + (4x)(4) + (-2)(3x) + (-2)(4)$

(These are the individual products we need to find)

$= 12x^2 + 16x + (-6x) + (-8)$

$= 12x^2 + 16x - 6x - 8$

$= 12x^2 + 10x - 8$

5. Multiply $(2x - 4)(3x^2 + 3x - 6)$

Multiply the first term, 2x, times each of the terms in the second expression. Then do this with the second term, -4. Finally, combine like terms to get the answer:

$2x(3x^2) + 2x(3x) + 2x(-6) + (-4)(3x2) + (-4)(3x) + (-4)(-6)$

$= 6x^3 + 6x^2 + (-12x) + (-12x^2) + (-12x) + 24$ (group like terms)

$= 6x^3 + 6x^2 - 12x^2 - 24x + 24$

$= 6x^3 - 6x^2 - 24x + 24$

This can be factored using the distributive rule to simplify:

$6(x^3 - x^2 - 4x + 4)$

Multiplying Algebraic Expressions

Calculate all possible products of the terms in the expressions and then combine them.

Fractional exponents

An exponent can be a fraction, either positive or negative. A fractional exponent describes which root to find for the base

Examples:

$2^{1/2} = \sqrt{2}$, $2^{-1/2} = \frac{1}{\sqrt{2}}$, $4^{1/3} = \frac{3}{\sqrt[3]{4}}$

Unit 2 – Working with Equations

Topic 2.D – Dividing Expressions

Background

The recommended approach is to write the division problem as a fraction with the divisor in the denominator. This will help you see that each term in the numerator will be divided by the denominator and the results will then be combined.

Division is really the process of simplifying an algebraic fraction using the tools covered in earlier topics.

Examples:

1. **Divide 3a into 9ab**

   Rewrite as a fraction: $\frac{9ab}{3a}$

   You can divide 3 into 9 to get 3, and a into ab to get b. This fraction simplifies to 3b. You can check by multiplying the quotient, 3b, times the divisor, 3a. 3a x 3b = 9ab.

2. **Divide $12x^3$ by $3x^2$**

   Writing this as a fraction will simplify the algebra:
   $\frac{12x^3}{3x^2}$

   You can see that 3 goes into 12 four times, and x^3 divided by x^2 equals x. This simplifies the fraction based on this division problem and gives the answer of 4x.

   Or, you can rewrite $12x^3$ as two fractions being multiplied in order to separate the coefficients from the variables.

   $$\frac{12x^3}{3x^2} = \frac{12}{3} \times \frac{x^3}{x^2}$$

   Simplify $\frac{12}{3} = 4$ to get $4 \times \frac{x^3}{x^2}$

   Now subtract the exponents (3 – 2 = 1) to simplify $\frac{x^3}{x^2} = x$

   The answer is the product of 4 and x, or 4x.

   Check by multiplying the divisor times the quotient:
   $(3x^2)(4x) = 12x^3$

Topic 2.D – Dividing Expressions

3. $10m^2n^3 \div 40m^2n^5$

Divide the coefficients = 10 ÷ 40 = 1/4

Subtract exponents of like terms, $m^2 - m^2 = m^0 = 1$, and $n^3 - n^5 = n^{-2}$

Write the resulting expression;

$1/4n^{-2}$ recall that a negative exponent means the base is the denominator of a fraction with one as the numerator, therefore $1/4n^{-2} = 1/4 \times 1/n^2$

$= \frac{1}{4n^2}$

4. Divide $8x^3 - 10x^2$ by $2x$

Rewrite as a fraction:

$$\frac{8x^3}{2x} - \frac{10x^2}{2x}$$

Now simplify each term in the subtraction problem.

$$\frac{8x^3}{2x} - \frac{10x^2}{2x} = 4x^2 - 5x$$

You can finish by factoring out x to get:

$x(4x - 5)$

Unit 3

Practice Exam Questions for Math – Module 3 – Variables and Equations

There are four modules in the common core required for all trades entrance math exams. Each module has a set of practice exam questions with an answer key. Each topic in the table of contents has sample questions to test your preparation for the trade's entrance exam.

You should aim for 100%, and study the sections of the curriculum for any topics that you do not get right. After each answer the units and topics you should review are identified. Turn to the appropriate part of the curriculum whenever you need help.

The core math curriculum is based on "need to know" competencies that are important in all trades. You may want to use the following sample exam questions both as a way of assessing what you need to learn before you work on the curriculum, and as a test of what you know after you have completed your preparation for the exam.

Unit 3 – Practice Exam Questions

Question 1

Solve 14 = 2(5x – 3)

a) x = 7
b) x = 13
c) x = 2
d) x = 3

Question 2

Solve 4(2y + 1) – 7 = 1

a) $y = \frac{1}{2}$
b) y = 2y
c) y = 9
d) $y = \frac{1}{4}$

Question 3

Find y when x = 8 in the formula 2x + 4y =18

a) $y = \frac{1}{2}$
b) y = 16 – x
c) y = 16
d) y = 8

Question 4

Solve 3x – 4 < 5x

a) x > -2
b) x > -4
c) x < 2
d) x < 4

Question 5

Ray worked five hours more than Martha each day for five days in a row. Which expression describes how long Ray worked over the five days if M = the hours Martha worked each day?

a) 5M
b) 25M
c) M + 5
d) 5(M + 5)

Unit 3 – Practice Exam Questions

Question 6

Simplify this expression: $9x - 3y + 10y^2 - 4x$

a) $5x + 13y^2$
b) $5x + 3y - 10y^2$
c) $10y^2 - 3y + 5x$
d) $5x - 3y + 10y^2$

Question 7

Solve $-10(3k + 4) = 0$

a) $k = -3/4$
b) $k = -4/3$
c) $k = 3/4$
d) $k = -3/4$

Question 8

What is the product of $10r$ and $15r - 1$?

a) $15r - 1$
b) $150r - 10r$
c) $150r^2 - r$
d) $150r^2 - 10r$

Question 9

Subtract $(15k + 6)$ from $(32x + 30k + 3)$

a) $17xk - 3$
b) $15k + 32x + 3$
c) $32x - 15k - 3$
d) $32x + 15k - 3$

Question 10

Solve $\frac{1}{3}(x - 4) = \frac{1}{2}(x - 6)$

a) $x = 15$
b) $x = -26$
c) $x = -5/3$
d) $x = 10$

Question 11

Solve $A = 1/2bh$ for h

a) $h = 1/2b$
b) $h = A/2$
c) $h = 2A/b$
d) $h = 2A/h$

Unit 3 – Practice Exam Questions

Question 12

Which choice is the simplest form of this expression: 8a + 6b + 10a?

a) 3(10ab)
b) 8aa + 6b
c) 18b + a
d) 18a + 6b

Question 13

Write an equation for a variable x, that is equal to three times itself, plus 5.

a) $x = 3 + 3x$
b) $x = 3 + 5x$
c) $x = 3x + 5$
d) $x = 15x$

Question 14

If we know that xy = 16, and y = 2, what is the value of x?

a) 16
b) 6
c) 8
d) 12

Question 15

Divide $16x^5 + 8x^2 + 12x$ by $12x^3$

a) $\frac{4x^2}{3} + (2/3)x + 1/x^2$
b) $\frac{4x^2}{2} + (4/3)x + x$
c) $2x^2 + x^2 + 12$
d) $3x^2 + (2/3)x + 1/x^2$

Question 16

Solve $0.06 + 0.08(100 - x) = 6.5$

a) .48
b) -.48
c) 19.5
d) -19.5

Unit 3 – Practice Exam Questions

Question 17

The area of a circle is given by a formula that says we multiply the square of the radius by pi = 3.14. What is the area if the radius is 3?

a) 9.12
b) 28.23
c) Can't tell from this information
d) 18.62

Question 18

Simplify $7(x - 2) - (2x + 5)$

a) $5x - 19$
b) $14x - 10$
c) $5x - 10$
d) $14x - 4$

Question 19

What is the sum of $2x$ and $4x^2 - 3x$?

a) $8x^2 - 6x^2$
b) $4x^2 - 5x$
c) $4x^2 - x$
d) $2x^2 - x$

Question 20

Solve this equation for r, $2r + 3y + 12 = 5r$

a) $r = 2y + 2$
b) $r = 8y - 12$
c) $r = y + 4$
d) $r = 2y + r$

Question 21

What is the result of simplifying $3(a - 6b) - 2a$?

a) $a - 18b$
b) $3a - 2ab$
c) $3a - 18ab$
d) $a - 6b$

Question 22

Simplify $18y + 5y - y - 6$

a) $6 - 23y$
b) $22y - 6$
c) $23y - 6y$
d) $23y - 6$

Unit 3 – Practice Exam Questions

Question 23

Simplify $5k - 2(k + 1)$

a) $3k + 1$
b) $3k - 2$
c) $3k + 2$
d) $3k - 1$

Question 24

Which equation will not have a graph that is a straight line?

a) $y = 3k + 12$
b) $H = AB$
c) $d^2 = 42 + \frac{1}{2}d$
d) $y = 15 - 4y$

Question 25

Which of the following describes $y + 5 = \frac{1}{2} y + 3$?

a) Some number equals one half of itself plus three more.
b) A number is equal to itself plus five then times one half plus three.
c) A number plus five equals three added to one half of itself.
d) Some number increased by five equals another number divided by 2 and then 3 added on.

Question 26

Subtract $x^2 - 4$ from $4x^2 + 2$

a) $3x^2 - 2$
b) $4x - 2x^2$
c) $-2x^2$
d) x^2

Question 27

Which inequality describes all of the numbers that are larger than 2k, but not including 2k?

a) $A > 2k$
b) $A \geq 2k$
c) $A \leq k$
d) $2k < A$

Unit 3 – Practice Exam Questions

Question 28

What is the value of the expression $2h + h^2$ when $h = 3$?

a) 18
b) 81
c) 33
d) 15

Question 29

Which point will be on the graph of $y = 2x$

a) 1, 3
b) 3, 1
c) 200, 400
d) $\frac{1}{2}$, $\frac{1}{4}$

Question 30

The formula $A = 2h$ has

a) One variable and two constants
b) Three terms
c) An inequality
d) One constant and two variables

Unit 3 – Practice Exam Questions

Answer Key

Answers	Study Topics
1) c	Unit 1, Topics 1 and 3
2) a	Unit 1, Topics 1 and 3
3) a	Unit 1, Topics 1 and 3
4) a	Unit 1, Topic 2
5) d	Unit 1, Topic 1
6) c	Unit 2, Topics 1 and 2.A
7) b	Unit 1, Topic 3
8) d	Unit 2, Topic 2.C
9) d	Unit 2, Topics 1 and 2.B
10) d	Unit 1, Topics 1 and 3
11) c	Unit 1, Topics 1 and 3
12) d	Unit 2, Topic 1
13) c	Unit 1, Topics 1 and 3
14) c	Unit 1, Topics 1 and 3
15) a	Unit 2, Topic 2.D
16) c	Unit 1, Topic 3
17) b	Unit 1, Topics 1 and 3
18) a	Unit 2, Topics 1 and 2.C
19) c	Unit 2, Topics 1 and 2.A
20) c	Unit 1, Topics 1 and 3
21) a	Unit 2, Topics 1 and 2.C
22) b	Unit 2, Topic 1
23) b	Unit 2, Topic 1
24) c	Unit 1, Topic 3
25) c	Unit 1, Topic 1
26) a	Unit 2, Topic 2.B
27) a	Unit 1, Topic 2
28) d	Unit 1, Topic 1
29) c	Unit 1, Topics 1 and 3
30) d	Unit 1, Topic 1

Resources

Abbott, P., Algebra, Teach Yourself Books, NTC Publishing Group, 1999. (ISBN 0-340-65847-9)

Alberta Vocational College: Pre-Trades Math, 6 modules

B.C. Ministry of Education, ABE Math modules

Barry Maurice, Small Marian, et al, Constructing Mathematics, Nelson Learning, Thomson Publishing 2000. (ISBN 0-176-05983-0)

Cleaves, Cheryl, Hobbs, Margin, Dudenhefer, Paul, Introduction to Technical Mathematics, Prentice Hall 1988. (ISBN 0-13-498700-4)

Doran, Edward, Fundamental Mathematics For College and Technical Students, Finesse Publishing, Broomfield Colorado 1988. (ISBN 0-941775-02-X)

Elchuck, Larry et al., Interactions 7,8,9,10, Teacher's Resource Books, Prentice Hall Ginn Canada, 1996. (ISBN 0-13-273129-0)

Gage Canadian GED Study Guide Series: Mathematics GED, Gage publishing 1998, (ISBN 0-7715-5141-X)

Garfunkel, Solomon, project director, For All Practical Purposes: Introduction to Contemporary Mathematics, fourth Edition, Freeman and Co. NY, 1997. (ISBN 0-7167-2841-9)

Hopkins, Nigel, et.al., The Numbers You Need, Gale Research, 1992. (ISBN 0-8103-8373-X)

Jacobs, Harold, Geometry, W.H. Freeman and Co., San Francisco, 1974. (ISBN 0-7167-0456-0)

IBID, Mathematics: A Human Endeavor, 2nd edition, Freeman and Co, NY. (ISBN 0-7167-1326-8)

McKeague, Charles P., Elementary Algebra, fifth edition, Harcourt Brace, 1995. (ISBN 0-03-097356-2)

Mathematics GED, Gage Canadian Study Guide Series, 1998, (ISBN 0-7715-5141-X)

O'Daffer, Phares, Clemens, Stanley, Geometry: An Investigative Approach, Addison-Wesley Publishing, 1977. (ISBN 0-201-05420-5)

Unit 4 – Resources

Rosenberg, Robert, R., Business Mathematics, Canadian Edition, Book I, McGraw Hill Ryerson, 1977. ISBN 0-07-082599-8)

Rudolf, Gordon, B., Introductory Trigonometry, Vancouver Community College Press, 1993. (ISBN 0-921218-54-0)

Shanks, Et al, Pre-Calculus Mathematics, Addison Wesley, 1965. LC 65-19230)

Trivieri, Lawrence, Essential Mathematics, 2nd edition, McGraw Hill 1994. (ISBN 0-07-065229-5)

Washington, Allyn J., Basic Technical Mathematics with Calculus, Fourth Edition, metric version, Benjamin Cummings Publishing, 1985. (ISBN 0-8053-9545-8)

Appendix A

The following topics from the Alberta Entrance Level Competencies are covered in **Math – Module 3 –Variables and Equations** of this curriculum.

A. Problem Solving Using Variables And Equations

Outcome: Use variables and equations to express, summarize and apply relationships as problem-solving tools in a restricted range of contexts. (1, 2, 3, 4, 5)

1. Write mathematical expressions that arise from problem-solving contexts.
2. Evaluate expressions with and without concrete models.
3. Illustrate the solution process for a one-step, single-variable, first-degree equation, using concrete materials or diagrams.
4. Solve and verify one-step linear equations, using a variety of techniques.
5. Explain how to solve simple problems, using informal algebraic methods.

B. One And Two Step Linear Equations

Outcome: Solve and verify one-step and two-step linear equations with rational number solutions. (1, 2, 3, 4, 5)

1. Illustrate the solution process for a two-step, single-variable, first-degree equation, using concrete materials or diagrams.
2. Solve and verify one- and two-step, first-degree equations with integer coefficients.
3. Create and solve problems, using first-degree equations.

C. Linear Equations And Inequalities In One Variable

Outcome: Solve and verify linear equations and inequalities in one variable. (1,2,3,4,5)

1. Illustrate the solution process for a first-degree, single-variable equation, using concrete materials or diagrams.
2. Solve and verify first-degree, single-variable equations with rational coefficients (with a focus on integers), and use equations of this type to model and solve problem situations.
3. Solve, algebraically, first-degree inequalities in one variable, display the solutions on a number line and test the solutions.

D. Generalize Arithmetic Operations From Rational Numbers To Polynomials

Outcome: Generalize arithmetic operations from the set of rational numbers to the set of polynomials. (1, 2, 3, 4, 5)

1. Identify constant terms, coefficients and variables in polynomial expressions.
2. Evaluate polynomial expressions, given the value(s) of the variable(s).
3. Represent and justify the addition and subtraction of polynomial expressions, using concrete materials and diagrams.
4. Perform the operations of addition and subtraction on polynomial expressions.
5. Represent multiplication, division and factoring of monomials, binomials, and trinomials using concrete materials and diagrams.
6. Find the product of two monomials, a monomial and a polynomial, and two binomials.
7. Determine equivalent forms of algebraic expressions by identifying common factors and factoring trinomials.
8. Find the quotient when a polynomial is divided by a monomial.