

NWT Apprenticeship Support Materials



Math



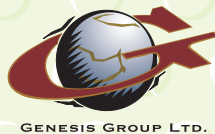
Reading Comprehension



Science

- * *Module 1 – Foundations*
- * *Module 2 – Patterns and Relations*
- * *Module 3 – Variables and Equations*
- * *Module 4 – Measuring Time, Shapes and Space*
- * *Module 5 – Special Topics*

PARTNERS



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The Apprenticeship Support Materials Project has been a true partnership. The challenge has been to develop a set of study materials, which covers every competency in the "Entrance Level Competencies for Apprenticeship Programs" developed by Alberta Advanced Education, Apprenticeship and Industry Training. These study materials although written for Northerners using northern examples, issues and problems, are appropriate for any individual entering the trades. The goal is to equip Northerners for the Trades Entrance Exam as well as to support them through the apprenticeship process.

The following partner organizations have all contributed to the development of these materials:

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Table of Contents

Module Two – Patterns and Relations

Introduction 5



Unit 1 – Number Patterns 7

Topic 1 – Background: Patterns and Formulas 9

Topic 2 – Finding Values in a Series 10

Topic 3 – Using Patterns to Make Predictions 14

Topic 4 – Working with Averages 16

Topic 5 – Reading a Value from a Table 18

Practice Exam Questions 20



Unit 2 – Relations, and Graphs 27

Topic 1 – Writing Expressions 29

Topic 2 – Equivalent Forms for Expressions 32

Topic 3 – Tables and Graphs 34

Topic 4 – Bar Graphs 36

Topic 5 – Line Graphs 40

Topic 6 – Interpolating and Extrapolating 45



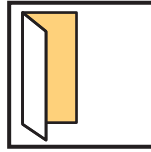
Unit 3 – Practice Exam Questions 47

Practice Exam Questions 49



Resources 57

Appendix A: List of Alberta Competencies Covered in this Module 59



Introduction

Entrance Level Competencies in Mathematics (Levels 1, 2, 3, 4, 5)

Module 1 – Foundations: Number Concepts and Operations

Module 2 – Patterns and Relations

Module 3 – Variables and Equations

Module 4 – Measuring Time, Shapes and Spaces

Organization of Topics

The emphasis in trades is on using mathematics to solve practical problems quickly and correctly. Each topic in this curriculum guide includes:

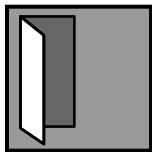
- 1) Background and theory
- 2) Examples with explanations
- 3) Practice Exam Questions with answers and explanations

This curriculum guide outlines competencies, but does not provide detailed lessons, as in a textbook or GED Study Guide. If you need more instruction on a particular competency, you may find these and other textbook resources helpful. If you want to build up speed as well as accuracy in a competency you will find these additional resources helpful as a source of additional practice questions. Need to know information for the trades entrance exam is singled out for your attention by the use of text boxes and bold type.

Examples are the focus

In this curriculum guide, examples with explanations are the primary tool used for review. Background for each competency is also given with a brief overview of what you need to know. Before any examples are given, the main ideas in each topic are explained and "need to know information" is summarized in rules and definitions.

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¹ A detailed introduction with sections on self-assessment and study tips is provided with Math – Module 1 – Foundations. Please review this material to supplement the brief overview provided here.



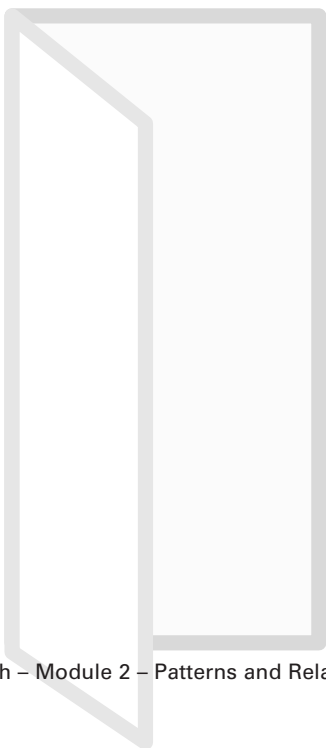
Introduction

Please Note:

When you work on an example, cover the text below with the laminated card provided so that you don't see answers and explanations prematurely.

You may want to skip the background given on a topic and go right to the examples to see how well you do. You can always go back to the theory if you find you need it.

In addition, a set of practice exam questions for Module 2 – Patterns and Relations accompanies this learning guide to enable you to assess yourself, decide what you need to study, and practice for the exam.





Unit 1

Number Patterns

Topic 1 – Background: Patterns and Formulas

What you need to know: Terminology for working with patterns

Numbers: 1, 2, 25%, .25, 1/2...

Can represent units: inches, metres, degrees, dollars...

Variables: x , y , a , b , c , ...

Letters can represent any quantity that has different values for different examples: income, number of storms, number of people dining, cooking time required, age, height, accident rates... In a problem we define what variable each letter refers to. Changing the letters used to represent a variable will not change the meaning of an expression. For example, income can be represented by l , or m , or x , or a , etc.

Operations: add, subtract, multiply, divide, find a square root, raise to a power.

Follow the order of operations and observe brackets (See Module 1 Foundations, Unit 3, Topic 4.D).

Expressions: $2x$, $3a-1$, $M-25\%$ of M , $\frac{a+b}{2}$, P ...

Expressions combine numbers, variables and operations to describe relations. An expression can also simply be one number or letter. $2y$ is an expression that multiplies any value chosen for y by 2, $3a-1$ is an expression that first multiplies any value chosen for a by three and then subtracts one. P is an expression with one variable. Expressions are instructions that tell us what to do with numbers.

Relations: number patterns that can be described by expressions.

Relations assign one value for each example governed by a pattern. For example: the expression $\$12.00 + (\$2.00 \times N)$ can be used to describe the salary for a worker who is paid $\$12.00$ an hour plus $\$2.00$ for every ton of earth, N , that is moved each hour. N is the variable in this expression and pay will vary according to how many tons are moved each hour.

Equations: relate two expressions with an equal sign.

In the equation for the salary earned by the worker discussed above, $S = \$12.00 + (\$2.00 \times N)$, stands for "salary" and N for "tons of earth moved per hour".

Topic 1 – Background: Patterns and Formulas

A numerical pattern can be described by using algebra. **Algebra** combines numbers with operations (+, -, ÷, ×) and variables that are represented by any choice of letters (x, y, i.e. m, a, b, c...). **An algebraic expression describes a pattern.** For example, recipes describe patterns that relate one quantity to another. A recipe is a formula, and a formula can be written as an equation.

Recipes are Formulas

For example, an omelette recipe requires two eggs per person. If 10 people are being served, 20 eggs will be needed. Math – Module 1, Unit 3, Topic 8 – Ratios, Rates and Proportions introduced problems that use patterns of this kind. This pattern can be expressed by the formula $E = 2N$ if we choose E to represent the number of eggs needed, and N to represent the number of people being served. $2N$ is an algebraic expression, and $E = 2N$ is an equation that uses this expression. Notice that these letters are convenient, but others could have been chosen.

Other patterns that are useful in trades include geometric patterns (tiles, bricks), and probabilities (accident rates, trends in business, averages). In many cases a formula can describe these patterns.

Examples:

1. The area of square = s^2 where s = the length of a side.
2. The distance a falling object covers = $32t^2$, where 32 refers to feet, and t to seconds.
3. The formula for the rise (tread height) and run (tread width) relationship on a stair is rise + run = 17 inches.

Substitute a value for a variable when you apply a formula

Example: find the area of a square room that measures 10 feet on a side.

Formula for area of a square = (length of a side)².

Substitute $s = 10$ because s represents the length of one side.

Find area = $10^2 = 100$ square feet.

Topic 1 – Background: Patterns and Formulas

Any letter can represent a variable

Example: The time it takes to travel from Inuvik to Yellowknife can vary depending on weather, means of transportation, and speed. This time can be represented by any choice of letter- a, b, c, z, y, k etc.

Choose letters that are convenient

A letter can be chosen that suggests the kind of quantity that is being represented. T can be chosen for time, d for distance, w for weight. However, any letter can be used.

One person can express the time it takes to travel from Inuvik to Yellowknife with "T", another with "m", and another with "x". Once we know that these letters have been chosen to represent the same thing, they will mean the same thing in an expression.

A recipe is a formula based on a pattern

An omelette recipe requires 2 eggs for each serving. If we let N equal the number of people being served, we can predict that $2N$ will be the number of eggs required. This relation will hold no matter how big or small N is. An equation, also known as a formula, $E = 2N$, shows the relationship between the number of people served (N) and the number of eggs required (E) for them.

This equation is read, "E equals 2 times N, where E is the number of eggs, and N is the number of people. The number of eggs is equal to two times the number of people." The same equation can be used to find how many people we can serve omelettes to if we know the number of eggs on hand. If $E = 2N$, then $N = E/2$. Dividing both sides of $E = 2N$ by 2 allows us to solve for N in terms of E. **The equation has two variables, N, and E, and if we know one we can calculate the other by using the equation.** The relationship between N and E is described by either of these equations.

Algebra uses letters, numbers, and operations to describe patterns. **The main idea in algebra is that the values chosen for examples can vary but patterns do not.** The expression that allows us to calculate the number of eggs needed to prepare omelettes will not change, even though the number of people we serve can change from one to several hundred.

Note:

$2n$ means 2 times n, $35p$ means 35 times p.

An expression with a number next to a variable refers to their product.



Topic 2 – Finding Values in a Series

A series is an ordered list of numbers. Each input value produces one output value. A formula, for example a recipe, can produce a series, and a series can be represented in a table.² For example, the formula for omelettes would produce the following table and series if we choose N to be the number of people being served, and then calculate E for the number of eggs needed.

N = number of people (input value)	E = number of eggs (output value)
1	2
2	4
3	6
4	8

The recipe tells us that there will be one correct answer to the question, "How many eggs will be needed to make omelettes for N people?" The value of E will depend on the choice of N in this table.

Some "Pure Math"

Now notice that there is one correct output value (number of eggs) for each position in the series of values for N (number of people).

Each output value forms a term in the series 2, 4, 6, 8... The first term is 2, the second term is 4, and so on. When we use a formula, we can produce a series of output values for each position in a series of input values.³

Numbers can follow patterns that allow us to predict the values for terms in any position in a series. Business and industry make decisions based on patterns. When numbers form a series where the order counts, the difference between a position in a series and the value that occupies that position must be clear.

The Series of Prime Numbers

Prime numbers are defined as those numbers that are evenly divisible only by one and themselves. 3 is a prime number, and so is 17. Numbers that are evenly divisible by other numbers are composite. For example, 6 is composite because it is divisible by itself, 1, and also 3 and 2. The factors of 6 are 6, 1, 3, 2. The definition of a prime number gives us a way to test any natural number. We can take any number and factor it to see if it satisfies the definition for a prime number. The result of using this rule on the natural numbers is the series of prime numbers, 1, 2, 3, 5, 7, 11, 13, 17, 19.....

The rule allows us to filter the natural numbers to see which ones belong in the series of prime numbers. We can see that the third prime number is 3, the fifth is 7 and so on.

² In this curriculum a functional interpretation of both series and expressions has been chosen given our focus on trades related problems.

³ If we look at the relationship by setting the number of eggs as our input numbers, the value of $N=E/2$ and the series produced will be 0, 1, 1, 2, 2, 3, 3 because one egg makes an omelette for 0 people, 2 eggs for one person, 3 eggs for one person with one egg left over, four eggs for two people and so on. In this situation, the value of $E/2$ must be a whole number.

Topic 2 – Finding Values in a Series

If you are presented with a series on the exam, you may be asked to find the pattern that is involved, and predict the value of a term in the series. Knowledge of the pattern will allow you to describe how to find the value of any term in the series. If a series ends with three dots, "...", it means that the series can go on indefinitely, or infinitely.

What You Need to Know

A number pattern can form a series whose terms are related by a rule or formula. A series that follows a pattern is made of numbers that have a position (first, second, third etc.), and a value assigned for each position according to the rule or formula.

- 1. A series of numbers forms a pattern when a formula can be found to predict the value of any number in the series.** For example, the series 2, 4, 8, 16... is the result of multiplying each term by two to get the next term. The series 1, 2, 3, 5, 7, 11, 13... is the result of removing all composite numbers and leaving only the prime numbers. A rule can be simple: a single operation is involved, or complex: several operations may be involved.
- 2. The pattern in a series can be expressed algebraically:**
In the series 2, 4, 8, 16... the value of the n th term = value of $2(n - 1)$. For example, the value of the 16th term ($n = 16$) is equal to two times the value of the 15th term. The value of the fourth term, 16, equals 2 times the value of the third term, or 2×8 .
- 3. If you know the pattern that creates a series, then you can predict the value of a term in any position, known as the n th term.** "Nth" term uses the letter "n" to stand for the position of any term you care to name. "n" is a variable that awaits a value. For example, if you want to know the value for the third term ($n = 3$), in the series 2, 4, 8, 16... you would multiply the value of the second term by 2. The value of the third term is 8, and can be found from 4×2 . If you want to know the value for the fifth term, you let $n = 5$ and multiply 2 times the value of the fourth term to get $2 \times 16 = 32$ for the value of the fifth term.



Topic 2 – Finding Values in a Series

Practice Examples: Predict the n th value in a series

1. What is the 30th term in this series?

10, 15, 20, 25,...

We can see that the value of each term is five more than the value of the previous one. The value of the 30th term will be five plus the value of the 29th term. This series is the result of counting "by fives". We can also see that the first term is 5×1 , the second is 5×2 , and the third is 5×3 and so on. In a table we can show this pattern:

Position number (n)	Value of $n =$	Multiple of five
1	5	1×5
2	10	2×5
3	15	3×5
4	20	4×5
5	25	5×5

This implies that the 30th term will be equal to $5 \times 30 = 150$. Put algebraically we can express the formula for the value of any term, n , in this series as: value of $n = 5n$. The position numbers can also be thought of as input values, and the values assigned to each position as output values.

In a series, we can relate the value of any term to the value of the previous term or to some formula. From this point of view, the value of $n = 5 +$ value of $(n-1)$ in the series 5, 10, 15... In words this says, "The value of any term in the series equals five plus the value of the previous term". Both formulas are true and produce the same results.

Additional Examples

2. What is the pattern in this series:

1, 2, 3, 4, 5...?

Here you can see the series of natural numbers. 1 is added to the previous term to reach each new term in the series. The series of natural numbers is the result. The formula for the value of any term in the series is $(n - 1) + 1$. Here the value of each term is also its position number. The first term is one, the second is two and so on. $3 = ((3 - 1) + 1)$, $15 = ((15 - 1) + 1)$ etc.

3. What rule describes the pattern in this series:

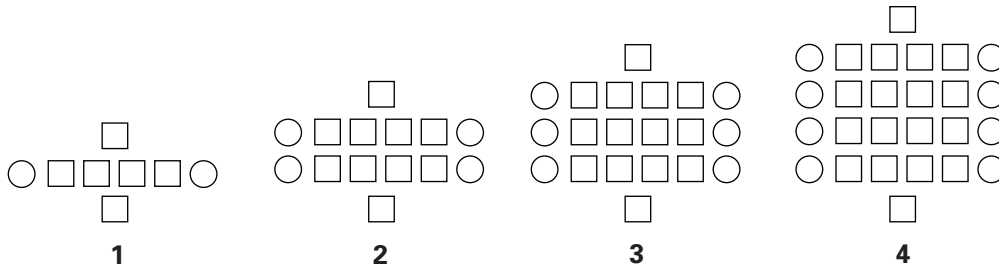
1, 1, 2, 3, 5, 8, 13, 21...?

Each term is the result of adding the two preceding terms. This pattern produces a series known as the Fibonacci series. Patterns in nature, for example the usual number of clockwise and counter clockwise spirals from the centre of a sunflower, 13 and 21, follow the rule for this series.

Topic 2 – Finding Values in a Series

Some series have several values for each position

Find an expression that describes this pattern:



Look at this sequence of diagrams to see that each one has a number of squares and circles. Each diagram shows a position in a series. You may be able to spot the pattern that relates the number of squares to circles in each position, or you may use a table to organize the information this way:

Position number	# of circles	# of squares
1	2	6
2	4	10
3	6	14
4	8	18
5	?	?

You can see that the number of squares increases by four for each position, while the number of circles increases by two. These increases apply to the first diagram which begins the series with two circles and 6 squares. An expression that relates the number of squares to circles in the first diagram is: number of squares = $2c + 2$, or $2 \times 3 + 2 = 6$ where c = the number of circles. This formula works for every position.

This question asks us for a formula that will predict the value of the n th term in a series using this information. In this series, each term has two values, one for circles, and one for squares. We can see that each term adds two circles and four squares to the value of the previous term. The values for the n th term will be the number of squares = $(n - 1) + 4$ squares, and the number of circles = $(n - 1) + 2$ circles.

4. Beginning with 1 as the value of the first term, write the next three terms of the series described by this formula: value of n = value of $(n - 1) + 2$.

In words this formula says that the value of any term, n , is equal to the value of the previous term $(n - 1)$, plus two. We start with the second term and use the formula: the value of the second term = $1 + 2 = 3$, the value of the third term is $3 + 2 = 5$, and the value of the fourth term is $5 + 2 = 7$. This formula describes the series of odd numbers: 1, 3, 5, 7...



Unit 1 – Number Patterns

UNIT 1

Topic 3 – Using Patterns to Make Predictions

In the trades, patterns are helpful for making predictions about time, materials, money, production, and maintenance. In what follows you will review what you need to know about patterns and their representation in mathematical language, including tables, graphs and equations.

Many predictions are based on tables of observations that can be put on graphs to reveal patterns or trends. A number pattern is a relationship between two or more quantities. An expression uses letters, numbers, and operations to describe the relationship. These are useful in helping us make predictions about the future.

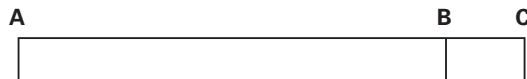
Examples

1. If we know that an aircraft engine component fails after five years of service, we can predict that the component will fail in 2008 if we are in 2003 and a new component has just been installed.⁴ If we want to use algebra to state the pattern for component failure based on this information, we could let f = a new installation year, and N = the next failure year. Since we want to know when future failures will occur in order to do preventive maintenance, we write the formula (equation) for the next failure year as $N = f + 5$. We choose the convenient letter N to represent the next failure year. This is an **equation** that uses the **expression** $f + 5$ to find N whenever we know f .

⁴ This unrealistic example is intended to focus on the mathematics of patterns- any pattern can be expressed by an equation, whether or not it is based on correct information.

An Example: How to Spot a Number Pattern

Numbers can reveal patterns without referring to events in the world such as salaries or predictable failure years for engine parts. For example, when we look at any line divided into three parts (segments) by points on it, we will see that the number of different segments we can make from them is three. We might also notice that this is equal to 3×2 divided by two, which equals three. This is the product of the position number of the last point $C = 3$ and the next-to-last point $B = 2$, divided by 2.



Line AC contains three possible line segments: AB, AC, and BC. There are no other combinations possible. This fact doesn't depend on the length of the segments. Studying this pattern in several simple examples helps us to see the bigger picture about combinations that are possible on a line with any number of points. **This is one way to discover useful patterns: examine a series of simple situations to find a general rule and then test the rule on more complex situations.**

We will see that the pattern for the number of different combinations of segments for any line will equal the product of the last point's position number and the next-to-last position number divided by two. This is a general statement of the pattern that was observed for line AC.

For example, a line with 20 points on it will have 20×19 divided by two, or 190 possible combinations. Understanding this pattern will allow us to find the possible combinations for a line with any number of points on it. Many patterns can be discovered by looking at simple examples and then making a general statement of what is true for all of them. This pattern is useful in trades dealing with wiring circuits and transportation routes.

Algebra allows us to make general statements about patterns by using letters for variables. In this example, the number of points on a line can vary, but whatever that number turns out to be, the relationship between them and the number of possible combinations will always be the product of the position number of the last point and the next-to-last point divided by two. If we let $C =$ the number of possible combinations of segments, and $n =$ the position number of the last point, then $C = \frac{n(n-1)}{2}$

2

This equation describes the relationship we are interested in.



Topic 4 – Working with Averages

The average of a list of numbers is the sum of the numbers divided by the number of items in the list.

To find the average of a list of numbers:

1. Add the items in the list
2. Divide the total by the number of items in the list. The formula for an average could be described by this equation:

$A = S/N$, where A = average, S = sum of the numbers in a list, and N = the number of items in a list. **An average is also known as the mean value of a list of data.**

Examples

1. Find the average cost of a plane ticket to Edmonton from the following prices:

$$\begin{array}{r} \$560 \\ \$455 \\ + \$890 \\ \hline \$1905 \end{array}$$

Divide \$1905 by 3, the number of items in the list:

$$\$1905/3 = \$635$$

The average cost = \$635.

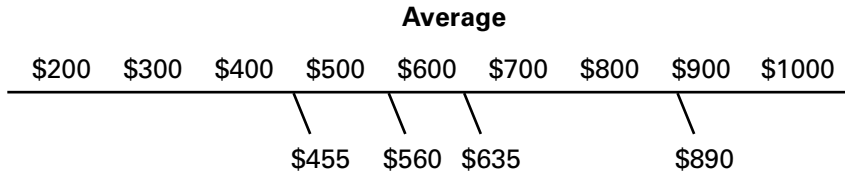
This information can guide consumers. We can compare the difference between the average price and a particular price to see if it costs more or less than the average. In the trades, purchasing and pricing decisions are influenced by average prices. In the following problems you will see how a list of observations can produce an average that may be useful for making predictions and making decisions.⁵

⁵ The distribution of numbers in a list can affect the meaning of an average. Some values may be unusually high or low and "skew" the data. Each problem must be considered in a way that is sensitive to the distribution pattern involved. The level five math curriculum discusses the display and interpretation of data.

Topic 4 – Working with Averages

2. Which ticket price is furthest away from the average price?

The average price is \$635, and \$560 is \$75.00 less, \$455 is \$180 less, and \$890 is \$255 more. The ticket priced at \$890 is furthest away, or deviates from the mean, the most.



3. Rainfall amounts in Gameti for the month of May are:

May 1998 = 2.5 cm
 May 1999 = 3.0 cm
 May 2000 = 2.7 cm
 May 2001 = 3.2 cm

What is the average amount of rainfall in May from 1998-2001?

Add the amounts for a total of 11.4 cm. Divide the total by 4, the number of items. Four monthly rainfall amounts for May were added.

$$11.4 \text{ cm} \div 4 = 2.85 \text{ cm.}$$

A weather forecaster in Gameti can now report May rainfalls and compare them with the average of 2.85 cm that can be expected based on the 1998-2001 period.

4. What is the average number of employees who may need to be replaced at the diamond mine each month based on the following information?

Month	Number of employees leaving
January	10
February	8
March	3
April	0
May	3

Add the number leaving for a total of 24. Divide by the number of months.
 $24 \div 5 = 4.8$

The human resource department can expect an average turnover of 4.8 employee positions each month based on this information.

Topic 5 – Reading a Value from a Table

A table organizes data in columns and rows. The titles given to the rows and columns tell us what the number in each cell means. A cell, or entry, is the intersection of a row and a column. In order to read a value from a table, you need to know what each cell tells us.

Example

Month	Income A	Income B
January	13,000	10,000
February	25,000	13,000
March	18,000	11,000
April	12,000	15,000
May	14,000	17,000
June	16,000	14,000
July	19,000	15,000
August	21,000	16,000
September	23,000	19,000
October	20,000	22,000
November	19,000	24,000
December	12,000	30,000

This table tells us the income from two sources labelled income A and income B for each month of a year. Let us consider these sources as two businesses. You can use the table to look up the income for either business in any month.

If you want to know the income for Business B in October, locate October in the vertical column and read across the row to the entry under Income B. The number there is \$22,000.

If you want to know which months either business earned more than \$20,000, you will read down each column and identify the month that corresponds to any values greater than \$20,000. In February, August, September, October, November, and December at least one of the two businesses earned more than \$20,000.

Tables can contain many columns and many rows. The key to reading a value from a table is knowing what each number represents. Once you know the headings for column and row, you can find the value for any desired combination of the two.

A table can be used to find a value that links two items, for example the distance between two communities. In this table the horizontal and vertical headings are communities, and the cells are (imaginary) distances in kilometres between the two communities that intersect in the cell.

Topic 5 – Reading a Value from a Table

	Inuvik	Wekweti	Tulita	Hay River
Inuvik	0	900	450	1800
Wekweti	900	0	300	650
Tulita	450	300	0	850
Hay River	1800	650	850	0

To find the distance between Tulita and Hay River according to this fictional table, look for the intersection of the Tulita row and the Hay River column or vice versa. The number is 850 km.



Unit 1 – Practice Exam Questions

Read a Value from a Graph

Use this table to answer the following questions.

	Fort Smith	Yellowknife	Inuvik	Hay River
Population now	6,200	16,000	8,500	4,300
Projected Population 2010	6,700	24,000	12,000	5,000

Question 1

Which community or communities projects a change of less than 1000 between now and 2010?

- a) Fort Smith and Inuvik
- b) Hay River and Fort Smith
- c) Yellowknife and Fort Smith
- d) Hay River and Inuvik

Answer: b

Explanation

The answer is found by looking at the difference in population projected for each community. Hay River and Fort Smith predict changes under 1000, with 700 for Hay River, and 500 for Fort Smith.

Question 2

What is the population projected for Fort Smith in 2010?

- a) 6200
- b) 5000
- c) 6700
- d) 8500

Answer: c

Explanation

The answer is found in the intersection of the second row and first column= 6700.

Unit 1 – Practice Exam Questions

Question 3

How many people live in Inuvik?

- a) 6700
- b) 5000
- c) 6200
- d) 8500

Variable means changeable.

Constant means unchanging.

Answer: d

Explanation

The question asks for the present population in Inuvik. The answer is found in the intersection of the first row and third column.

Question 4

Which community has the smallest projected increase in population?

- a) Inuvik
- b) Hay River
- c) Fort Smith
- d) Yellowknife

Answer: c

Explanation

The smallest change is 500 for Fort Smith, all other locations project larger increases.

Question 5

Which community will have a population of 5000 in 2010?

- a) Fort Smith
- b) Hay River
- c) Yellowknife
- d) Inuvik

Answer: b

Explanation

The answer is found in the second row, fourth column.



Unit 1 – Practice Exam Questions

Number Patterns

Question 1

Which number will come next in the following series: 2, 5, 8, and 11...

- a) 13
- b) 19
- c) 15
- d) 14

Answer: d

Explanation

The pattern can be found by looking at the difference between terms. Each term is the result of adding three to the previous term.

Question 2

A recipe calls for $\frac{1}{4}$ cup of butter for each cup of flour used in a cake. Which formula will allow us to predict how much butter to use when we know the amount of flour that we have on hand, if B = cups of butter, and F = cups of flour?

- a) $B = \frac{F}{4}$
- b) $F = \frac{B}{4}$
- c) $F = \frac{B}{2}$
- d) $B = \frac{F}{2}$

Answer: a

Explanation

This question asks for a formula that will express the ratio between flour and butter in terms of a known amount of flour. We are looking for B when we are given F . From the facts given, we can see that one cup of flour requires $\frac{1}{4}$ cup of butter, or amount of flour = four times the amount of butter. The ratio of flour to butter is four to one.

However, the question asks us to find a formula for the amount of butter when we know the amount of flour. We need a formula for B in terms of F . $B = \frac{F}{4}$ because the amount of butter is always one-fourth the amount of flour. You can also find this formula by working from a table based on the problem:



Unit 1 – Number Patterns

UNIT 1

Unit 1 – Practice Exam Questions

Amount of flour (F)	Amount of Butter (B)
1	1/4
2	1/2
3	3/4
4	1

The amount of butter is always $\frac{1}{4}$ of the amount of flour, this is the pattern, so

$$B = \frac{1}{4} F = \frac{F}{4}$$

Question 3

If a series were produced based on the table in problem two, what would be the value of B for the 10th term in the series?

- a) $\frac{5}{8}$
- b) $3 \frac{1}{3}$
- c) $2 \frac{1}{2}$
- d) $\frac{7}{8}$

Answer: c

Explanation

The pattern in the series based on this formula is "value of $n = \frac{1}{4}n$ ", where n is the position number for each term in the series. In the table, flour is given in whole numbers that label each position as well as the pounds of flour for that position. The value of any term will be that term's position number divided by four. The tenth term will have a value of $\frac{10}{4} = 2.5$ pounds, or 2.5 pounds of butter for ten pounds of flour.

Question 4

Margaret is offered a seat sale to Edmonton from Inuvik for \$930 in August. She researches recent airfares from Inuvik to Edmonton and finds the following information:

May \$1000
June \$890
July \$1200

If she purchases the ticket, which of following statements will be true?

- a) She is paying an average price for the May-July period.
- b) She is paying an above average price for the May-July period.
- c) She is paying a below average price for the May-July period.
- d) She cannot decide if the seat sale is above or below average price for the May-July period.

Answer: c



Unit 1 – Number Patterns

UNIT 1

Unit 1 – Practice Exam Questions

Explanation

First calculate the average price for the period she investigated. Add the three fares and divide by three to get \$1030. This is the mean or average price, and the seat sale of \$930 is \$100 less, therefore it is below the average price for the May-July period.

Question 5

What are the first five terms in the series beginning with 1 that are described by this formula: "the value of $n = 2n - 1$ "

- a) 1, 2, 5, 9, 11...
- b) 1, 3, 5, 7, 9...
- c) 1, 2, 3, 5, 7
- d) 2, 5, 9, 9

Answer: b

Explanation

The series can be found by building a table that uses the formula to match a value to each position number:

Position Number (n)	Formula ($2n - 1$)	Value
1	$2 \times 1 - 1$	1
2	$2 \times 2 - 1$	3
3	$2 \times 3 - 1$	5
4	$2 \times 4 - 1$	7
5	$2 \times 5 - 1$	9

Question 6

A ball bounces half as high on each bounce. If the ball reaches 5 feet on its first bounce, what height will it reach on the third bounce?

- a) 2.5 feet
- b) Can't tell from this information
- c) 10 feet
- d) 1.25 feet

Answer: d

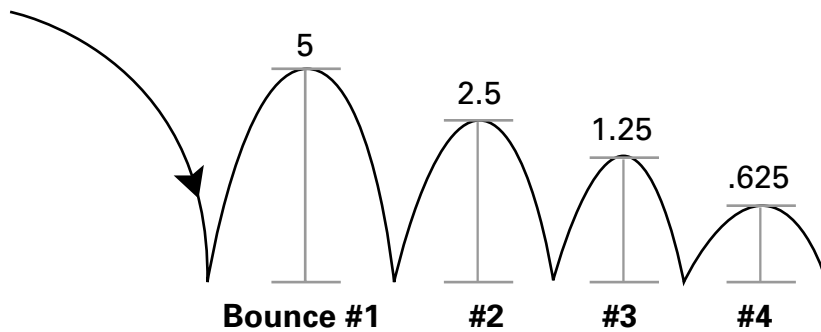


Unit 1 – Practice Exam Questions

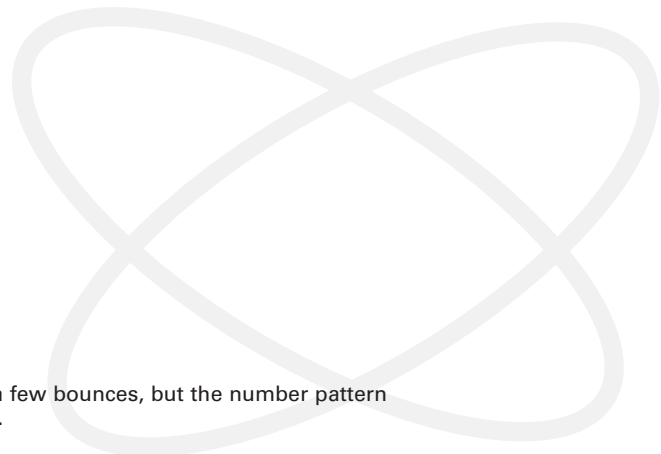
Explanation

The problem describes a pattern. A table using the information given will predict the height on the third bounce:

Bounce number (n)	Formula	Output value
1		5 feet
2	$5 \div 2$	2.5 feet
3	$2.5 \div 2$	1.25 feet
4	$1.25 \div 2$.625 feet



This series divides the value of each position (i.e. the height of each bounce) by 2 to give the value of the next term (i.e. the height of the next bounce). The bounces are the input position numbers, and the heights are the output values. The formula for this pattern can be expressed as "The value of bounce n = value of $(n - 1)$ divided by 2", or The value of bounce n = (value of $(n - 1)$) \div 2.⁶



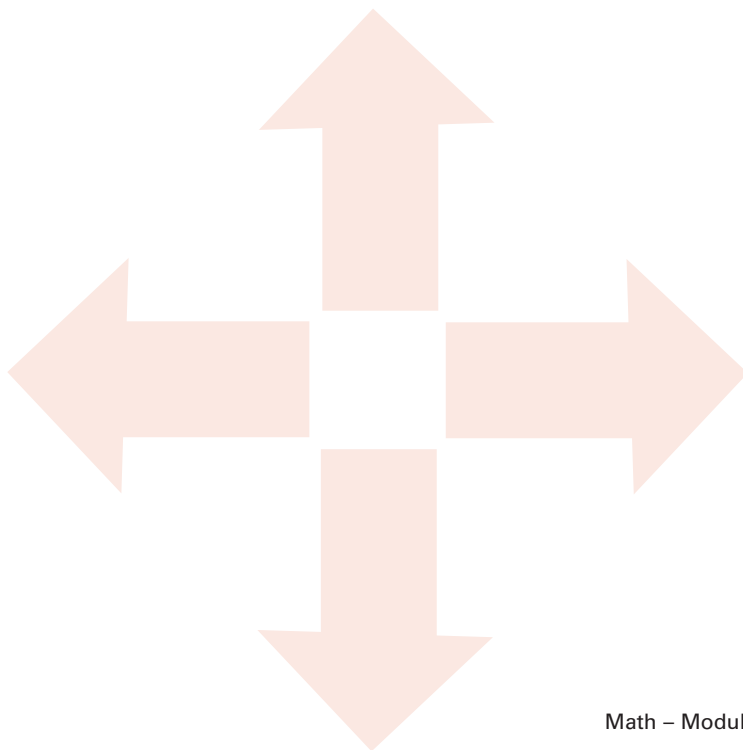
6 In the real world the bouncing ball will come to rest after a few bounces, but the number pattern can go on and on- approaching zero but never equalling it.



Unit 2

Relations and Graphs

In the preceding unit you have seen how number patterns can be discovered from tables and expressed using formulas and series. Formulas are equations that set one expression equal to another. The relationships that are described by formulas can be put on graphs to help us better understand and use patterns.



Topic 1 – Writing Expressions

Many situations in the trades require the use of formulas, or algebraic expressions. A verbal description of a relationship between numbers can be written as a mathematical expression.

Examples

1. The building code requires that residential woodstove chimneys must be 3 feet above the highest part of a roof. If a chimney is installed at ground level, write an expression that describes the total height required for a residential woodstove chimney.

Answer

In this problem the height of a roof above ground level can vary, but there will always be three feet added on for a chimney. Choose letters to represent the quantities in the problem. The highest point on a roof from ground level, which can vary, could be represented by h . The number 3, a constant, represents the height of a chimney above the highest point on a roof. The operation we need to find the total chimney height is addition. $3 + h$ will equal total chimney height.

Additional Explanation

The sum of three feet and the roof height will equal the number we seek. Unless we are told otherwise, the measurement of the height of a roof from ground level automatically tells us the highest point of the roof. The expression $h + 3$ describes the total chimney length required. This number can be represented by T , (or any other letter we choose). Then we can set the expression for chimney height equal to $h + 3$. A chimney installer can use this formula, $T = h + 3$ to find the total length of chimney that will be needed. Notice how the expression for chimney height was set equal to a variable we are interested in to create a formula that we can use in a trade.



Topic 1 – Writing Expressions

2. Peter burns 2 litres of gas for every 10 kilometres when he drives his truck from Rae Edzo to Yellowknife. If his trip is 120 kilometres, write an expression for the amount of fuel he will need.

We can calculate the fuel he will need by dividing 120 by ten = 12, to see how many ten kilometre segments he covers. Each segment uses 2 litres; therefore he will need 24 litres (2×12). We can also work with the unit ratio that compares one litre of fuel to the distance it will cover using his truck.⁷ In this case, one litre will cover five kilometres because two litres covers ten kilometres. By dividing the total distance by five we find once again that 24 litres will be required.

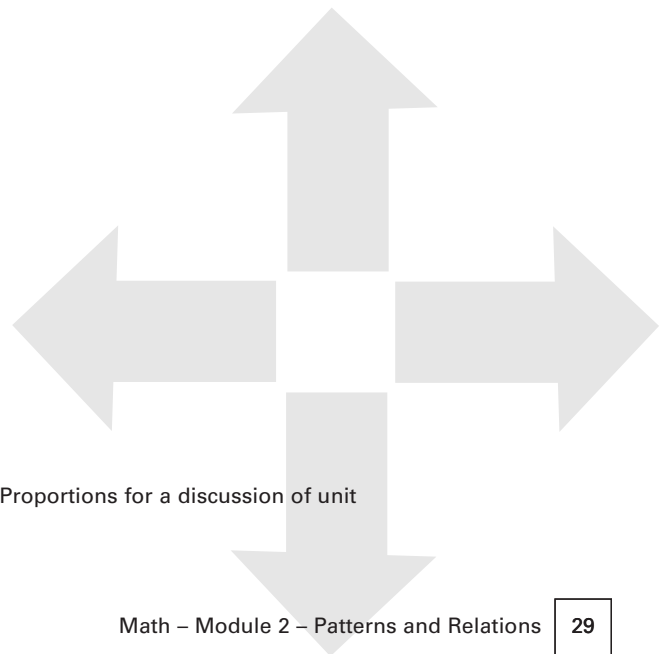
The expression for the total fuel required if we choose T to represent the total amount of fuel and D the total distance to be covered is $T = D/\text{volume of fuel per unit of distance}$.

A general formula for finding total fuel needed = distance travelled \div distance per litre of fuel. For example, if we drive 200 miles at the rate of 20 miles per gallon, then we will need 10 gallons = $200 \div 20$. This example uses the formula $T = D/\text{distance per unit of fuel}$.

3. Some problems provide the letters that will be used in an expression. Then we have to figure out how to operate with them to find the quantity we seek. Remember to think of an algebraic expression as a set of instructions.

What is the value of $3x - 4$ when $x = 3$, when $x = 4$, and when $x = 5$?

We are asked to evaluate the expression $3x - 4$ by putting each value in place of x and finding the result. When $x = 3$, $3x - 4 = 5$, when $x = 4$, $3x - 4 = 8$, and when $x = 5$, $3x - 4 = 11$. This process is also described as substituting a value for a variable.



⁷ See Math – Module 1, Unit 3, Topic 8 – Rations, Rates and Proportions for a discussion of unit ratios and rates.

Topic 1 – Practice Exam Questions

Question 1

Write an expression that tells us to add five to the product of two numbers, a and b .

- a) $a + 5 + b$
- b) $ab5$
- c) $a + 5b + 5$
- d) $ab + 5$

Answer: d

Explanation

Remember the order of operations to see that multiplication comes before adding and subtracting in evaluating an expression. Choice d tells us to multiply a by b and then add 5 to that result.

Question 2

What is the value of $5ab - 2$ when $b = 2$ and $a = 3$?

- a) 28
- b) 32
- c) 36
- d) 30

Answer: a

Explanation

Substitute $a = 3$ and $b = 2$. Multiply ab to get 6. Now multiply by 5 to get 30. Finally subtract 2 to get 28.

Question 3

Bernice is paid \$30.00 for every 5 patterns she sews. What expression describes how much she earns?

- a) \$30.00 times the hours worked
- b) \$30.00 divided by the number of patterns she sews
- c) 5 times \$30.00
- d) \$30.00 times the total number of patterns divided by five

Answer: d

Explanation

We need a formula that relates the total number of patterns she sews to \$30.00 for every five that are completed. For example if she sews 50, she will earn $50 \div 5 = 10 \times \$30.00$, which equals \$300.00. If S equals her salary, and P is the total number of patterns she completes, then $S = (P/5) \times \$30.00$.

Topic 1 – Practice Exam Questions

Question 4

Which expression tells us to add three to the product of 8 and 5?

- a) $3 + 8 - 5$
- b) $3 \times 8 + 5$
- c) $3 + 5 \times 8$
- d) $8 \times 3 + 5$

Answer: c

Explanation

Multiplication is done before adding and subtracting. Choice c correctly says to add three to 5×8 .

Question 5

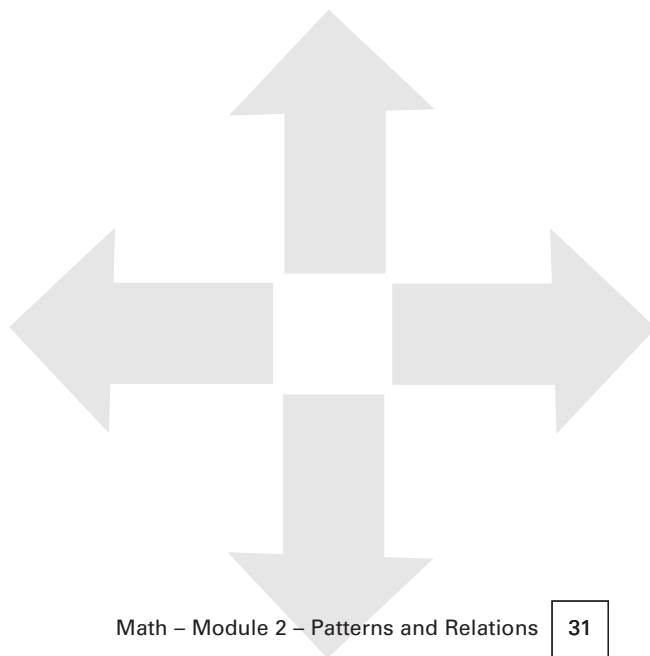
Write an expression that says that Bill earns three times as much as Michael.

- a) $B \times 3 = M$
- b) $M \times 3 = B$
- c) $B - M = 3M - 3B$
- d) $B = 3M$

Answer: d

Explanation

B is used to represent Bill's earnings, and M is used to represent Michael's earnings in the choices given. Choice d says that Bill's earnings equal three times Michael's.





Topic 2 – Equivalent Forms for Expressions

Once an expression is put into algebraic form, it can be expressed in equivalent ways as long as the rules of arithmetic are followed. Expressions that are equal to the same thing are equal to each other. The following examples show how the order of terms in an expression can be rearranged to produce equivalent expressions. You can check to see if two expressions are equivalent by substituting the same value for the same variables in each expression.⁸ When you evaluate the two expressions the results will be identical if they are equivalent expressions.

Examples

1. $3b = b3$ (although it is usual to put a variable after a constant, and $3b$ is usually the preferred form)

Check: let $b = 1$, then $3b = 3 \times 1 = 3$, and $b3 = 1 \times 3 = 3$

2. $4M - 3 = -3 + 4M$

Check: let $M = 1$, then $4M - 3 = (4 \times 1) - 3 = 1$ and $-3 + 4M = -3 + (4 \times 1) = 1$

3. $6x + 2 = 2 + 6x$

Check: let $x = 1$, then $6x + 2 = 8$ and $2 + 6x = 2 + 6 \times 1 = 8$

4. $a/b = a \times 1/b$ (check this yourself)

5. $b \times a/5 = ab/5$

Rearrange Equations to Solve for Each Variable

When expressions are used to describe a formula, they are equations that can be rearranged to give an output number for any variable or constant in the expression.⁹

Recall the earlier example of a recipe for omelettes. $E = 2N$, where the number of eggs equals twice the number of people being served. This formula can be rearranged in terms of N to give: $N = E/2$. Dividing both sides of the equation by two isolates N so that we can see what it is equal to in this formula. Both of these equations are based on the same relationship.

Examples

1. If we know that $d = 32T^2$, (the distance an object falls equals 32 feet times the square of the time) then:

$$32 = \frac{d}{T^2} \text{ (divide both sides of } d = 32T^2 \text{ by } T^2\text{)}$$

⁸ See Math – Module 3 – Variables and Equations for more on simplifying expressions and equivalence.

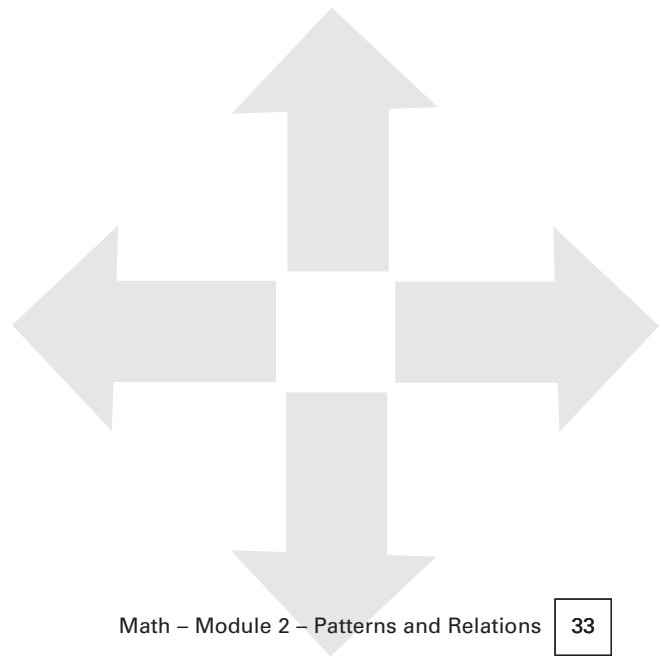
⁹ See Math – Module 3 – Variables and Equations, for more on rearranging equations.

Topic 2 – Equivalent Forms for Expressions

And,

$$T = \sqrt{\frac{d}{32}} \quad (\text{divide both sides of } d = 32T^2 \text{ by } 32, \text{ then take the square root of both sides})$$

2. The equation $y = 14b + 6$ can be rearranged to show that $b = (y - 6) \div 14$, and also that $6 = y - 14b$.





Unit 2 – Relations and Graphs

UNIT 2

Topic 3 – Tables and Graphs

Tables and graphs can show us the relationship between a position and a value assigned to that position. By comparing these pairs of numbers (aka ordered pairs) we can study patterns and predict the value of output numbers for any input numbers. For example, if we know that $y = 3x - 1$, then we can write a table that finds y for every choice of x :

x (input)	y(output) = $3x - 1$
1	2
2	5
3	8

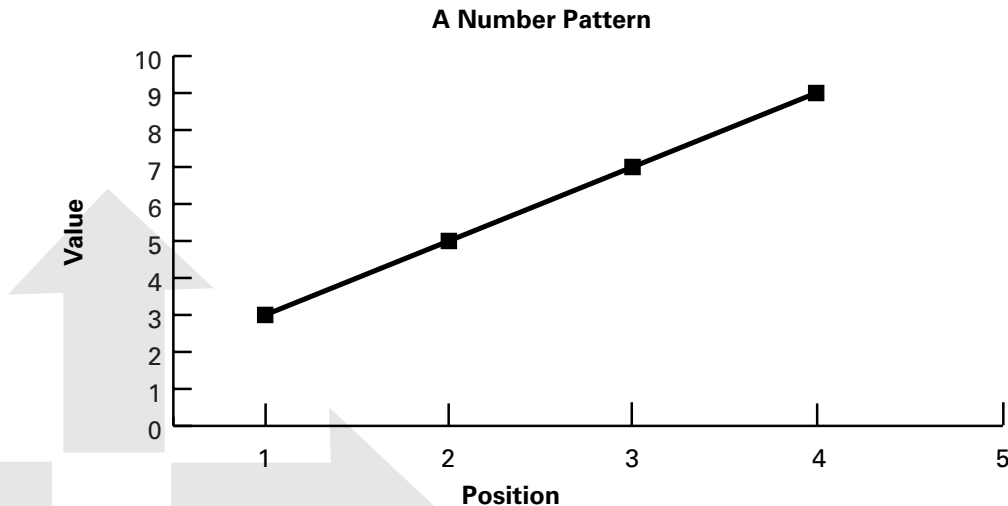
Examples

- 3, 5, 7, 9...

Each term in the series after the first term is reached by adding two to the value of the previous term. In a table this series would look like this:

Position	Value
1	3
2	5 $5 = 3 + 2$
3	7 $7 = 5 + 2$
4	9 $9 = 7 + 2$

In a line graph this series would look like this:





Unit 2 – Relations and Graphs

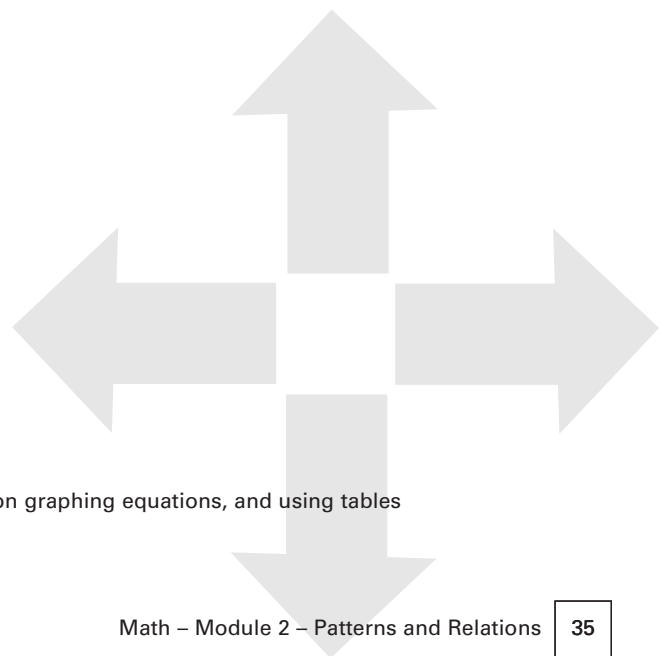
UNIT 2

Topic 3 – Tables and Graphs

The graph shows the position of each number in the series on the x axis. The y axis shows the value assigned to each position. We can read the graph by looking at the position number in the series and finding the corresponding value on the y axis. The first term has a value of 2, the second term a value of 5, the third term a value of 7, and so on.

This graph is a straight line. Equations that have this kind of graph are called **linear equations**.¹⁰

This pattern might describe the relationship that was observed in a survey of all car dealerships in the Northwest Territories between years of experience for a salesperson and number of sales per month. If this were the case, then the x axis could be labelled years of experience, and the y axis could be labelled sales per month. The graph would show that success in selling increases with years of experience for the dealerships that were surveyed.



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¹⁰ See Math – Module 3 – Variables and Equations for more on graphing equations, and using tables to find solutions for equations.

Topic 4 – Reading Values from a Bar Graph

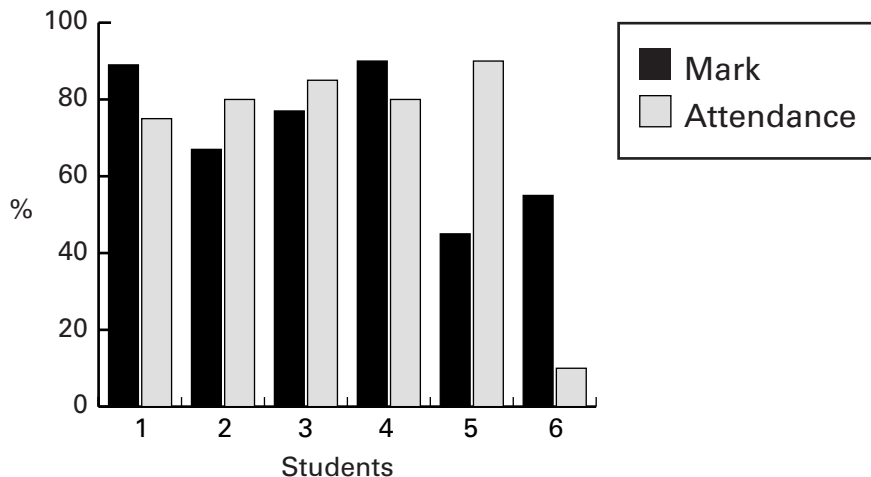
A bar graph allows us to compare information from several sources and understand relationships. We can discover useful and important relationships by studying bar graphs. Each graph will have two axes, one horizontal, and one vertical. The information on each axis will be labelled to show the units or items involved. The bars will show the values on one axis that describe something about the items on the other axis. A bar graph relates information from several sources in one picture.

An explanation, or key, also called a legend, is provided to show what each bar in the graph represents. Sometimes a bar graph with vertical bars is called a column graph.

Example 1: Looking for a Pattern

This bar graph compares the marks at the end of an apprenticeship class for 6 students, with their attendance.

The numbers on the horizontal axis refer to students. There are six students who



have their grades compared with their attendance. Their names could have been used instead of numbers. The vertical scale is in percent. Grades and amount of attendance are both expressed in percent. Each student has two bars, one to show the grade and one to show the attendance. The data used to create this graph has two percent numbers to enter for each student. The data used to create the graph could be displayed in a table:

Student #	Grade %	Attendance %
1	90	75
2	67	80
3	77	85
4	90	80
5	45	90
6	55	10

Topic 4 – Reading Values from a Bar Graph

The graph can help us learn if there is a connection (aka a correlation) between how high a student scored and how high their attendance rate was. We might think that those who attend more also got higher scores, but is this true according to the graph?

A **correlation** connects two facts, but does not prove that one causes the other.

The answer is "yes and no". Students 5 and 6 show the opposite. Number 5 had a 90% rate of attendance, but only scored 45%. Student 6 only attended 10% of the time, but received a grade of 55%. Students 1-4 show a positive relationship between attendance and marks, but with some variation. A positive relationship is one that proves or supports a pattern that we are testing. A negative relationship does the opposite. In this example the pattern is supported by the results for two thirds (four out of six) of the class.

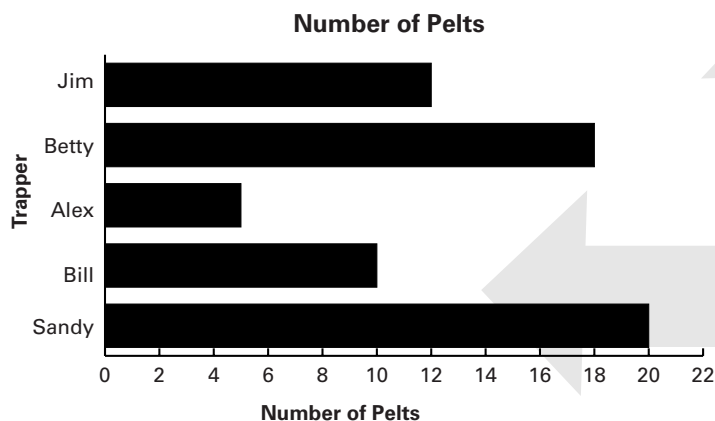
We can also conclude that attendance is not the only factor determining how well a student does in this class because two exceptions to the pattern occurred. Bar graphs are useful tools for exploring questions of this kind.

Example 2: Comparing Items on a Graph

A simpler bar graph would have only one bar for each item on one of the axes. Here is a table showing how many pelts each of five trappers brought in this year:

Trapper	# of Pelts
Jim	12
Betty	18
Alex	5
Bill	10
Sandy	20

Here is a bar graph using horizontal bars to show this information.



Topic 4 – Reading Values from a Bar Graph

What You Need to Know:

In order to read values from a bar graph you need to answer the following questions:

1. What units or items are represented on each axis?
2. What do the bars (or columns) tell you about the items?

In the last example, the vertical y axis shows the names of the trappers. The horizontal x axis is a number line. The bars tell the number of pelts (horizontal axis) each trapper brought in.

The values we read from this graph refer to numbers of pelts. Questions about values on a bar graph can be direct or comparative. A direct question asks how many pelts a particular trapper got, or which trapper got a particular number of pelts. For example:

1. How many pelts did Jim get?

We look at the bar for Jim and see that it ends at the 12-point mark on the x axis. Jim got 12 pelts.

2. Which trapper got 20 pelts?

We look at the bar that ends at 20 on the x axis. The trapper this information refers to is Sandy.

Comparative questions ask us to reach a conclusion based on the information on the graph. For example:

1. Which trapper had the poorest results?

We need to know who brought in the fewest pelts. The shortest bar will belong to the trapper with the fewest pelts; this is Alex who got only 5 pelts.

2. Who brought in more pelts, Bill or Alex?

To answer this question, see which bar is longer. Alex's bar is shorter than Bill's. Bill has 10 pelts and Alex has 5.

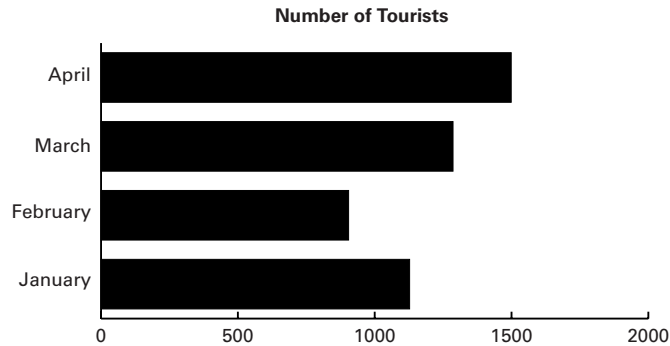


Unit 2 – Relations and Graphs

UNIT 2

Topic 4 – Practice Exam Questions

Use this graph to answer the questions. The graph shows the number of tourists who came to view the northern lights in Yellowknife between January and April.



Question 1

How many tourists came in April?

- a) 1500
- b) 1000
- c) 1200
- d) 1300

Answer: a

Question 2

Which month had the smallest number of tourists?

- a) April
- b) February
- c) March
- d) January

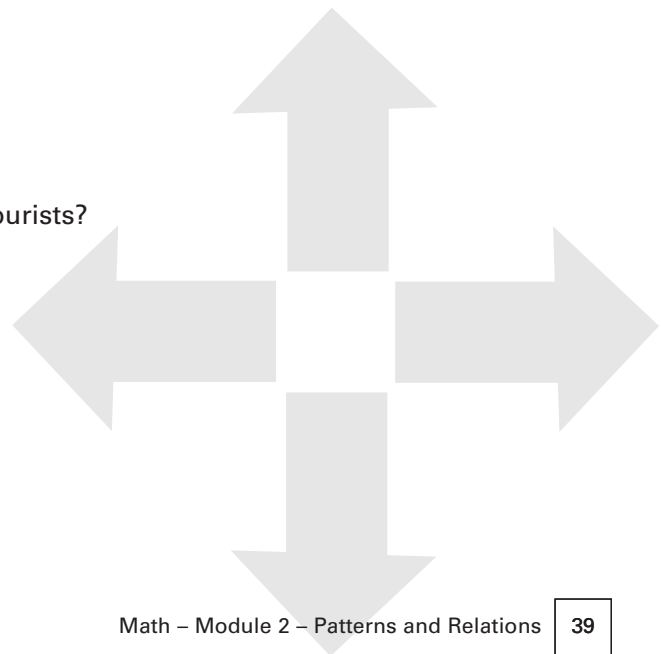
Answer: b

Question 3

Which month has the second largest number of tourists?

- a) January
- b) February
- c) March
- d) April

Answer: c





Unit 2 – Relations and Graphs

UNIT 2

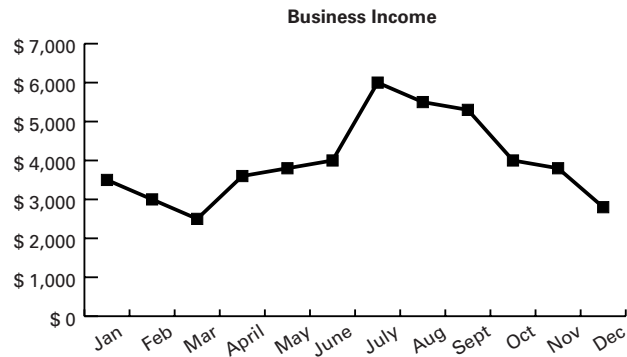
Topic 5 – Reading a Value from a Line Graph

A line graph connects data points on two axes. More than one line can be put on the same graph to permit comparisons between the data points represented by each line.

A line graph is useful when we want to track changes over time.

Example

A new tourism business in Inuvik is keeping track of income each month. A line graph shows the changes in revenue each month.

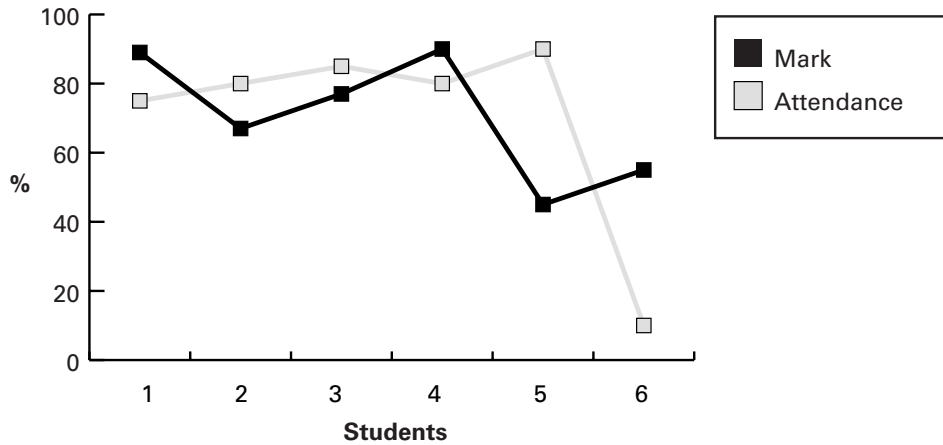


The data used to create this graph involves calculating the total money earned by the business each month. Each point on the graph represents the amount for a given month. By connecting the points over time we can see the changes in revenue. The summer period shows an increase in monthly income that peaked in July and then decreased to the same level in November that was reached in March. This information may suggest an annual trend in tourism that will continue to bring higher revenues in the summer months. The company will plan their supply orders accordingly.

A line graph can also compare two data points for each item being tracked. The graph used in the previous section on bar graphs to compare student grades with attendance, can be changed into a line graph using the same data.

Topic 5 – Reading a Value from a Line Graph

Grades and Attendance for Six Apprenticeship Students



Each student has two points, one for the grade, and one for attendance. Both points are expressed as percent numbers. By connecting the points we can compare attendance with grades. The same conclusions will be reached from the line graph that was reached from the bar graph. If attendance and grades were closely linked, the two lines should be close together. We can see that there are exceptions to this expectation in cases 5 and 6.

Sample Problems

In order to read values from a line graph you need to answer the following questions:

1. What units or items are represented on each axis?
2. What do the points on the line tell you about the items?

In the earlier example of a line graph showing business income, the vertical y axis, shows income amounts. The horizontal x axis shows the months of the year.

The values we read from this graph refer to amounts of income for each month. Questions about values on a line graph can be direct or comparative. A direct question asks how much income the business earned each month. For example:

1. How much was earned in July?
We look at the point above July and see \$6000.

2. Which months had less than \$5000 income?

Look at the points below the \$5000 line. They are February, March, April, May, November, and December.



Unit 2 – Relations and Graphs

UNIT 2

Topic 5 – Reading a Value from a Line Graph

Comparative questions ask us to reach a conclusion based on the information on the graph. For example:

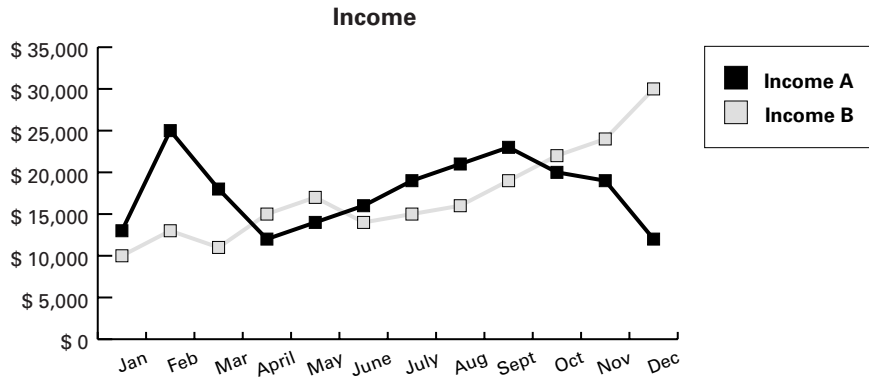
1. Which month had the lowest income?

March had \$2500; this is the lowest point on the line graph. All other months had higher income.

2. Which month had more income, November or February? November equals \$4000, and February equals \$3000. November has more income.

Topic 5 – Practice Exam Questions

The income from two businesses are compared on the following line graph. Use this graph to answer the following questions.



Question 1

Which business earned more money in July?

- a) Business B
- b) Business A
- c) Both are equal
- d) Can't tell from this graph

Answer: b

Explanation

In July, business A earned more than \$15,000 and business B earned less than \$15,000 according to the graph.

Question 2

What is the difference in income between October and February for business A?

- a) \$10,000
- b) \$5,000
- c) \$8,000
- d) \$12,000

Answer: b

Explanation

The data point for October is \$20,000, and for February \$25,000, which makes a difference of \$5000.



Unit 2 – Relations and Graphs

UNIT 2

Topic 5 – Practice Exam Questions

Question 3

In what two months did the two businesses earn almost identical amounts?

- a) January, December
- b) March, June
- c) July, October
- d) June, October

Answer: d

Explanation

In June and October the distance between the two income points is smaller than all other pairs of months that are given in the choices.

Question 4

In which months did business B earn more than \$20,000?

- a) October, November, December
- b) March, June, October
- c) April, May, December
- d) October, November, January

Answer: a

Explanation

In these months the data points are above the \$20,000 line.

Question 5

In what month did business A income first fall below business B income?

- a) October
- b) April
- c) May
- d) November

Answer: b

Explanation

In April the line for B crosses below the line for A for the first time.

Topic 6 – Interpolating and Extrapolating

A graph, a table, and a series, will each allow us to find values both between (interpolated) and beyond (extrapolated) those that are given in a problem or series of observations. For example, we can see that the earlier graph of a number pattern describes a linear doubling function that takes each number on the x axis and assigns a point on the y axis that is twice as large. $Y = 2x$ is the equation describing this relationship. We can interpolate and see that the value for 1.5 is 3 on the y axis, even though 1.5 is not a value given in the table that the x axis of the graph is based on. The line connecting the points that are given touches many other in-between points that can be related to points on the x axis.

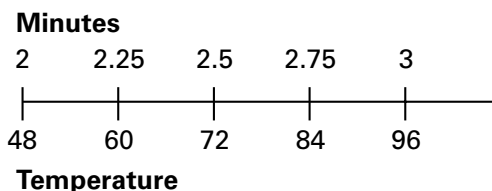
For example, if we know that the starting temperature of water is 6 degrees, and that the temperature will double each minute until the boiling point of 100 degrees is reached, we can find the temperature at 1.5 minutes, or 3.3 minutes by interpolating. This process assumes that the temperature is changing continuously and evenly. The temperature is going up at the same rate per unit of time, whatever unit we choose (seconds, half minutes, quarter minutes) between temperature measurements.¹¹

Example

- Find (Interpolate) the temperature of water being heated after 2.25 minutes using the information given in the following table:

Time (in minutes)	Temperature (in degrees Celsius)
0	12
1	24
2	48
3	96
4	100
5	100

We are given the temperatures for two minutes and for three minutes. 2.25 lies between these values. 2.25 is a decimal number that is .25 greater than two, and .75 less than 3. .25 is 25% or 1/4 of the time between minute two and minute three. We know that the change in temperature from minute two to minute three is $96 - 48 = 48$. One fourth, or 25% of 48 is 12. This means that the temperature at 2.25 minutes will be the temperature at two minutes plus 12 more degrees. $48 + 12 = 60$ degrees. We can easily interpolate the values for 2.5 and 2.75 minutes using the fact that 1/4 of the temperature change between minutes two and three equals 12 degrees:



¹¹ Interpolated values must be checked in the real world with observations. It could turn out that the rate of temperature increase is not even between minutes two and three. For example, it could turn out that most of the 48 degree difference between minutes two and three is reached in the first 15 seconds. Observations are needed to settle this question.



Topic 6 – Interpolating and Extrapolating

Notice that this method assumes that the rate of change in temperature is constant between intervals.¹²

Extrapolating

When we have a number series or a table that continues beyond a given list of values, we can extend the information that is given to predict values further out in the series or table.

Examples

- Bruce has kept records of his recreational vehicle sales for the past six months. He notices that each month has included an increase of at least 15% over the previous month's sales. Assuming that that the future will continue this trend, what can he predict about next month's sales?

Answer: at least a 15% increase over the previous month.

Other things being equal, his records show a trend, such that each month's sales are 15% greater than the previous month's. However, there are no guarantees that this will actually continue to happen. Predictions based on trends assume that the future will resemble the past. Because trends based on human behaviour involve many factors, this assumption can sometimes fail. A number pattern must be interpreted carefully in the light of all factors that can affect a prediction based on the pattern. However, if nothing happens to change the trend, we can predict an increase of at least 15% over the previous month.

Extrapolation uses the pattern in a table or graph to predict "future" values.¹³ We can also extrapolate from a series when we know the formula for the value of any term in it. You have already worked on problems finding the value for the nth term in a series. These problems are examples of extrapolating.

¹² How much does the temperature increase each second until 100 degrees is reached?

Assuming that temperature rises continuously and evenly, we need to know how much the temperature increases each second. There are 60 seconds in a minute. However, we can see from the table that the rate of increase involves more temperature rise per second between minutes two and three (48 degrees), than between minutes one and two (24 degrees). Not all seconds are equal from the standpoint of the temperature change that occurs during each of them. Under these conditions, the best we can do is take an average change per second. Advanced methods using calculus are needed to calculate the rate of change in temperature at each second.

The average change per second will be the total change in degrees, divided by the total number of seconds in four minutes. $88/240 = .366\dots$ degrees. Statistical methods are needed to further refine this estimate of the change per second.

¹³ A future value can be one implied by a mathematical series and simply mean "further out in the series", or it can refer to a predicted value to be observed in the world based on a past pattern, for example the temperature of a heating liquid after 10 minutes given a record of changes occurring in the first three minutes. These predictions must be tested by experiments.



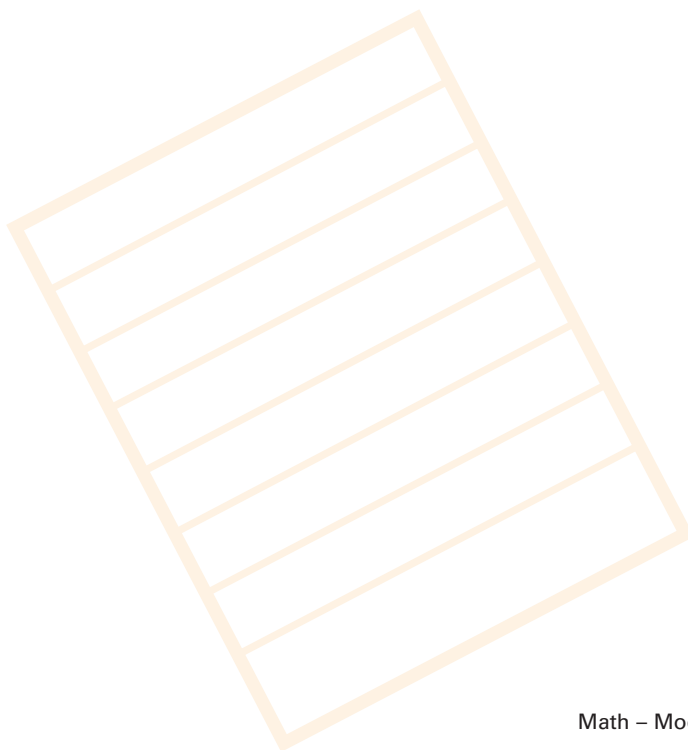
Unit 3

Practice Exam Questions Math Module 2 – Patterns and Relations

There are four modules required for all trades entrance math exams. Each module has a set of practice exam questions with an answer key. Each topic in the table of contents has sample questions to test your preparation for the trades entrance exam.

You should aim for 100%, and study the sections of the curriculum for any topics that you do not get right. After each answer the sections you should review are identified. Turn to the appropriate section of the curriculum whenever you need help.

The core math curriculum is based on "need to know" competencies that are important in all trades. You may want to use the following sample exam questions both as a way of assessing what you need to learn before you work on the curriculum, and as a test of what you know after you have completed your preparation for the exam.



Practice Exam Questions

Question 1

The expression $(\$12.00 \times \text{hours worked}) + (\$2.00 \times N)$ describes the salary for a worker who is paid \$12.00 an hour plus \$2.00 for every ton of earth, N , that is moved each hour. How much will a worker earn if he moves 10 tons of earth in one hour?

- a) \$14.00
- b) \$140.00
- c) \$32.00
- d) \$64.00

Question 2

How much earth will a worker have to move in 12 hours to earn \$500 using the expression in question one (above)?

- a) 178 tons
- b) 144 tons
- c) 10 tons
- d) 50 tons

Question 3

Sally earned \$320.00, \$560.00, \$210.00, and \$750.25 each week in January. What is the average weekly amount she earned?

- a) \$560.06
- b) \$360.06
- c) \$460.06
- d) \$466.00

Question 4

What is average height of people in this group: Bill = 6', Mary = 5' 3", Jim = 5' 9"?

- a) 5' 6"
- b) 5' 8"
- c) 5'
- d) 5' 5"

Question 5

The average test score for three apprentices is 75. Which scores produce this average?

- a) 80, 60, 75
- b) 73, 83, 63
- c) 74, 75, 76
- d) 78, 77, 73

Practice Exam Questions

Question 6

What is the average wage paid to a retail clerk based on the following (imaginary) information: Inuvik Co-op \$15.00/hour, Fort Simpson store \$17.00/hour, Fort Smith supermarket \$12.00, Centre Square Mall \$13.00/hour.

- a) \$16.25
- b) \$15.25
- c) \$14.25
- d) \$14.75

Question 7

The average rainfall in Fort Liard for June is 3 mm not including this year's June amount. This year 4.2 mm fell in June. How much does this year's amount deviate from the average?

- a) 3.2 mm
- b) 1.2 mm
- c) 3.05 mm
- d) can't tell from this information

Question 8

What is the next number in this pattern: 3, 6, 9, 12...

- a) 13
- b) 15
- c) 16
- d) 14

Question 9

Which expression says "add two to the product of a and five"

- a) $2 \times 8a$
- b) $2 + 25a$
- c) $5a + 2$
- d) $2a + 5$

Question 10

Peter earns a different amount of money each week, but he saves 10% of what is left each week after paying his bills. Write an expression that describe how much he saves each week if S is chosen to represent what he earns each week and B represents his weekly bills.

- a) $.10S - B$
- b) $B - .10S$
- c) $.10(S - B)$
- d) $(B - .10S) + S$



Unit 3 – Practice Exam Questions

UNIT 3

Practice Exam Questions

Question 11

Every third year the caribou have migrated away from town. If the herd migrated away from town in 2002, will they migrate away from town in 2022?

- a) can't tell from this information
- b) yes
- c) no
- d) maybe

Question 12

A stairway has a tread width (run) of 7 inches. If the formula for stairs is $\text{rise} + \text{run} = 17$ inches, which expression will describe the rise?

- a) $17 + \text{run}$
- b) $17 - \text{rise}$
- c) $7 - \text{run} + \text{rise}$
- d) $17 - \text{run}$

Use this table to answer questions 13, 14 and 15.

Year	Apprentices	Employed	Employed in mines
1998	23	20	10
1999	26	18	14
2000	32	22	16
2001	45	40	22
2002	44	42	30

Question 13

Which year showed the greatest increase in the number of apprentices employed in mines?

- a) 1999-2000
- b) 2000-2001
- c) 2001-2002
- d) 1998-1999

Question 14

What percentage of apprentices were employed in 1998?

- a) 20%
- b) 87%
- c) 13%
- d) can't tell from this table



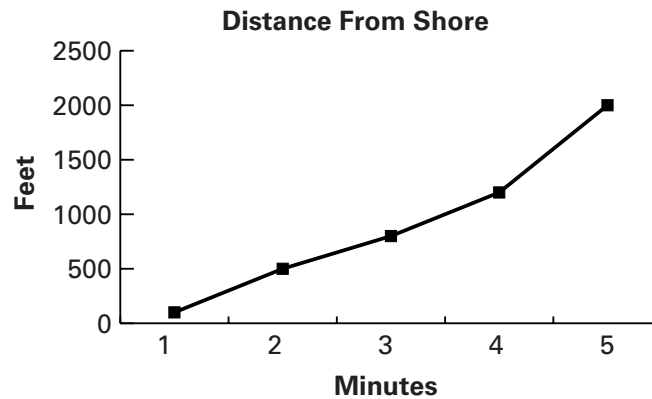
Practice Exam Questions

Question 15

Which statement is not supported by information in this table?

- a) employment for apprentices was greater in 2000 than in 1998
- b) employment in mines has steadily increased
- c) apprentices always find work
- d) In 2001 18 apprentices were not employed in mines

Use this graph to answer questions 16, 17 and 18.



Question 16

At the end of the fourth minute, how far is the boat from shore?

- a) 1000 feet
- b) 4000 feet
- c) 1500 feet
- d) 2000 feet

Question 17

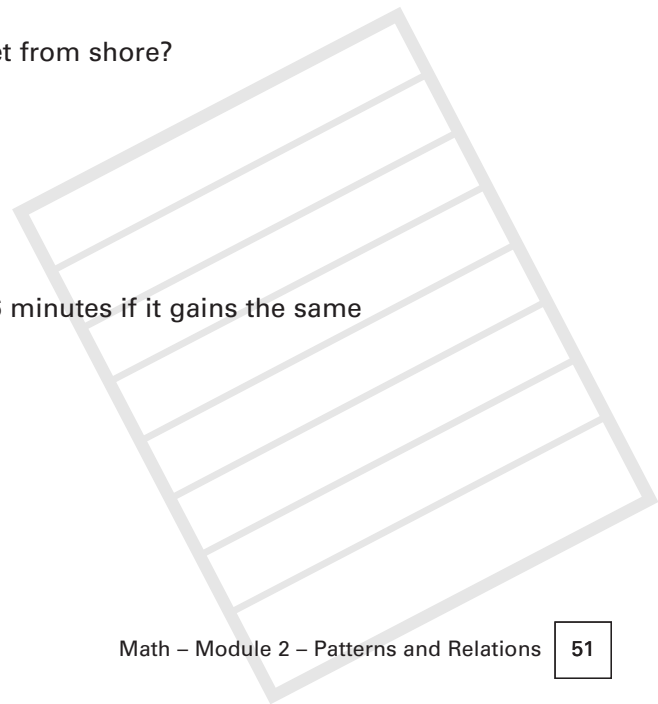
How long did it take for the boat to reach 1000 feet from shore?

- a) 1 minute
- b) 1.5 minutes
- c) 2 minutes
- d) 3 minutes

Question 18

How far from shore will the boat be at the end of 6 minutes if it gains the same distance as it did during the fifth minute?

- a) 2750 feet
- b) 3000 feet
- c) 2300 feet
- d) 2700 feet





Practice Exam Questions

Question 19

What is the sixth term in this series: 5, 20, 80, 320, ...?

- a) 380
- b) 5120
- c) 640
- d) 1280

Question 20

If the value of the first term in a series is defined as 5, what formula will give the value of any term after the first term in this series: 5, 20, 80, 320, ...?

- a) $5^n = \text{value of } n$
- b) $\text{value of } n \times 4 = \text{value of } n$
- c) $40 + 2(n - 1) = \text{value of } n$
- d) $\text{Value of } n^{\text{th}} \text{ term} = (\text{value of } n - 1 \text{ term}) \times 4$

Use this table to answer questions 21 and 22.

Time	Temperature
6:00 am	-35
7:00 am	-33
8:00 am	-31
9:00 am	-29

Question 21

Estimate the temperature at 8:30 am.

- a) -30
- b) -29
- c) -28
- d) -27

Question 22

If the pattern in this table continues, what would the temperature be at 11 am?

- a) -27
- b) -25
- c) -22
- d) -23



Practice Exam Questions

Question 23

A truck is being loaded with gravel at a steady rate on a scale. At the end of one minute it weighs 3 tons, and at the end of two minutes it weighs 4 tons. What did it weigh at the end of 1.25 minutes?

- a) 3.5 tons
- b) 3.25 tons
- c) 3.05 tons
- d) 3.75 tons

Question 24

Which expression is equivalent to $3\left(\frac{a-4}{2}\right)$?

- a) $\frac{3}{2}(a - 4)$
- b) $-\frac{a}{2}$
- c) $3a - 8$
- d) $2a - 12$

Question 25

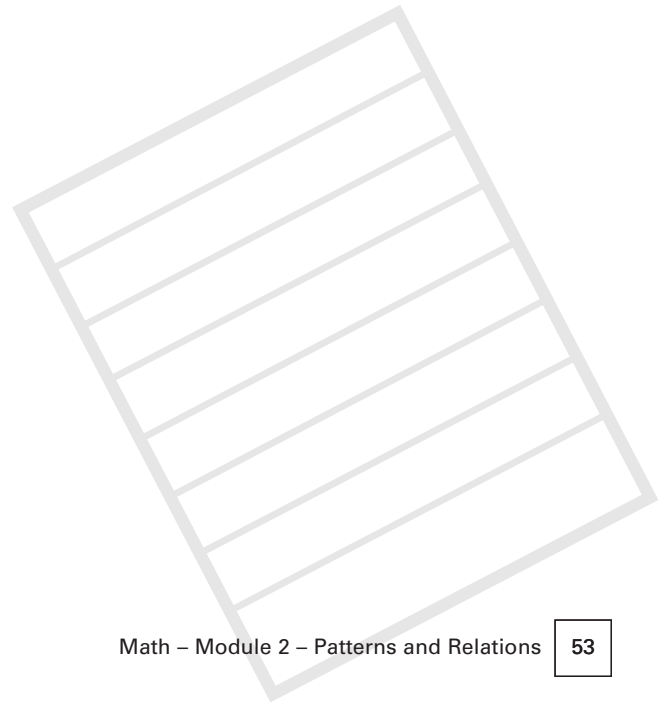
What is the value of the expression $2h + 12$ when $h = 17$?

- a) 46
- b) 32
- c) 41
- d) 28

Question 26

Rearrange $P = r + 12$ to find the expression that is equal to r ,

- a) $r = 12 - P$
- b) $r = 12/r$
- c) $r = P - 12$
- d) $r = 12p$



Answer Key

Answers	Study Topics
1) c	Unit 1, Topic 1 and Unit 2, Topic 1
2) a	Unit 1, Topics 1 & 3 and Unit 2, Topic 2
3) c	Unit 1, Topic 4
4) b	Unit 1, Topic 4
5) c	Unit 1, Topic 4
6) c	Unit 1, Topic 4
7) b	Unit 1, Topic 4
8) b	Unit 1, Topic 2
9) c	Unit 2, Topic 1
10) c	Unit 1, Topic 1 and Unit 2, Topic 1
11) c	Unit 1, Topic 3
12) d	Unit 1, Topic 1
13) c	Unit 1, Topic 5
14) b	Unit 1, Topic 5
15) c	Unit 1, Topic 5
16) c	Unit 2, Topics 3 and 5
17) d	Unit 2, Topics 3 and 5
18) a	Unit 2, Topics 3 and 5
19) b	Unit 1, Topic 2
20) d	Unit 1, Topic 2
21) a	Unit 2, Topic 6
22) b	Unit 2, Topic 6
23) b	Unit 2, Topic 6
24) a	Unit 2, Topic 2
25) a	Unit 2, Topics 1 and 2
26) c	Unit 2, Topic 2



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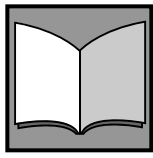
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Appendix A

The following topics from the Alberta Entrance Level Competencies are covered in **Math – Module 2 – Patterns and Relations** of this curriculum.

A. Patterns

Outcome: Express patterns, including those used in business and industry, in terms of variables, and use expressions containing variables to make predictions. (1, 2, 3, 4, 5)

1. Predict and justify possible n th values of a number pattern.
2. Interpolate and extrapolate number values from a given graph.
3. Graph relations, analyze the result and draw a conclusion from a pattern.
4. Use patterns and relations to represent simple oral and written expressions as mathematical symbols, and vice versa.

B. Problem Solving Using Patterns

Outcome: Use patterns, variables and expressions, together with their graphs, to solve problems. (1, 2, 3, 4, 5)

1. Generalize a pattern arising from a problem-solving context, using mathematical expressions and equations, and verify by substitution.
2. Substitute numbers for variables in expressions, and graph and analyze the relation.
3. Translate between an oral or written expression and an equivalent algebraic expression.

C. Design And Justify Mathematical Procedures

Outcome: Generalize, design and justify mathematical procedures, using appropriate patterns, models and technology. (1, 2, 3, 4, 5)

1. Use logic and divergent thinking to present mathematical arguments in solving problems.
2. Model situations that can be represented by first-degree expressions.
3. Write equivalent forms of algebraic expressions, or equations, with rational coefficients.

