NWT Apprenticeship Support Materials

Math

Reading Comprehension

Science

* Module 1 – Foundations
* Module 2 – Patterns and Relations
* Module 3 – Variables and Equations
* *Module 4 – Measuring Time, Shapes and Space*
* Module 5 – Special Topics

PARTNERS

Snap Lake Diamond Project De Beers

Aurora College

Northwest Territories Education, Culture and Employment

Skills Compétences Canada Northwest Territories Nunavut

Genesis Group Ltd.

Canada

Alberta Advanced Education Apprenticeship and Industry Training

First Printing: February 2003
Second Printing: March 2004

Acknowledgements:
The Apprenticeship Support Materials Project has been a true partnership. The challenge has been to develop a set of study materials, which covers every competency in the "Entrance Level Competencies for Apprenticeship Programs" developed by Alberta Advanced Education, Apprenticeship and Industry Training. These study materials although written for Northerners using northern examples, issues and problems, are appropriate for any individual entering the trades. The goal is to equip Northerners for the Trades Entrance Exam as well as to support them through the apprenticeship process.

The following partner organizations have all contributed to the development of these materials:

De Beers Canada Mining Inc. – Snap Lake Diamond Project
Government of the Northwest Territories – Education, Culture and Employment
Government of Canada – Indian and Northern Affairs Canada
Government of Canada – Human Resources Development Canada
Government of Canada – Canadian Rural Partnership
Aurora College
Skills Canada NWT/NU
The Genesis Group Ltd.

Author
Thomas O'Connor M.A., Education Consultant with the Genesis Group Ltd., is the author of the NWT Apprenticeship Support Materials.

The Partnership Steering Committee wishes to acknowledge his dedication to the goals of northern adult education.

NWT Apprenticeship Support Materials Math Module 4 Measuring Time, Shapes, and Space
ISBN : 0-7708-0118-8

Table of Contents

Module Four – Measuring Time, Shapes and Spaces

Table of Contents

Measuring Time, Shapes and Spaces

Introduction[1]

Entrance Level Competencies In Mathematics (Levels 1, 2, 3, 4, 5)

Module 1 – Foundations: Number Concepts And Operations

Module 2 – Patterns And Relations

Module 3 – Variables And Equations

Module 4 – Measuring Time, Shapes, And Spaces

Organization of Topics

The emphasis in trades is on using mathematics to solve practical problems quickly and correctly. Each topic in this curriculum guide includes:

1. Background and theory
2. Examples with explanations
3. Practice Exam Questions with answers and explanations

This curriculum guide outlines competencies, but does not provide detailed lessons, as in a textbook or GED Study Guide. If you need more instruction on a particular competency, you may find these and other textbook resources helpful. If you want to build up speed as well as accuracy in a competency you will find these additional resources helpful as a source of additional practice questions. Need to know information for the trades entrance exam is singled out for your attention by the use of text boxes and bold type.

[1] A detailed introduction with sections on self-assessment and study tips is provided with Math – Module 1 – Foundations. Please review this material to supplement the brief overview provided here.

Introduction

Examples are the focus

In this curriculum guide, examples with explanations are the primary tool used for review. Background for each competency is also given with a brief overview of what you need to know. Before any examples are given, the main ideas in each topic are explained and “need to know” information is summarized in rules and definitions.

Please Note:

When you work on an example, cover the text below with the laminated card provided so that you don’t see answers and explanations prematurely.

You may want to skip the background given on a topic and go right to the examples to see how well you do. You can always go back to the theory if you find you need it. The abbreviation “aka” (also known as) is used in the background sections to point out different, but equivalent, expressions for the same idea. For example “perpendicular aka normal” in Unit 2, Topic 1.B – Bisecting Angles and Lines.

In addition, a set of practice exam questions for Module 4 – Measuring Time, Shapes and Spaces accompanies this learning guide to enable you to assess yourself, decide what you need to study, and practice for the exam.

Unit 1

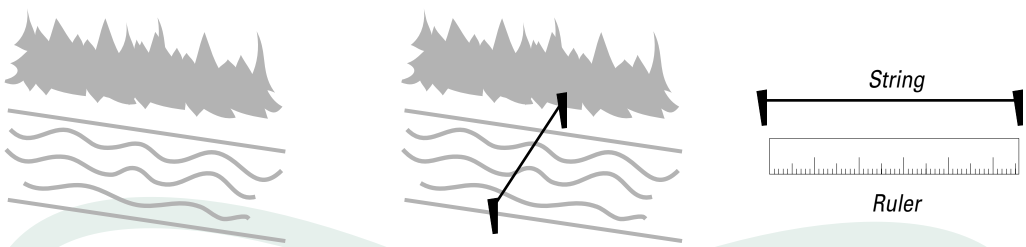
Measurement Systems

Topic 1 – Units of Measurement

Background

Numbers that measure something are used in all trades. A measurement system gives us the units that give a physical meaning to a number.

Units of measurement include feet, meters, degrees, minutes, gallons and liters. These units are the result of human decisions about what to use as standards. Nature has no units attached to itself until people decide that they want to compare one thing with another.



A River | **A String = Width of River** | **Standard Units Measure the String**

In this diagram you can see the difference between an object in nature, a river, a measurable quantity, the string stretched across the river, and a measuring tool that uses standard units to assign a number, ie a measurement, to the width of the river.

Units are chosen for convenience, for historical or traditional reasons, and for scientific practicality. For example, the inch is an arbitrary unit, approximately equal to the length of the second joint of a person's pointer finger. The metric system is practical because we use a base ten number system. The binary system is effective for electronics and computers because two-position switching (i.e. on, off) is the basic physical device behind even the most complex operations. Work in the trades and science depends on agreements about what units mean.

Unit 1 – Measurement Systems

Topic 1 – Units of Measurement

Rulers, tape measures, scales, protractors, thermometers, and clocks are used in the trades to get information about a quantity that is measurable. Rulers measure length, protractors measure angles, thermometers measure temperature, and clocks measure time. Readings from each of these measurement tools will be based on units that come from a system of measurement.

Two measurement systems are important

In the trades in Canada, **the international, or metric S.I. system** is important as well as the **Imperial system**. Time is measured in both systems by means of the same units: seconds, minutes, hours, days etc.

What you need to understand about measurement

1. **All measurements are based on a measurement system:** the S.I or international metric system (centimetres, kilograms, litres etc), and the Imperial system (feet, pounds, quarts etc.) are both used in the trades.
2. **All measurement systems have standard units** that are used to compare a measurable quantity to the number of standard units it equals. Measurements tell us "how much" and "how fast".

   Examples:

   A dog team takes 10 days to complete the Yukon quest

   A furnace increases room temperature 15 degrees in 20 minutes

   The river is 350 meters across at Fort Simpson
3. **Numbers with no units attached are "dimensionless" or "scalar"** numbers. The constants in a formula are scalar numbers. The variables in a formula have measurement units.

Examples:

1. d = vt (distance equals velocity multiplied by time)

   Each variable describes a measurable quantity that will have units attached. D could be measured in miles or kilometres, v in miles per hour or kilometres per hour.
2. A = 2h (Average adult height equals twice the average height of a four year old)

   2 is a scalar number, but A and h are variables expressed by a unit of length, for example centimetres, or inches. If we find that h = 3 feet, then A = 6 feet. **A scalar factor in a formula is also known as a constant.**

Unit 1 – Measurement Systems

Topic 1 – Units of Measurement

Optional: Some More Theory

A measurement is the result of comparing a known standard to something that has a measurable dimension. For example, every time you measure something with a ruler you are comparing a known standard (the intervals marked on the ruler) to the length of an object. You might find that the object is 6 inches long. This is the same thing as saying that the length of the object is equal to the length of six standard inches placed next to each other. A measurement is only meaningful when we know what the standard unit (for example one inch)
looks like.

> **Formulas Based on Measurements Save Work**
>
> Measurable quantities that are used in the trades include (but are not limited to) weight, height, length, width, temperature, time, distance, speed, electric current, and degrees. These measurable quantities are important inputs into formulas that give us perimeter, area, volume, density, heat capacity, electrical resistance, and many other outputs that are useful on the job.

A number that measures something will have a unit of some kind attached to it. Carpenters use a tape measure to measure inches, meters, feet, or centimetres. Cooks use a thermometer to measure degrees of temperature in Fahrenheit or Celsius degrees, and scale operators use scales to measure pounds, tons, tonnes, and kilograms.

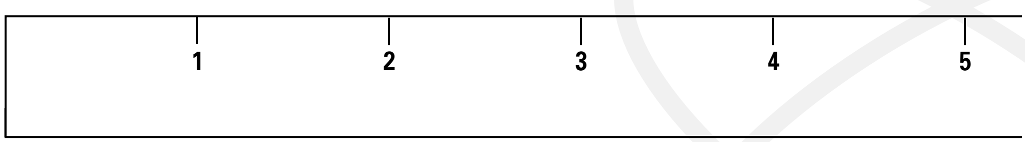
Scales and measurement

A scale is a relationship that compares two kinds of measurements. A scale is based on a unit ratio.[2]

Examples:

1. **Within the same measurement system:** the scale between feet and inches is 1 foot = 12 inches.

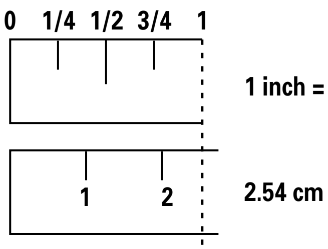
One Foot = Twelve Inches



This diagram shows five of the 12 inches in a foot. The scale in this diagram is 1:1, each space in the diagram equals the space it represents in the real world. An inch in the diagram = an inch anywhere in the world.

2 See Math – Module 1, Unit 3, Topic 8 for a review of unit ratios.

2. **Between different measurement systems:** the scale between inches and centimetres is 1 inch = 2.54 centimetres.



3. **A scale drawing or map** will indicate the relationship between a length on the drawing or map and the length it represents in the world.

   1 inch = one mile, 1 cm = .5 km, 1 inch = 100 feet, etc.

   A number without a unit description attached to it is not a physical measurement. 3 is a number, 3 inches is a measurement. 1/2 is a number, 1/2 ton is a measurement.[3]

> A unit ratio is also known as a scaling factor because we multiply or divide with it to find the relationship between a map or drawing and what it represents.

Example

A lumber grader measures length, width and thickness of the boards in a pile of lumber to find the volume in cubic feet. This will tell him how many cubes measuring one foot on a side his pile of lumber is equal to.

The same calculations could be based on metres and give a different but equivalent answer. Many trades use formulas to find costs, amounts of materials, and to estimate production.

Input	Formula	Output
L = Length = 10' W = Width = 1' T = Thickness = .25'	Volume = L x W x T	2.5 Cubic Feet
Measurement	**Operations**	**Solution**

3 Numbers can represent the total reached by counting the number of items in a group, for example "there were 13 people in the cafeteria". Counting to a total does not require that a unit be attached, but a description or the context will tell us when a number refers to people, dogs, mountains etc.

Unit 1 – Measurement Systems

Topic 1 – Units of Measurement

Measurements are inputs into formulas

Taking measurements allows us to solve problems by using formulas. In sections two and three of this curriculum measurable quantities are used in many of the examples and problems. For example, pounds of an ingredient in a recipe, the width of a rafter for a given span, and the time needed for a trip when the velocity and distance are known. The input numbers in the tables, graphs, and relations studied in sections two and three often result from measurements in applied situations.

Use a Calculator – And Know the Units Involved

Many calculators have formulas, or can have formulas programmed into them, that are useful in the trades. Effective use of these formulas requires inputs based on a correct choice of units.

Examples:

1. An electrician has programmed his calculator to find resistance in ohms when he enters voltage in volts, and current in amps.
2. A hairdresser has programmed her calculator to find the amount of shampoo to order in litres when she enters (inputs) the number of customers she estimates that she will serve in a month.

Unit 1 – Measurement Systems

UNIT 1

Topic 1 – Practice Exam Questions

Question 1

A drawing of a house floor plan has a scale of 1 inch = 3 feet. If the length of the living room floor is 12 feet, how long will the line on the drawing be that represents this floor?

a) 6 inches
b) 3 inches
c) 3 feet
d) 4 inches

Answer: d

Explanation
The scaling factor is one inch for every three feet of floor. We divide the total floor length by 3 to find the number of inches that will represent it in the drawing.
$12 \div 3 = 4$.

Question 2

Which of these measurements are not made with the same measurement system?

a) 12 inches and one foot
b) 3 liters and 4 gallons
c) 1 meter and 10 centimeters
d) 4 cubic yards and 1 ton

Answer: b

Explanation
Liters are metric (S.I. system) units of volume, and gallons are imperial units of volume.

Unit 1 – Measurement Systems

Topic 1 – Practice Exam Questions

Question 3

Which answer goes from units to subunits in the same measurement system?

a) inches, centimetres, millimetres
b) decimetres, centimetres, millimetres
c) gallons, quarts, liters
d) hours, seconds, minutes

Answer: b

Explanation
Read from left to right. Centimetres are a subunit of decimetres (1dm = 10cm), and millimetres are a subunit of centimetres (1cm = 10mm). The other choices either mix measurement systems (choices a and c) or put the order from subunit to unit (choice d, minutes are not a subunit of seconds.)

Question 4

If you want to use a formula that has constants and variables in it, what will you measure?

a) the variables
b) the constants
c) the measurement system
d) the unknown

Answer: a

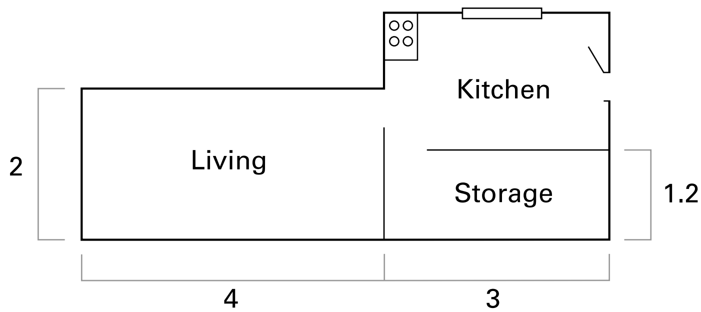
Explanation
In a formula the variables are quantities that you can measure directly, or calculate by using the formula after one or more variables are replaced with measurements. For example in the formula for the area of a triangle, $A = \frac{1}{2}bh$, there are three variables and one constant ($\frac{1}{2}$). We measure b (the base) and h (the height) in linear units (feet, centimetres etc.) and then use the formula to find A in square units (square feet, square centimetres etc).

Topic 1 – Practice Exam Questions

Question 5

What is the actual length of the living room in this floor plan:

Scale 1 cm = 1 m



a) 8 metres
b) 8 feet
c) 12 feet
d) 4 metres

Answer: d

Explanation
Each cm in the drawing represents one meter in the world. The living room measures 4 cm in the drawing. Multiply 4 x 1 to find 4 meters as the actual length of the room.

Unit 1 – Measurement Systems

Topic 2 – Units and Subunits

Background

Two measurement systems are commonly used today: the **S.I, or international system** based on metric units, (litres, centimetres, grams etc.) and the **Imperial or British system** based on inches, pounds, ounces etc. You need to know how to convert units into subunits and subunits into units in both systems, for example feet into inches, inches into feet, and meters into centimetres and centimeters into meters.[4]

> **Apples and Oranges…**
>
> Notice that we cannot compare units that refer to different types of quantities. For example, units of time measure something that is different from units of distance. Units of time cannot be converted into units of distance or volume, currency, or rates. Units of time can, however, be related to these other units in a formula. For example, time equals distance divided by velocity in the formula: t = d/v

Units are what we measure with. Units include inches and feet, yards and miles for distance in the imperial system, and centimetres, meters, and kilometres for distance in the metric or S.I. (System International) system. Units also refer to the basis for measuring time: minutes and seconds, hours, and days. Other units used for measurements in the trades include units of weight (pounds, kilograms), units of volume, (quarts, litres) and units of currency (dollars, cents).

There are many kinds of units, and we need to relate them to each other when they refer to the same type of quantity. For example, time units can be changed into other units of time, seconds into minutes, and hours into minutes. Seconds are subunits of minutes and minutes are subunits of hours.

Another example:

Units of volume can be changed into each other. Quarts into pints, cubic yards into cubic feet, and cubic kilometres into cubic metres. The relationship between units and subunits in the same measurement system is described by a **scaling, or exchange, factor.**[5]

4 For clarity we refer to **conversion factors** when going **between** measurement systems, and **scaling factors** when converting within a measurement system. Both are referred to in this curriculum as **exchange factors.**

5 In this curriculum scaling factor and exchange factor mean the same thing.

Unit 1 – Measurement Systems

Topic 2 – Units and Subunits

The exchange factor for minutes and seconds is 1 minute = 60 seconds. The exchange factor for money is 1 dollar = 100 cents. These exchange factors are also **unit ratios**[6] that tell us what one unit of some measured quantity is worth in comparison to other units in the same measurement system. The scaling factor can be used to go either from unit to subunit, or from subunit to unit, “ = ” works both ways.

Telling units apart from subunits

A unit of measurement can be both a unit and a subunit in a measurement system. For example, notice that minutes are subunits of hours, but also that minutes are units of which seconds are subunits. A yard is a subunit of a mile, and a foot is a subunit of a yard. When we exchange a yard for the number of feet it equals, we are going from a unit to a subunit of itself (whole to parts). When we exchange a yard for the fraction of a mile it equals, we are going from a subunit to a unit (parts to whole).

	UNIT	SUBUNIT
TIME	Hour	Minutes, seconds
	Minute	Second
DISTANCE	Mile	Yards, feet, inches
	Yard	Feet, inches
	Foot	Inches
	Kilometre	Metre, centimetre, millimetre
	Metre	Centimetre, millimetre
	Centimetre	Millimetre

6 See Math – Module 1, Unit 3, Topic 8 – Ratios and Proportions.

Unit 1 – Measurement Systems

Topic 2 – Units and Subunits

Examples

The unit ratios (i.e. scaling factors) for the following quantities (time and length) are used in many trades:

1. **Time:** scaled from units to subunits

   1 day (unit) = 24 hours (hours are subunits of days)

   1 hour (unit) = 60 minutes (minutes are subunits of hours and days)

   1 minute (unit) = 60 seconds (seconds are subunits of minutes, hours, and days)

Some exchanges take several steps

Example
To change days into seconds, first convert days into hours, then hours into minutes, then minutes into seconds.

2 days = 2 x 24 = 48 hours = 60 x 48 = 2880 minutes = 60 x 2880 = 172,800 seconds

To change seconds into days, do the reverse, and divide seconds by 60 to get the equivalent number of minutes. Then divide the minutes by 60 to get the number of hours, and finally divide the number of hours by 24 to get the number of days.

2. **Imperial Lengths:** scaled from subunits to units

   12 inches (subunit) = 1 foot (a foot is a unit based on subunits of inches)

   3 feet (subunit) = 1 yard (a yard is a unit based on subunits of feet and inches)

   1760 yards (subunit) = 1 mile (a mile is a unit based on subunits of yards, feet, and inches)

3. **Metric lengths:** scaled from units to subunits

   1 kilometre (unit) = 1000 metres (metres are subunits of kilometres)

   1 metre (unit) = 100 centimetres (centimetres are subunits of metres)

   1 centimetre (unit) = 10 millimetres (millimetres are subunits of centimetres)

Unit 1 – Measurement Systems

Topic 2 – Units and Subunits

An exchange factor describes the relationship between 1 unit and 1 subunit and vice versa. These are also **unit ratios** because they express the rate of exchange between one 1 of something (time, length, volume etc.) and an equivalent number of subunits, and vice versa.

To convert units within a measurement system means to change 1 measurement of a quantity, for example weight in pounds, into something of equal value within that system, for example weight in ounces. We do this with multiplication or division and a scaling factor.

You need to know the relationship that defines the exchange of smaller units for larger ones and vice versa in the same system of measurement. With this information you can **divide to change from smaller units to larger, and you can multiply to change from larger units to smaller units.**

An example: Inches and Feet

The scaling, or exchange, factor between feet and inches is 12. 12 inches = 1 foot. The inch is the smaller unit. You must **divide** inches by 12 to get the number of feet a given number of inches is equal to. 144 inches = 12 feet. If you want to change from feet to inches, going from the larger unit to the smaller, you multiply the number of feet by 12 to get the equivalent number of inches.
12 feet = 144 inches.

What you need to know:

Going from units to subunits (from big to small: multiply)

1. Find the exchange relationship between the units and subunits.

   Examples:
   1 foot = 12 inches.
   1 gallon = 4 quarts

2. Multiply the units by the exchange factor

   Examples:
   12 feet = 12 times 12 inches = 144 inches
   8 gallons = 8 times 4 quarts = 32 quarts

Going from subunits to units (from small to big: divide)

1. Find the exchange relationship between the subunits and units

   Examples:
   12 inches = 1 foot
   1 quart = .25 gallons

2. Divide the subunits by the exchange factor

   Examples:
   144 inches ÷ 12 = 12 feet
   32 quarts ÷ .25 gallons = 8 gallons

Unit 1 – Measurement Systems

Topic 2 – Units and Subunits

Tip…

Write down the name of the units you are going from with an arrow pointing to the name of the units you are going to. Then write "big to small" (subunit to unit), or "small to big" (unit to subunit), to help you plan the next step.

Examples
Kilogram → gram (big to small) (multiply)

Centimetre → metre (small to big) (divide)

Remember…

Only convert the units or subunits that are not already in the desired form.

Example

Change 3 hours and **10 minutes** into minutes.

Step 1) 3 hours = 3 x 60 minutes = 180 minutes.

Step 2) add the 10 minutes already given in the desired subunit
180 + 10 = 190 minutes.

Unit 1 – Measurement Systems

Topic 2 – Practice Exam Questions

Exchanging Units for Subunits and Vice Versa

Question 1

What is the dollar value of 525 dimes?

a) $52.50
b) $5.20
c) $525.00
d) $1,525.00

BSM
Big → **S**mall
More: **M**ultiply

Answer: a

Explanation
Talk yourself through it this way:

1. What kind of quantity is involved?
   Answer: Canadian currency
2. Are we going from big to small or small to big?
   Answer: we are going from subunits of dimes to units of dollars, small to big.
3. What is the exchange factor?
   10 dimes = 1 dollar
4. Divide
   525 dimes ÷ 10 = $52.50

SBD
Small → **B**ig
Divide

Question 2

Convert 3 lbs. 3 ounces into ounces

a) 48 ounces
b) 92 ounces
c) 51 ounces
d) 1lb. and 35 ounces

Answer: c

Topic 2 – Practice Exam Questions

Explanation

1. What kind of quantity is involved?
   Answer: Imperial weight measurement: ounces
2. Are we going from big to small or small to big?
   Answer: we are going from units of pounds and ounces to subunits of ounces, big to small: multiply.
3. What is the exchange factor?
   1 pound = 16 ounces
4. Multiply the number of pounds by 16
   3 pounds x 16 ounces = 48 ounces
5. Add the 3 ounces that didn't need to be converted
   48 + 3 = 51 ounces

Question 3

Convert 69 miles into yards

a) 351,440 yards
b) 181,440
c) 15,514 yards
d) 121,440 yards

Answer: d

Explanation

1. Quantity: Imperial distance measurement: miles
2. Going from miles to yards, big to small, multiply
3. Exchange factor: 1 mile = 1760 yards
4. Multiply the number of miles by 1760

   69 x 1760 = 121,440 yards

Unit 1 – Measurement Systems

Topic 2 – Practice Exam Questions

Question 4

What is the number of metres in 341 centimetres?

a) 34.1 metres
b) 3140 metres
c) 3.41 metres
d) 124.1 metres

Answer: c

Explanation

1. Quantity: Metric length measurement: metres and centimetres
2. Going from centimetres to metres, small to big: divide
3. Exchange factor: 100 centimetres = 1 metre
4. Divide the number of centimetres by 100
   341/100 = 3.41 metres

Question 5

How many weeks are in 223 days?

a) 31 weeks
b) 52 weeks
c) 18 weeks
d) 42 weeks

Answer: a

Explanation

1. Quantity: Time, weeks and days
2. Going from days to weeks, small to big: divide
3. Exchange factor: 7 days = 1 week
4. Divide the number of days by 7
   223/7 = 31 6/7 weeks = 31 weeks and 6 days (you may see that 6/7 is equal to 6 days, or look at question 6)

Topic 2 – Practice Exam Questions

Question 6

How many days are 6/7 of a week?

a) 5 days
b) 4 days
c) 13 days
d) 6 days

Answer: d

Explanation

1. Quantity: Time, days and weeks
2. Going from weeks to days, big to small, multiply
3. Exchange factor: 1 week = 7 days
4. Multiply the number of weeks by 7
   7/1 x 6/7 weeks = 6 days

Unit 1 – Measurement Systems

Topic 3 – Converting Between Measurement Systems

Background

In the trades you will need to know how to convert quantities from one system of measurement to equivalent amounts in another system by multiplying or dividing with conversion factors. You will convert metres to feet, ounces to litres, pounds to kilograms and vice versa. If you use a calculator to do this, the conversion factors are stored for you, and you will need to know which numbers to input and how to describe their relationship to each other as well as the measurement system involved.

Each metric unit has a corresponding imperial unit it can be converted into and vice versa.

Conversion factors are the exchange factors for the units you are changing into each other. Now instead of changing between units and subunits within the same system, you will be changing between systems that use different standards for measuring the same quantities.

In every case, identify the conversion factor that changes one of the units you are **going from** into one of the units you are going to. Next multiply this factor times the number of units you are going from to get the equivalent number of units in the other system that you are going to. Your exam will include a handout of formulas and conversions similar to this example:

Mathematical Formulas
You may make use of the following formulas when answering certain questions in this examination.

- $\pi = 3.14$
- Circumference of a circle = π D
- Area of a rectangle = Length x Width
- Area of a circle = πr^2
- Area of a triangle = $\dfrac{\text{Altitude x Base}}{2}$
- Volume of a cylinder = $\pi r^2 h$
- Volume of a cube = Length x Width x Height

Metric Conversions

Distance

Imperial		**Metric**	**Metric**		**Imperial**
1 inch	=	2.540 centimetre	1 centimetre	=	0.3937 inch
1 foot	=	0.3048 metre	1 metre	=	3.281 feet
1 yard	=	0.9144 metre	1 metre	=	1.094 yards
1 rod	=	5.029 metres	1 metre	=	0.20 rods
1 mile	=	1.609 kilometres	1 kilometre	=	00.6214 mile

Capacity

Imperial			**US**		
1 pint	=	0.568 litres	1 pint (U.S.)	=	0.473 litre
1 gallon	=	4.546 litre	1 quart (U.S.)=		0.946 litre
1 bushel	=	36.369 litres	1 gallon	=	3.785 litres
1 fluid oz.	=	28.41 ml	1 barrel oil	=	158.99 litres
1 quart	=	1.137 litres	1 cup - 8 fl. oz	=	227 mil

Unit 1 – Measurement Systems

Topic 3 – Converting Between Measurement Systems

Metric

1 litre	=	1.76 pints
1 litre	=	0.220 gallon
1 litre	=	.88 quart

US

1 tablespoon	=	14.21 ml
1 teaspoon	=	4.74 ml

Weight
Imperial

1 ounce (troy)	=	31.103 grams
1 ounce (avoir)	=	28.350 grams
1 pound (troy)	=	373.242 grams
1 pound (avoir)	=	453.592 grams
1 ton (short)	=	453.592 grams
2000 lb	=	0.907 tonne*

**1 tonne = 1000 kilograms*

Metric

1 gram	=	0.032 ounce (troy)
1 gram	=	0.035 ounce (avoit)
1 kilogram	=	2.679 pounds (troy)
1 kilogram	=	2.205 pounds (avoir)
1 tonne	=	1.102 ton (short)

Examples

1. How many litres are in 12 imperial gallons?
    1. **Write down your target units,** "litres", as a reminder of where you are headed. We want our answer in litres, gallons → litres
    2. **What is the exchange factor** (aka "conversion factor") between 1 gallon and 1 litre?
       1 gallon = 4.546 litres
    3. **Multiply** 12 gallons times 4.546 litres to find the number of litres in 12 gallons.
    4. **Solution:** 54.552 litres, rounding off to the nearest hundredth = 54.55 litres.

2. How many miles are in 603 kilometres?
    1. **Write down** "from kilometres to miles" as a reminder of where you are headed. We want our answer in miles. Kilometres ➔ miles
    2. **What is the exchange factor** between 1 kilometre and 1 mile?
       Answer: 1 kilometre = 0.6214 miles
    3. **Multiply** 603 x 0.6214 miles to find the number of miles
    4. **Solution**: 603 kilometres = 374.70 miles

3. Change 350 grams into pounds (avoir).
    1. We want our answer in pounds (avoir), **write down** "pounds (avoir)"
    2. 1 gram = 0.035 ounces (avoir) (metric to imperial), and,
       16 ounces = 1 pound (subunit into unit)
       This is a two step problem: first convert grams into the Imperial subunit of weight (ounces) that we have a conversion factor for. Next, change the subunit of ounces to the unit of pounds in the imperial system.
    3. Multiply 350 grams x 0.035 = 12.25 ounces (grams ➔ ounces)
    4. We have less than 1 pound or 12.25/16 = .7656 pounds (ounces ➔ pounds).

Unit 1 – Measurement Systems

Topic 3 – Practice Exam Questions

Question 1

How many kilometres are in 43 miles?

a) 39.2 km
b) 59.2 km
c) 69.1 km
d) 59.1 km

Answer: c

Explanation
1. We want our answer in kilometres.
2. 1 mile = 1.609 kilometres
3. Multiply 43 x 1.609 = 69.187 kilometres

Question 2

How many metres are in 30 yards?

a) 27.4 m
b) 180 m
c) 90.4 m
d) 210.4 m

Answer: a

Explanation
1. We want our answer in metres
2. 1 yard = 0.9144 metres
3. Multiply 30 x 0.9144 = 27.43 metres

Unit 1 – Measurement Systems

Topic 4 – How to Read a Metric Rule

Background

A metric ruler has divisions based on powers of ten. Metric rulers can be tape measures, metre sticks, or shorter rulers. A ruler is a number line. Measuring with the ruler amounts to adding and subtracting numbers. Review the Foundations section for a discussion of number lines and basic operations including operations with fractions on an imperial ruler (adding and subtracting fractions of inches.)

The lines on the ruler mark units with longer vertical lines than the subunits. A metric rule marks three subunits: decimetres = tenths of a metre, centimetres = hundredths of a metre, and millimetres = thousandths of a metre.

What You Need to Know About Metric Rulers:

1 metre = 10 decimetres

1 decimetre = 10 centimetres

1 centimetre = 10 millimetres.

The ruler will have short lines marking each millimetre, slightly longer lines to mark every fifth millimetre, and slightly longer lines again to mark every centimetre. A centimetre mark occurs every tenth millimetre. A number is given for every centimetre on the ruler or tape measure.

Unit 1 – Measurement Systems

Topic 4 – How to Read a Metric Rule

A number on a metric ruler can be expressed in centimetres and millimetres. **A decimal point can be used to show the relationship between units and subunits.** 3.2 cm = 3 cm and 2 millimetres. 1.52 decimetres (dm) = 15.2 centimetres, or 152 mm.

Example

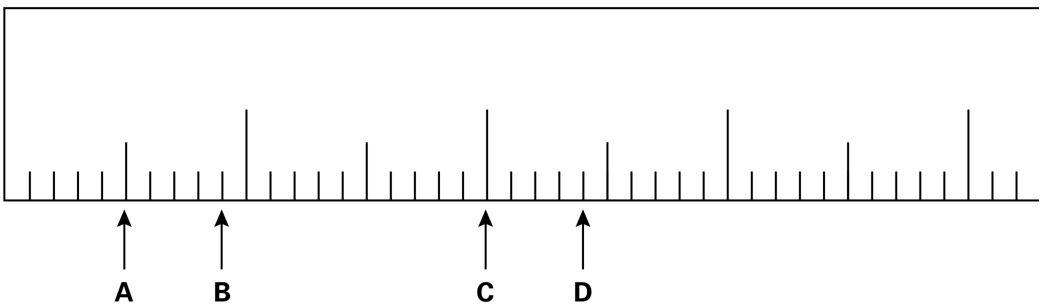
1) Find the following measurements in centimetres and millimetres.

   A = 5 mm

   B = 9 mm

   C = 2 cm

   D = 2 cm + 4 mm



Unit 1 – Measurement Systems

Topic 4 – Practice Exam Questions

Question 1

Which of the following is not a unit in the S.I. system of measurement?

a) Kilograms
b) Ounces
c) Milligrams
d) Litres

Answer: b

Explanation
Ounces are a subunit of pounds in the Imperial system.

Question 2

Convert 12 inches to centimetres. Round to the nearest tenth.

a) 4.7 cm
b) 14 cm
c) 30.5 cm
d) 10 cm

Answer: c

Explanation
The answer requires a conversion between measurement systems, ie., from imperial to metric. The target unit is centimetres. The exchange factor (aka conversion factor) is 1 inch = 2.54 cm. Multiply 12 x 2.54 to find 30.48. This rounds up to 30.5.

Question 3

How many feet are in 3.2 miles?

a) 148,000
b) 23,896
c) 14,800
d) 16,896

Answer: d

Explanation
This problem requires an exchange between a subunit (feet) and a unit (miles) in the imperial system of measurement. We are going from unit to subunit. We will multiply 3.2 miles by the exchange factor: 1 mile = 5280 feet. 3.2 x 5280 = 16,896 feet.

Unit 1 – Measurement Systems

Topic 4 – Practice Exam Questions

Question 4

How many seconds are in 1.5 days?

a) 36,000
b) 360,000
c) 129,600
d) 132,600

Answer: c

Explanation
This is a three step problem. We convert from unit to subunit three times. First days into hours, 1 day = 24 hours, then hours into minutes, 1 hour has 60 minutes, and finally minutes into seconds, 1 minute has 60 seconds. 1.5 days has $24 + (\frac{1}{2} \times 24) = 36$ hours. 36 hours has 36 x 60 minutes = 2,160 minutes. 2,160 minutes has 60 x 2,160 seconds = 129,600 seconds.

Question 5

How many centimetres are in 2.5 decimetres?

a) 25
b) 250
c) .25
d) 1.25

Answer: a

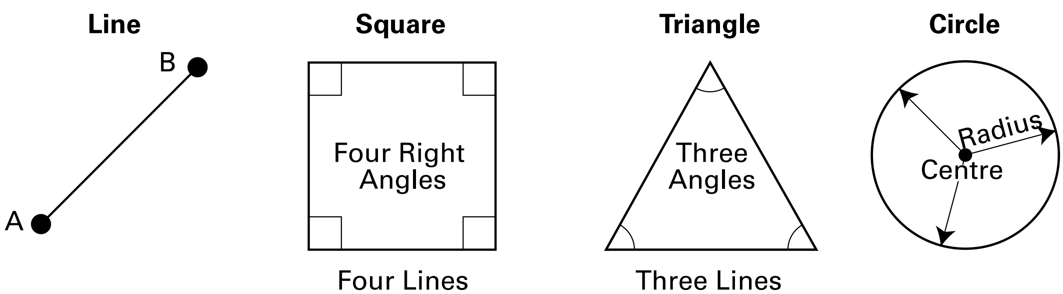
Explanation
The relationship between centimetres and decimetres is:
1 decimetre = 10 centimetres. 10 x 2.5 = 25 centimetres in 2.5 decimetres.

Unit 2

Measuring Shapes

Background

Circles, squares, and triangles are basic shapes that are used in the trades. They can be described by measurements in two dimensions, for example length and width. They can be drawn on a plane, for example a piece of paper.



Basic shapes are defined by lines and angles. A line is a series of points that are next to each other. Every line has two endpoints, that can be labeled by letters.[7] A line can be measured. A square is described by the length of one side of a four sided figure, and the knowledge that all four sides are equal to each other. A triangle is defined as a plane figure with three sides and three angles. Triangles can have different shapes that depend on the length of the sides and the size of the angles. A circle is defined by the curved line that is the same distance from the center at all points.

> A **plane** contains all of the points on a flat surface.

> A **dimension** is a measurable quantity.
>
> Length, width, and height are dimensions that measure shapes.

Problems in the trades use **geometric relationships** to complete measurements, and to calculate perimeter, area, and volume.

[7] Some texts place a horizontal line above the letters that label the endpoints of a line. In this curriculum the horizontal line is omitted. AB means the line between point A and point B.

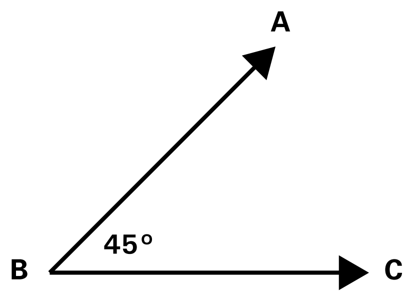
Unit 2 – Measuring Shapes

Topic 1 – Lines and Angles

Geometry is a Greek word meaning "the measurement of the earth". Measurement is used in every trade. Geometry deals with shapes and their properties. On the trades entrance exam, four sided figures, the circle and the triangle are the most important shapes to understand.

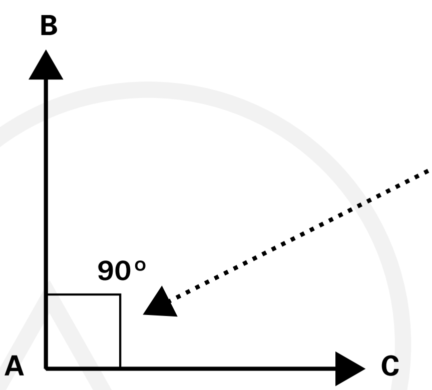
Angles are measured in degrees. The unit of measurement for an angle is the degree. An angle is measured at the **vertex**, the point where the rays of the angle meet. The rays of an angle are lines that can be extended without limit.[8] **As the rays extend, the distance between them increases, but the angle between them doesn't change.**

Angle ABC is written as ∠ABC or ∠CBA. The middle letter, B, in this angle, names the point at the vertex. Notice that the angle of 45 degrees won't change when the rays, BA, and BC are made longer. Every angle can be described by three letters with the vertex point in the middle. The outer letters can be switched and still describe the same angle. An angle can also be labeled by a single letter or number on a diagram.



Any choice of three letters can be used to label an angle, but the middle letter will always name the vertex.

∠ABC = ∠CBA It doesn't matter which ray is named first, but the middle letter must always label the vertex.



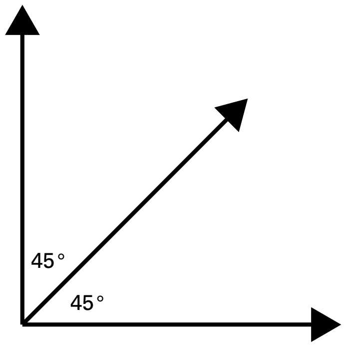
This symbol indicates that the angle is a right angle.

[8] **Angles that have equal measures are congruent angles.** The symbol ≅ means "is congruent to". Congruence means that two separate shapes share the same measure of angles or sides. The use of the "=" sign is often used in the trades to mean "equal in size", but in pure geometry the idea of congruence is separated from the idea of equality. In geometry, equality between two shapes would mean that they are the same, or identical, object. This is why an angle and the size of an angle are treated as separate ideas. Geometry texts use, "the measure of an angle" to refer to the size of an angle. In line with the practical purpose of the trades, the name of an angle will also refer to its size unless otherwise noted in this curriculum.

Unit 2 – Measuring Shapes

Topic 1 – Lines and Angles

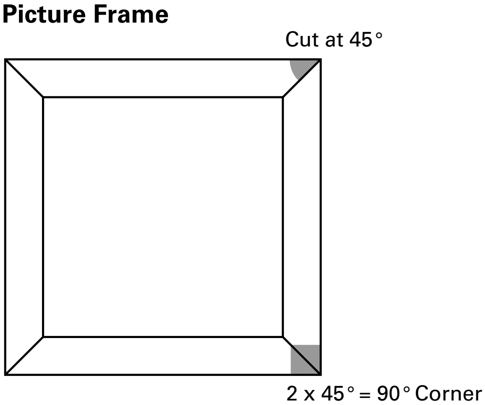
A right angle is also the sum of two 45 degree angles:



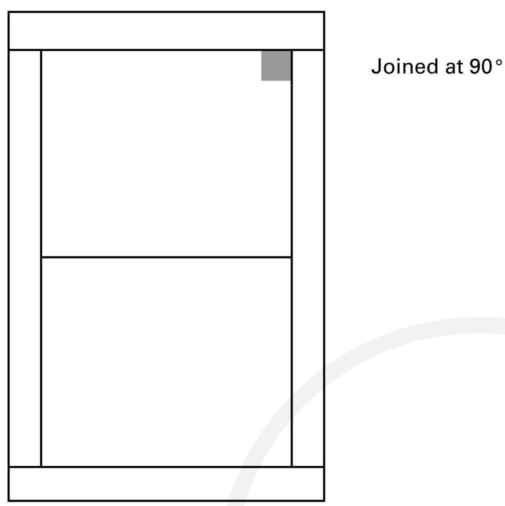
A 45 degree angle is half as big as a right angle. The corners of a window or picture frame are often joined at a 45 degree angle. A right angle is equal to the sum of two 45 degree angles.

In carpentry, corners meet at a 90 degree angle, but they can be cut on a diagonal that divides the corner into two 45 degree angles next to each other as shown in the picture frame.

Picture Frame



Window Frame



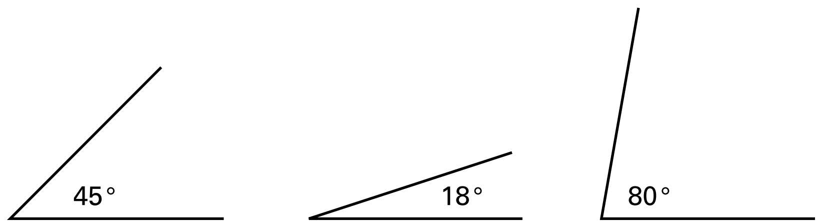
Unit 2 – Measuring Shapes

Topic 1 – Lines and Angles

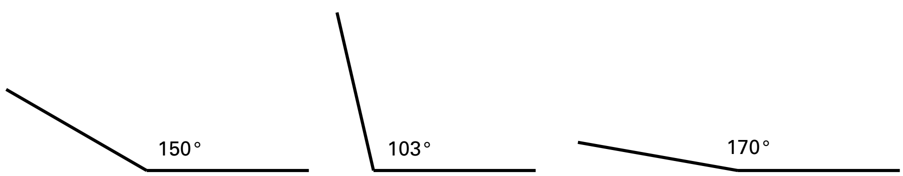
Angles you need to know

A right angle measures 90 degrees, and a straight angle measures 180 degrees. Other angles are described by the range of values that they can have:

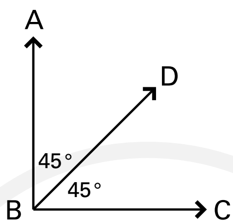
1. Acute angles are less than 90 degrees.



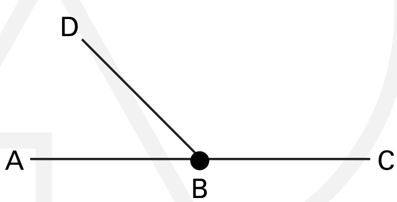
2. Obtuse angles are more than 90 degrees but less than 180 degrees.



When two angles share a vertex and one ray in common they are called **adjacent angles**.



Angle ABD is adjacent to angle DBC. The line DB is a bisector of right angle ABC. These angles are also **complementary**. Adjacent angles are called complementary when they add up to 90 degrees, and **supplementary** when they add up to 180 degrees.

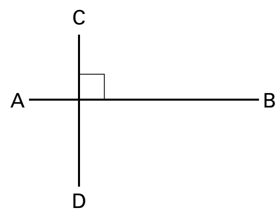


Angle ABD is adjacent to angle DBC, and vice versa. They share a vertex at point B. They are supplementary because they add up to a straight angle of 180 degrees.

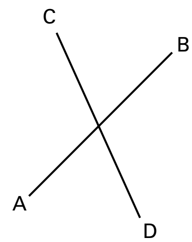
Unit 2 – Measuring Shapes

Topic 1 – Lines and Angles

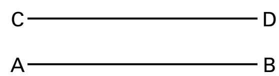
Important lines: perpendicular, intersecting, and parallel



AB is perpendicular to CD and vice versa

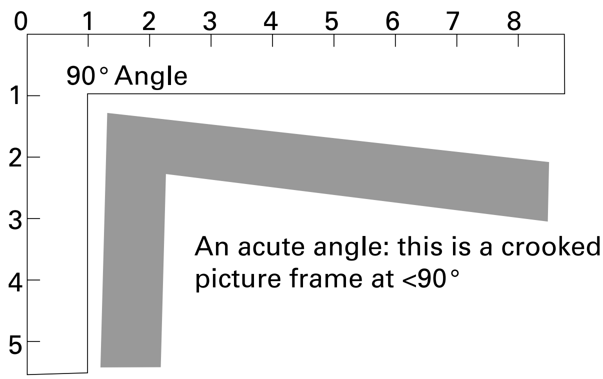


CD intersects AB and vice versa

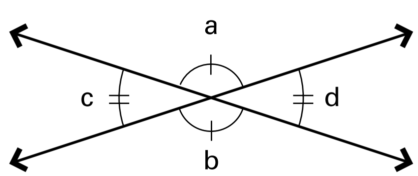


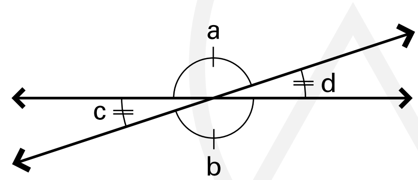
CD is parallel to AB and vice versa

When two lines cross each other (aka “intersect”) at 90 degrees, they are called **perpendicular** lines. The symbol ⊥ is used to show that two lines meet at a right angle. The perpendicular to a given line can also be called the normal line.[9] In the trades keeping things “straight, level and square” requires proof that things meet at right angles to each other. **There is only one line that will be perpendicular to a given point on a line**. Carpenters use a square to find perpendicular lines.



When two lines cross each other, they **intersect**. The angles that are formed on opposite “sides” of the intersection are equal in size to each other. The angles across from each other are equal. The angles can vary when lines intersect, but the relationship of equality between opposite angles won’t change.





Angle a = angle b, and angle c = angle d in both diagrams. When you see intersecting lines, label the four angles and note these equalities.

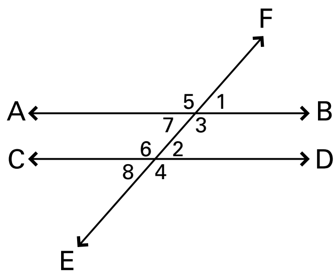
9 In physics and engineering the perpendicular can also be called an orthogonal line.

Unit 2 – Measuring Shapes

Topic 1 – Practice Exam Questions

Parallel Lines

When two lines in the same plane can never intersect, they are **parallel** to each other. When two lines are parallel the symbol II is used. A **transversal** is a line that crosses two or more parallel lines.



Line EF is a transversal and AB, CD

What you need to know about parallel lines and transversals:

1. **The corresponding angles** 1 and 2, 3 and 4, 5 and 6, 7 and 8 will be equal when a transversal crosses two parallel lines. When we know that any of these pairs are equal, we can prove that the lines being crossed are parallel.
2. Angles on opposite sides of the transversal and between the parallel lines are called **alternate interior angles**. There are two pairs of alternate interior angles: 7 and 2, 3 and 6. When we know that two alternate interior angles are equal we can conclude that the lines crossed by the transversal are parallel.
3. **Interior angles on the same side of the transversal** are 3 and 2, 7 and 6.
4. **Exterior angles on the same side of the transversal** are 1 and 4, 5 and 8.
5. **Vertically opposite angles are equal** to each other: 5 = 3, 1 = 7, 6 = 4, 2 = 8.
6. **Adjacent angles are supplementary**: The sum of the following pairs equal 180 degrees: 1 and 5, 7 and 3, 6 and 2, 8 and 4.

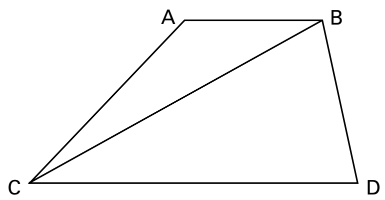
Unit 2 – Measuring Shapes

UNIT 2

Topic 1 – Practice Exam Questions

Question 1

Which of the following angles is obtuse?



CB is a diagonal
AB II CD

a) angle BCD
b) angle DCB
c) angle CAB
d) angle CDB

Answer: c

Explanation
An obtuse angle can have a range of values greater than 90 degrees but less than 180 degrees. Choices a, b, and d are less than a right angle. Only choice c is greater than 90 degrees but less than 180 degrees.

Question 2

When two lines intersect what can be said about the angles that equal each other (ie. Have the same measure)?

a) The angles next to each other are equal.
b) The angles across from each other are equal.
c) The angles that are adjacent are complementary.
d) The angles that are greater than 90 degrees are equal.

Answer: b

Explanation
When lines intersect they form four angles. The sizes depend on how "slanted" the lines are, but the angles across from each other will always be equal.

Question 3

What is the size of the angle that bisects a 72 degree angle?

a) Can't tell without a protractor or compass.
b) 36 degrees
c) 144 degrees
d) 12 degrees

Answer: b

Explanation
Divide 72 degrees in half to find the answer.

Unit 2 – Measuring Shapes

Topic 1 – Practice Exam Questions

Question 4

How many lines are perpendicular to each point on a line?

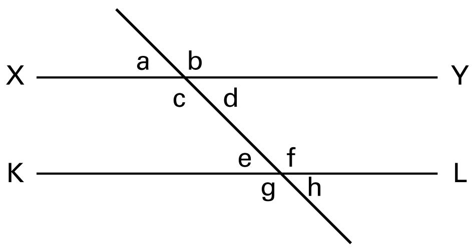
a) As many as you wish.
b) Only one.
c) Two intersecting lines.
d) A pair of parallels.

Answer: b

Explanation
A perpendicular crosses a given point at an angle of 90 degrees. There is only one line that can do this for each point on a line.

Question 5

Identify the alternate interior angles between the transversal and two parallel lines:



a) e and d, c and f
b) b and c, f and g
c) b and d, f and h
d) b and c, c and f

Answer: a

Explanation
Refer to the discussion and diagram of transversals and parallel lines given earlier. Interior means the space between the parallel lines, and alternate means on opposite sides of the transversal.

Unit 2 – Measuring Shapes

Topic 1 – Practice Exam Questions

Question 6

Which angles are supplementary?

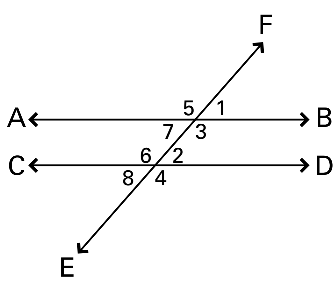
a) A right angle and a 90 degree angle.
b) 1 degree and 180 degrees.
c) 45 degrees and 145 degrees.
d) 60 degrees and 30 degrees.

Answer: a

Explanation
Any two angles that add up to 180 degrees are supplementary. A right angle = 90 degrees. Choice a adds two right angles to get a straight angle of 180 degrees.

Question 7

If AB is parallel to CD which angles will not be equal to each other?



a) 1 and 2
b) 1 and 3
c) 5 and 6
d) 8 and 7

Answer: b

Explanation
When a transversal crosses a pair of parallel lines, the corresponding angles are equal. Choice b describes supplementary angles and they are not equal unless the transversal is a perpendicular, which we cannot assume is the case based on the problem. Angles 1 and 3 do not correspond because they do not have locations that are in the same relationship to the transversal as the other pairs.

Topic 1 – Practice Exam Questions

Question 8

How can a contractor prove that the forms on opposite sides of a foundation are parallel to each other?

a) Place a straight edge that intersects both sides and show one pair of opposite angles are equal.
b) Place a straight edge that intersects both sides and show that no pairs of corresponding angles are equal.
c) Place a straight edge that intersects both sides and show that two alternate interior angles are equal.
d) Place a straight edge that intersects both sides and show that the sum of the exterior angles on the same side of the transversal is 180 degrees.

Answer: c

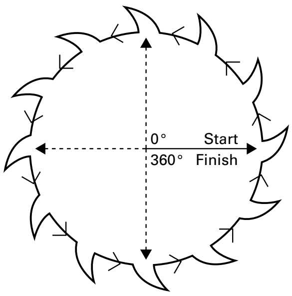
Explanation
Refer to the diagram used in question 7 and note that angles on opposite sides of the transversal and between the parallel lines are called alternate interior angles. There are two pairs of **alternate interior angles**: 7 and 2, 3 and 6 in the diagram. When we know that any two alternate interior angles are equal we can conclude that the lines crossed by the transversal are parallel. This will prove that the contractor's forms are parallel, or an equal distance apart at all points.

Unit 2 – Measuring Shapes

Topic 1.A – Angles and Circles: How to Use a Protractor

There are 360 degrees in a circle. If you picture a ray moving anticlockwise, it will complete a circle of 360 degrees when it goes around one time and returns to its starting position. This is true for any circle, wheel, pulley, gear or propeller. A straight angle is halfway around a circle. Notice that the diameter of a circle is a line that makes a straight angle with its vertex at the center of the circle. Every time a circular saw blade rotates once, it describes a circle and goes through 360 degrees.

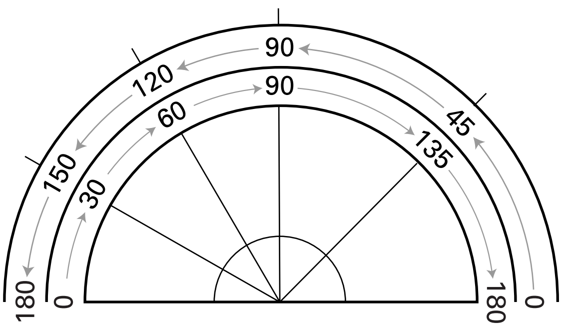
A protractor measures any of the angles in a half circle.



One Rotation = 360°

A protractor looks like this:

A protractor measures 180 degrees in a semicircle. The centre of the base is placed at the vertex of an angle with the edge on one ray. Angles are measured by looking

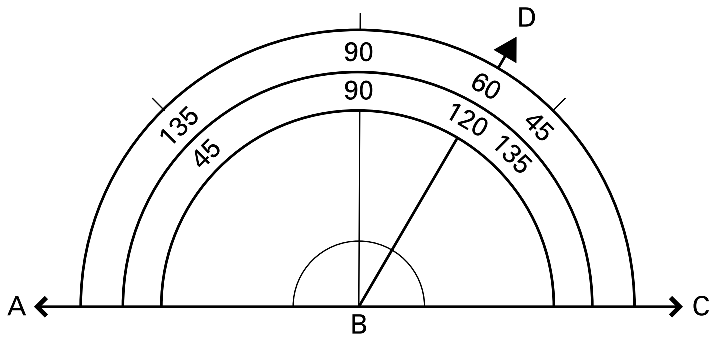


where the other line forming the angle crosses the protractor scale. This number is the number of degrees that measures the size of the angle. It is often useful to imagine the vertex of an angle as the center of a circle.

A protractor has two scales: an outer scale running anticlockwise from 0 to 180 (from right to left), and an inner scale running clockwise from 0 to 180 (from left to right). The sum of the angles measured by the same point on both scales will always = 180 degrees. For example, the line perpendicular to the base makes a ninety degree angle on both scales, and their sum is 180 degrees.

Unit 2 – Measuring Shapes

Topic 1.A – Angles and Circles: How to Use a Protractor



You can see that point B is the vertex shared by two angles that **supplement** each other because they add up to 180 degrees: a straight angle, or line.[10] $\angle$ABD = 120 degrees, and $\angle$DBC = 60 degrees. These are supplementary angles. **Complementary angles** add up to 90 degrees. Two 45 degree angles are complementary, a pair of angles that measure 30 degrees and 60 degrees are also complementary.

For Greater Clarity:

Notice that you can place the base of the protractor on either side (leg) of any angle, but the centre of the protractor base must be located at the joining point (the vertex) of the angle's sides.

The diagram above measures $\angle$ABD = 120 degrees, by putting the base of the protractor on the ray AB with the centre on B. Here you use the scale on the protractor that starts from 0 degrees on the left. However, if you choose to put the protractor on the ray DB you would use the outer scale beginning with 0 on the right to once again find that the angle measures 120 degrees.

The letters that describe the angle and your choice of position for the protractor tell you which scale to read. For example, if you are measuring $\angle$DBC = 60 degrees with your protractor's base on ray BC, (as in the diagram) you will use the outer scale on the protractor that starts from 0 degrees on the right. The sum of the inner and outer measures for any point on a protractor are supplementary and will always equal 180 degrees.

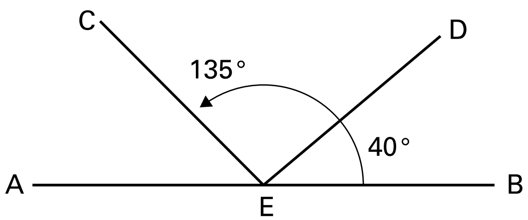
[10] Some math texts use $m\angle$ to mean "measure of angle". In this curriculum it is assumed that the symbol for an angle will mean the measure of the angle.

Unit 2 – Measuring Shapes

Topic 1.A – Angles and Circles: How to Use a Protractor

Examples

1. A protractor placed on line AB gave angle DEB = 40 degrees and angle CEB = 135 degrees. What is the size of angle CED?



Answer: 95°

The size of angle CED could be read directly by moving the protractor into a new position with the base of the protractor on DE. Or, we can use the information given in this problem to subtract 40 degrees from 135 degrees. The answer will tell us how many degrees apart 135 is from 40. This difference will equal angle CED. 135° – 40° = 95°. We can check this answer by adding 95° + 40° to get 135°.

2. What is the size of angle CEA?

Answer: 45 degrees.

You can see that the sum of angles CEB and AEC must be 180 degrees to equal the straight angle AEB. We can write an equation to solve for angle CEA: CEA = 180° – 135° = 45°. Angles AEC and CEB are supplementary. We can also see that angles DEB + DEC + AEC will equal 180 degrees as well. This statement will also be true: AEB = 40° + 95° + 45° = 180°.

Unit 2 – Measuring Shapes

Topic 1.B – Bisecting Angles and Lines

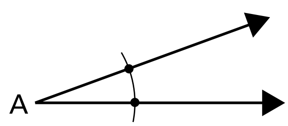
A line that is drawn at the halfway point of an angle divides it in half, or **bisects** it. For example, the bisector of a 90 degree angle is located at 45 degrees.

You can find the bisector of an angle by dividing it in half and using a protractor to draw the bisecting line or ray. You can also bisect an angle by using a straightedge and a compass.

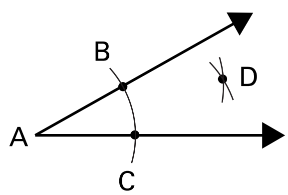
> A ray or line that divides an angle into two equal angles is called the **bisector** of the angle.

Construct an Angle Bisector

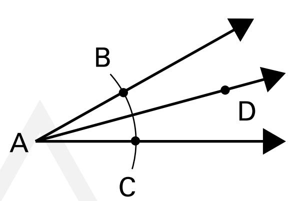
1. Put the point of the compass on the vertex A. Draw a circle that intersects (i.e. crosses) the sides of the angle you want to bisect.



2. Label the points of intersection B, and C, and then use each of these points as the center of a circle with the same radius. You can lock the compass to make sure the radii are equal. The circles will intersect at point D.



3. Draw a line from A to D and you will have the angle bisector. Notice that this method divides an angle without measuring the size of the angle.



Unit 2 – Measuring Shapes

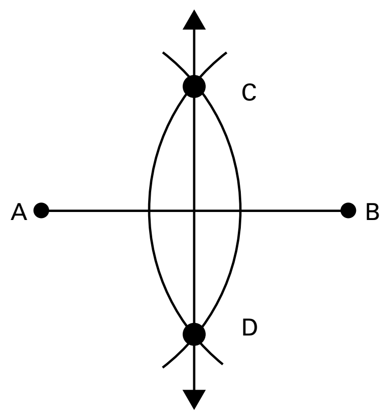
Topic 1B – Bisecting Angles and Lines

Bisecting a Line

A line that cuts another line in half by intersecting it at 90 degrees is a perpendicular bisector. You can draw a perpendicular bisector by measuring a line, dividing it in half, and then using a protractor or square to draw a line at right angles to this point. You can also construct a perpendicular bisector to a line by following these steps:

Construct a Perpendicular Bisector

1. Draw two circles that have their centers at the end points, A, B, of a line. Make each circle with the same radius, and label the points where these circles intersect C, and D. Draw a line connecting D and C and you will have the perpendicular bisector for line AB.

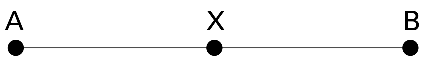


Unit 2 – Measuring Shapes

Topic 2 – Four Sided Figures

A figure with straight lines for sides (as opposed to curved lines) can range from a triangle with three sides to a figure with any number, n, of sides. An n sided figure is called a **polygon**, or "many angled and sided figure". If the sides of a polygon are all equal to each other, the figure is called a **regular polygon**. For example, a hexagon is a regular polygon with six sides. A polygon with sides that are not all equal is an **irregular polygon**.

When part of a line is labeled, it is called a line segment



Line AX is a segment of AB

Quadrilaterals are figures with four sides in the same plane. This means that all sides (lines) are on the same surface. A line called the **diagonal** can be drawn from two opposite "corners" of any four sided figure, examples are shown on the next page.

The sides of a four sided figure meet at vertices that form angles. **The sum of the four angles in any quadrilateral is 360 degrees.** You can see this by noticing that any quadrilateral is the combination of two triangles that share the diagonal as one of their sides.[11] Tick marks are used to show sides that have the same length in the examples that follow.

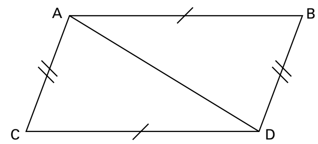
11 See Unit 2, Topic 3 – Triangles, for a demonstration that the sum of the angles in any triangle is 180 degrees.

Unit 2 – Measuring Shapes

Topic 2 – Four Sided Figures

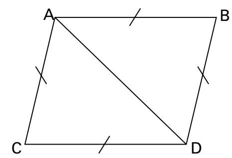
What you need to know about four sided figures:

1. A **parallelogram** has opposite sides that are parallel and congruent. The opposite angles are equal.



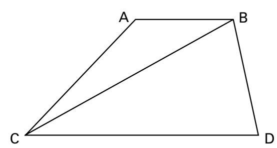
AD is a diagonal
AB II CD and AC II BD

2. A **Rhombus** is a parallelogram with sides that are equal in length.



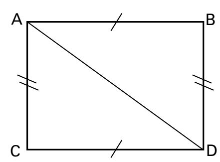
AD is a diagonal

3) A **trapezoid** has only one pair of parallel sides known as bases.



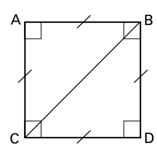
CB is a diagonal
AB II CD

4) A **rectangle** is a parallelogram with four right angles



AD is a diagonal

5) A **square** is an example of a rectangle that has equal sides. A square is also an example of a rhombus with four right angles.



BC is a diagonal
AB II CD and AC II BD

6) The sum of the four angles in any quadrilateral is 360 degrees.

Unit 2 – Measuring Shapes

Topic 2 – Four Sided Figures

Examples

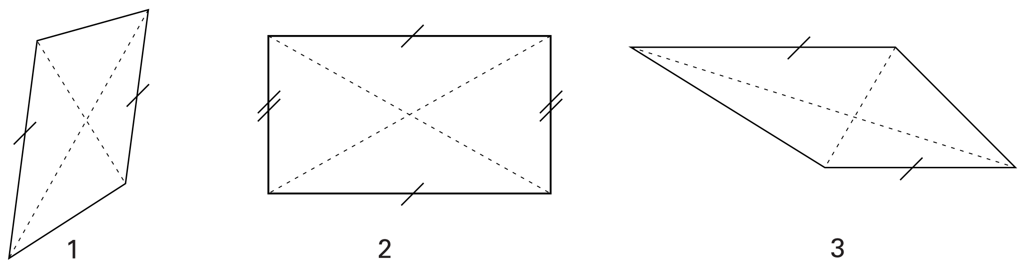
1. If quadrilateral has two sides that are parallel, what must be true?

   a) The diagonals are equal.
   b) The transversals are parallel.
   c) The parallel sides must be opposite each other.
   d) The bases are equal.

   Answer: c

   Explanation

   This question is typical of those that require a combination of knowing definitions and being able to sketch both examples and counter examples based on the problem. For example, choice a suggests something that could be true, but is it? Try several sketches that use the information given: namely that two sides of a four sided figure are parallel.



1 2 3

The tick marks show the parallel sides in each figure, and the dotted lines show the diagonals. You can see that some diagonals are equal (2), and some are not (1) and (3), yet each figure has at least two parallel sides. If you are testing the claim that diagonals are equal, (2) disproves this, and is known as a "counterexample".

Note the importance of thinking "at least" when you are given a problem that states a fact about a shape. Don't incorrectly assume that the description in this problem means a figure that "only" has two parallel sides. In this problem we know only that there are at least two parallel lines, and this doesn't rule out squares and rectangles, which have pairs of parallel sides.

The answer, c, is the result of logical thinking about what must be true based on what we are told. The definition of parallel requires that parallel lines be opposite each other in a quadrilateral, otherwise they wouldn't be an equal distance apart at all points. Choices b and d can be ruled out because they are irrelevant: there are no transversals in a quadrilateral, or they make assumptions that can be disproved. Choice d is disproved by counterexamples 1 and 3 in the diagram.

2. One angle of a quadrilateral measures 90 degrees. What will be the sum of the remaining angles?

    a) 180 degrees.
    b) 90 degrees.
    c) 270 degrees.
    d) Can't tell from this information.

    Answer: c

    Explanation
    The sum of the four angles in a quadrilateral is 360 degrees. 360° – 90° = 270°.

3. What makes a rectangle different from a square?

    a) A square has more diagonals.
    b) A square has two sets of parallel sides.
    c) A square is smaller.
    d) A square has equal sides.

    Answer: d

    Explanation
    The process of elimination will work on choices a, b, and c – they are false. The difference between a rectangle and a square is the fact that squares have all equal sides, while rectangles have pairs of equal sides. You can also see that a square is a "special case" of a rectangle because, like a rectangle, it has two pairs of parallel sides, however it is the equal length of these sides that defines a square.

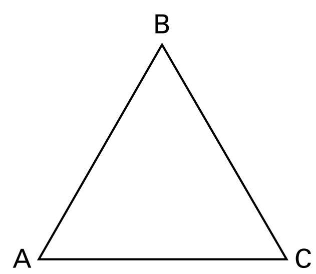
Topic 3 – Triangles

Types of Triangles

Triangles are three sided figures that can be described by their side lengths and/or their angle sizes. A triangle is a figure with three sides and three angles inside. The angles inside are called internal or interior angles. **The internal angles of a triangle add up to 180 degrees**. A triangle is labelled by giving a letter name for each angle. The triangle is named by the three letters, and each angle by the single letter at the point where two sides meet.

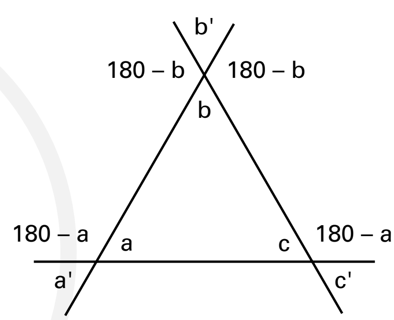
Example

Triangle ABC has three internal angles, angle A, angle B, and angle C.



Angle A + angle B + angle C = 180 degrees. The triangle chosen for this example is equilateral, meaning that the sides have equal lengths and the angles are = 60 degrees.

The three internal angles of a triangle open into the triangle's interior. If a line is drawn to extend the leg of an internal angle, we get an external angle that is supplementary to each internal angle: angles a, b, and c are internal angles. The opposite angles of intersecting lines are equal, and are labelled a′, b′, and c′. The supplementary angles for each internal angle are also shown. Their size will equal 180 degrees minus the size of each interior angle. An understanding of these relationships will help you identify types of triangles and solve problems on the exam.

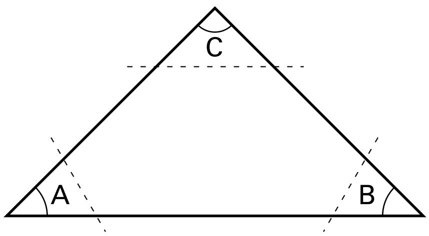


Unit 2 – Measuring Shapes

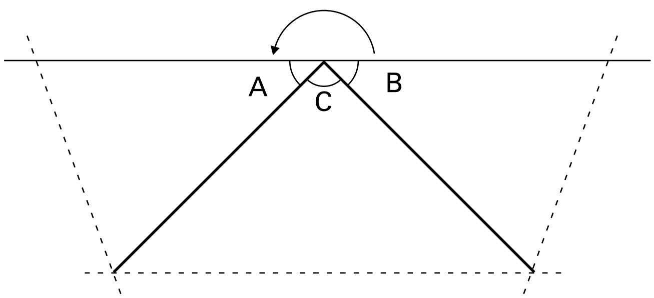
Topic 3 – Triangles

You can prove that the sum of the internal angles equals 180 degrees

1. Cut out a triangle of any shape and label each interior angle.
2. Cut the corner containing each interior angle off.



3. Arrange the corners in a line with the interior angles next to each other sharing a common point as their vertex. The angles add to give a straight line.



4. You will get a straight line along the edge of the corners when they are arranged this way that show that their sum is a straight line = 180 degrees. The diagram shows the addition of angles A + B + C makes a straight line, ie, a straight angle. Triangles of every shape produce this result.

Unit 2 – Measuring Shapes

Topic 3 – Triangles

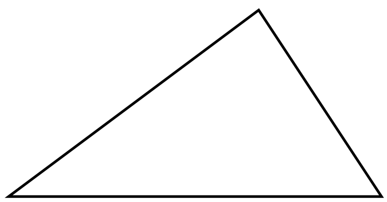
Learn to Recognize Six Types of Triangles

Any triangle can be described as an example of one or more of six types. Three types are based on side length relationships, and three are based on angle relationships.

Three Angle to Size Types

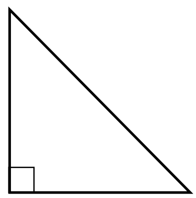
Type 1 Acute Triangles

The internal (or interior) angles are each less than 90 degrees.



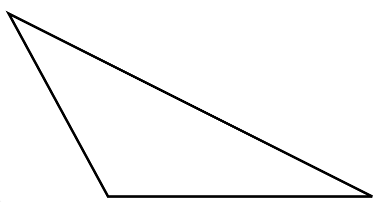
Type 2 Right Triangles

One internal angle = 90 degrees.



Type 3 Obtuse triangles

One angle is greater than 90 degrees.

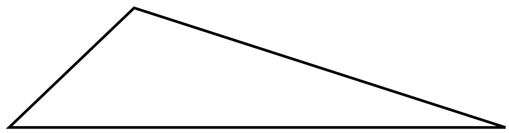


Topic 3 – Triangles

Three Side to Length Types

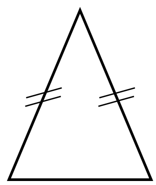
Type 4 Scalene triangles

No two sides are equal.



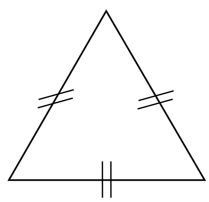
Type 5 Isosceles triangles (pronounced "eye-sah-so-lees")

At least two of sides are equal.

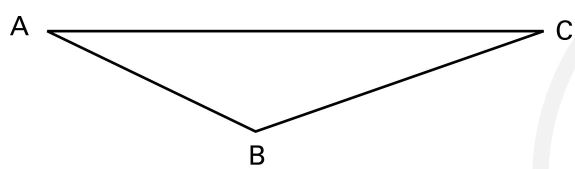


Type 6 Equilateral triangles

Three sides are equal in length and three angles are equal. Equilateral triangles have all three angles = 60 degrees.



Some triangles can be described in more than one of these ways. For example, an equilateral triangle is also an acute triangle. An obtuse triangle may also be a scalene triangle if, in addition to having one angle greater than 90 degrees, it also has no two sides of equal length.



This is an obtuse scalene triangle because angle ABC is greater than 90 degrees, and no two sides are equal.

Unit 2 – Measuring Shapes

Topic 3 – Practice Exam Questions

Question 1

A triangle has three equal sides. What will the size of its interior angles be?

a) Can't tell from this information
b) 45 degrees
c) 90 degrees
d) 60 degrees

Answer: d

Explanation
We know that the angles opposite a pair of equal sides will be equal in a triangle. An equilateral triangle has equal sides and equal angles. To find the size of each of three angles that must add up to 180, find 180 ÷ 3 = 60°.

Question 2

A right triangle must have one angle equal to:

a) 45 degrees
b) 60 degrees
c) 90 degrees
d) 180 degrees

Answer: c

Explanation
A right triangle has one angle equal to 90 degrees. No triangle can have more than one right angle.

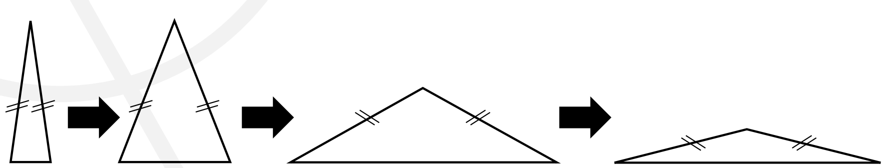
Question 3

An isosceles triangle has:

a) one right angle.
b) one pair of equal sides.
c) three equal angles.
d) none of the above.

Answer: b

Explanation
An isosceles triangle is one with two sides that are equal. They can range from very "thin" – not much more than a vertical line, to very "flat" – not much more than a horizontal line.



Unit 2 – Measuring Shapes

Topic 3 – Practice Exam Questions

Question 4

A triangle with one angle greater than 90 degrees is called:

a) binary.
b) equilateral.
c) acute.
d) obtuse.

Answer: d

Explanation
See the definitions for obtuse and acute. Any interior angle greater than 90 degrees makes a triangle an example of an obtuse triangle.

Question 5

Which type of triangle has no equal sides?

a) Acute
b) Isosceles
c) Scalene
d) Obtuse

Answer: c

Explanation
A scalene triangle has no sides equal to each other.

Question 6

In triangle XYZ, if angle X = 49 degrees, and angle B = 25 degrees, what will angle Z equal?

a) 90 degrees
b) 45 degrees
c) 60 degrees
d) 106 degrees

Answer: d

Explanation
We know the sum of the three angles in any triangle = 180 degrees. Add the two angles we know and subtract the result from 180 degrees to find the missing angle.

$25° + 49° = 74°$

$180° – 74° = 106°$

Angle Z = 106°

Topic 3 – Practice Exam Questions

Question 7

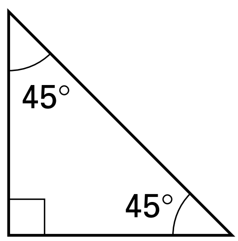
If we want to design a triangular yard that will have one angle = 90 degrees and the remaining two angles equal to each other, what will each of these angles have to be?

a) 30 degrees
b) 45 degrees
c) 70 degrees
d) 18 degrees

Answer: b

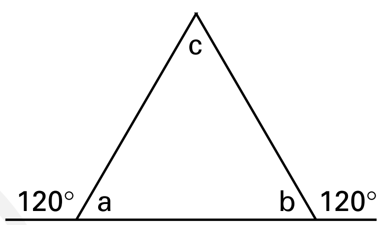
Explanation
Every triangle has only 180 degrees in it. 90 degrees are already used up by one angle given in the problem. This means 180° – 90° = the number of degrees left to be split equally between the two angles we are looking for. 180° – 90° = 90°.

Divide 90°/2 = 45° to get the answer. Each angle will equal 45 degrees.



Question 8

What is the size of the interior angles of this triangle?



a) 60 degrees, 30 degrees and 30 degrees
b) 30 degrees, 45 degrees and 55 degrees
c) 45 degrees, 45 degrees and 90 degrees
d) 60 degrees, 60 degrees and 60 degrees

Answer: d

Explanation
The interior angles a, b, and c, are inside the triangle. We are given two exterior angles. We know that a straight line = 180 degrees. Therefore the sum of 120 + b = 180°, and the sum of 120° + a = 180°. Angle a and angle b must each equal 60 degrees. The remaining interior angle, c, is 180° – 120° = 60°. We can see that this is an equilateral triangle.

Topic 3 – Practice Exam Questions

Question 9

A surveyor knows that one angle of a triangle is 60 degrees. He also knows that one of the two remaining angles is 3 times as large as the other. What are the sizes of the two angles?

a) 45°
b) 40° and 50°
c) 60° and 30°
d) 90° and 30°

Answer: d

Explanation
An equation built on what you know about the sum of the angles in a triangle will answer this question. We know that the sum of the remaining angles will equal 180° – 60° = 120°. We also know that one angle is three times the size of the other. If x represents one angle, then 3x will equal the other and their sum will equal 120.

$$x + 3x = 120°$$

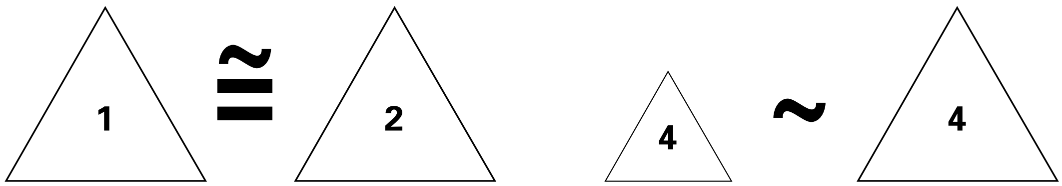
$$4x = 120°$$

$$x = 30°$$

If one angle is 30 degrees, then the other is 3x or 90°. The triangle will have angles that measure 60°, 30° and 90°.

Topic 3.A – Congruent Triangles and Similar Triangles

When two triangles have the same angles and sides they are congruent. The symbol ≅ means **congruent**. Congruence between triangles means that one could be placed on top of the other and their boundaries would be identical. **Congruent triangles have both the same shape, and the same size**. Similar triangles, however, only have the same shape. Their sizes will be proportional but not necessarily equal. The sides of similar triangles will be multiples of each other.



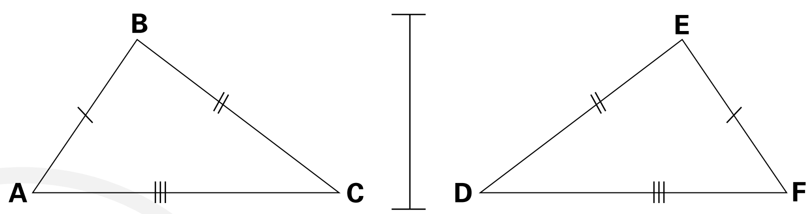
Triangle 1 is congruent (≅) with triangle 2. Triangle 3 is similar (~) to triangle 4.

It can save work to know that two triangles are congruent by comparing only some of their sides and angles. You can also apply what you know about work with proportions to solve problems with similar triangles.

There are three ways to prove that two triangles are congruent.

What You Need to Know About Congruent Triangles:

1. **(SSS) Side, Side, Side**: When three corresponding sides have the same measure the triangles are congruent, and you can conclude that the corresponding angles are also equal.

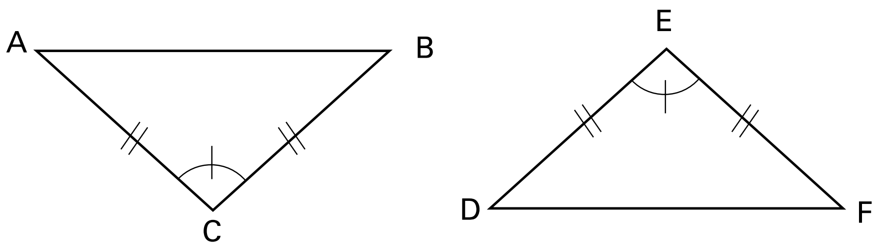


ΔABC ≅ ΔDEF because the length of AB = EF, BC = DE, and AC = DF. The tick marks show the sides that correspond to each other such that one triangle could be made to overlay the other if the corresponding sides were placed on top of each other.

Topic 3.A – Congruent Triangles and Similar Triangles

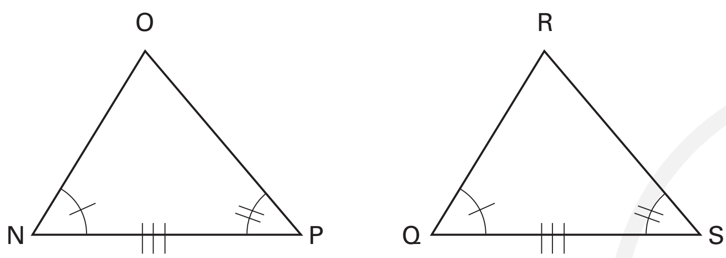
If you cut out triangle ABC you would have to flip it so that AB lies over EF to show congruence. Simply rotating the triangle on the paper won't allow you to place corresponding sides over each other. Triangle DEF is called a **reflection** of triangle ABC "through" a perpendicular line, and triangle ABC is a reflection of DEF. Sometimes a triangle has to be **transformed** by reflecting it, sliding it, or rotating it, to see congruence. You can also see that any plane figure can be transformed in one or more of these ways: sliding, rotating, flipping.

2. **(SAS) Side, Angle, Side:** when two sides and the angle between them have the same measure in two triangles, then the triangles are congruent and we can conclude that the third sides and angles are equal.



The angle marks and tick marks tell us what lines and angles have the same measure. $\Delta ABC \cong \Delta DEF$ because angle C = angle E, and the corresponding sides of angles ACB and DEF are equal: AC = DE (shown by three tick marks), and EF = BC (shown by two tick marks). Notice that one triangle would have to be **rotated** before it could be slid to overlay the other. Either triangle can be chosen to accomplish this. In this example, a flip to produce a reflection is not needed.

3. **(ASA) Angle, Side, Angle:** When two angles are equal and the side between them is equal we can conclude that the triangles are congruent. This means that the third angles will be equal, and the two other corresponding sides will be equal.

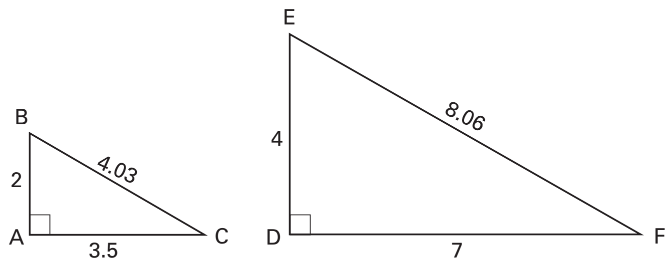


Δ NOP $\cong$ Δ QRS because angle N = angle Q, and angle P = angle S, and NP equals QS. In this diagram you can picture congruence by mentally sliding one triangle on top of the other. No flips or rotations are needed. Or, use scissors to cut one out and complete the slide.

Topic 3.A – Congruent Triangles and Similar Triangles

What You Need to Know About Similar Triangles:

1. **Similar triangles have the same shape** such that one can be slid, rotated, or flipped to match up 3 equal angles and corresponding sides inside the other. **The triangles will have different sizes, but the corresponding angles will be equal and the side lengths will be multiples of each other (ie be proportional).** As you will see in the examples below, proof of similarity allows measurements of a small triangle to be multiplied and transferred to the measurement of a larger triangle that we are interested in.



Here angle A = angle D = 90°. Recall that the same angle can have rays of different length. Here the lines from vertex D that form 2 sides of triangle DEF are longer than the lines from vertex A that form two sides of triangle ABC.

We can see that triangle DEF is an **enlargement** of triangle BAC, and BAC is a **reduction** of DEF. The triangles are similar because the corresponding sides are in proportion:

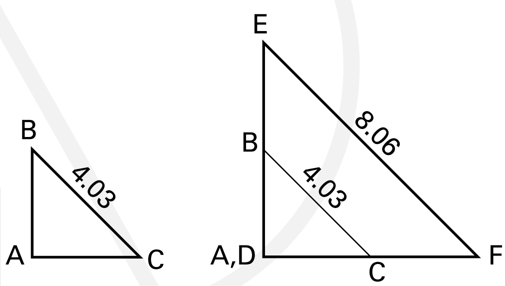
$$\frac{BA}{ED} = \frac{AC}{DF} = \frac{BC}{EF}$$

$$\frac{2}{4} = \frac{3.5}{7} = \frac{4.03}{8.06} = .5$$

(rounding off)

The scaling factor that relates triangle DEF to triangle ABC is 1:2 because triangle ABC is .5, or ½ the size of triangle DEF.

In this example no rotations or reflections are needed to see the similarity. One triangle can be slid over the other until the corresponding lines coincide:



Unit 2 – Measuring Shapes

Topic 3.A – Congruent Triangles and Similar Triangles

Triangle ABC and triangle DEF have been slid on top of each other to show their similarity.

You can see that a scaling factor relates two similar triangles.[12] In this example triangle DEF is twice the size of Triangle ABC. ED is twice as long as AB, DF is twice as long as AC, and EF is twice as long as BC.

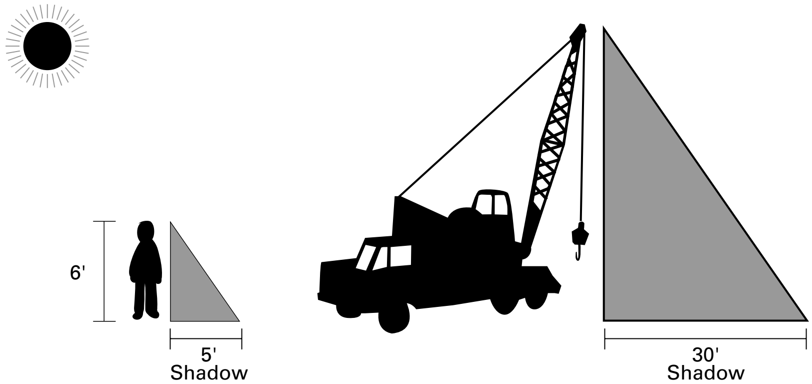
12 See Unit 1, Topic 3 – Converting Between Measurement Systems for a discussion of scaling factors and scaled drawings.

Unit 2 – Measuring Shapes

Topic 3 – Practice Exam Questions

Question 1

What is the height of the crane in this diagram:



a) 60 feet
b) 36 feet
c) 30 feet
d) 56 feet

Answer: b

Explanation
Use similarity of the two triangles to solve for the height of the crane. Both triangles have a right angle, and the corresponding bases are proportional because 30 = 5 x 6. 6 is the scaling factor that relates the two triangles. The large triangle is six times bigger than the small one. The triangles are proven similar by using the SAS rule.

Also note that the smaller triangle can be slid right to show the corresponding sides. You can also use the scaling factor of 6 to see that the height of the crane will equal 6 times the 6′ height of the person. The corresponding sides form this proportion:

$$\frac{6}{h} = \frac{5}{30}$$

$5h = 180$

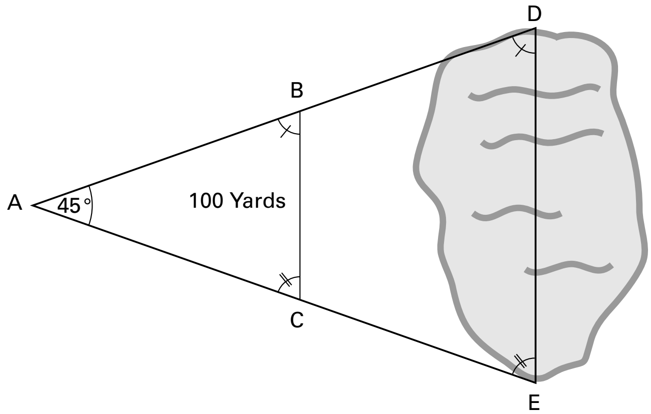
$h = 36$

Unit 2 – Measuring Shapes

Topic 3 – Practice Exam Questions

Question 2

A surveyor wants to measure the width of a lake, line DE, without getting wet. He draws the following sketch based on measurements he has made and uses the fact that alternate interior angles of a transversal between parallel lines are equal:



How can he find the distance across the width of the lake?

a) Use the Pythagorean relationship.
b) Multiply 100 x 2.
c) Solve the proportion: 100/DE = AB/AD.
d) Measure AE and divide by 2.

Answer: c

Explanation
This problem gives enough information to conclude that triangle ABC is similar to triangle ADE. We use the fact that both triangles have 45 degrees at one corresponding vertex, point A. It helps to realize that triangle ABC is overlaying (superimposed on) triangle ADE. The tick marks tell us that angle ABC is the same size as angle ADE, and that angle ACB is the same size as angle AED. Because the three angles of both triangles are equal, we conclude they are similar: and that means their sides will be in proportion. We are not given the measurements for AB, AD, AC, AE, but we can set up the proportion in choice c that will allow us to solve for DE after dry land measurements are made of AB, and AD. By cross multiplying we get

$$DE = \frac{100AD}{AB}$$

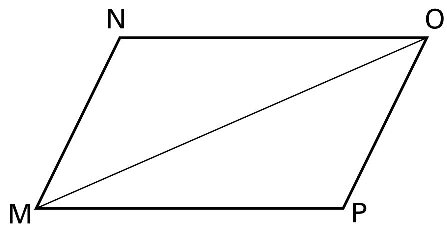
The surveyor will need to measure AD and AB to find the solution.

Unit 2 – Measuring Shapes

Topic 3 – Practice Exam Questions

Question 3

In the following diagram, ΔNMO ≅ POM. Which of the following do not correspond to each other:



a) MN and OP
b) Angle P and angle N
c) MP and NO
d) OP and ON

Answer: d

Explanation
We are told that the triangles are congruent. You can see this mentally by imagining a reflection and a rotation that puts point N over P, and side NM over side OP. We know the triangles are equal, and one will lie exactly over the other. Choice d names two sides that are not equal to each other in length. The other choices name equal and corresponding lines.

Topic 3.B – Right Triangles: Measuring the Hypotenuse

Background

Right triangles are important for surveying and building. A complete description of a right triangle can be given when we know the size of any two sides.

The **hypotenuse** is the side of a right triangle that is opposite the right angle. Hypotenuse is a Greek word meaning "stretched below". The length of the hypotenuse can be calculated by solving an equation that relates the squares of the sides to each other in a right triangle. No matter what the shape of the right triangle, the equation will work. You will need to understand squares and square roots to work with this equation. Review Math – Module 1, Unit 3, Topic 6 – Bases, Exponents and Square Roots.

The square of the hypotenuse = the sum of the squares of the remaining sides in a right triangle.

If we use the letter c for the hypotenuse (side opposite the right angle), and a and b for the remaining sides, this is the equation that will let us calculate the length of the hypotenuse:

$a^2 + b^2 = c^2$ aka Pythagoras' Theorem

The sum of the squares of the sides = the square of the hypotenuse.

Rearrange this equation to find the solution for the length of the hypotenuse:

$c = \sqrt{a^2 + b^2}$

Right triangles with whole number sides are useful in the trades.

A series of triangles with integer sides follows this pattern:

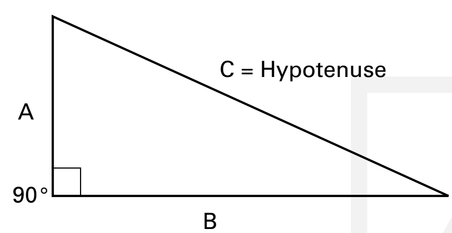
(leg, leg, side)

3, 4, 5

6, 8, 10

9, 12, 15

One leg increased by three, the other by four, and the hypotenuse by 5, will give the next three values in the series: 12, 16, 20. All integer right triangles are similar and are related by multiples of 3, 4, 5. However, many right triangles have integer sides but a hypotenuse that is not.



Topic 3.B – Right Triangles: Measuring the Hypotenuse

If we know a and b we can calculate c, the hypotenuse.[13] c will equal the square root of c^2, and c^2 = the sum of a^2 and b^2.

Sample problems:

1. In a right triangle with a = 4 and b = 16, what is the length of c?

   $4^2 + 16^2 = c^2 = 272$

   If $c^2 = 272$ then $c = \sqrt{272} = \sqrt{16}\sqrt{17} = 4\sqrt{17} = 16.49$

   The hypotenuse = $4\sqrt{17}$. We can estimate the square root of 17 as bigger than four but less than five. The length of the hypotenuse will be a little more than 16.

2. What is the hypotenuse of a right triangle with one side = 12, and one side = 20

   $144 + 400 = c^2$

   $c = \sqrt{544} = \sqrt{16}\sqrt{34} = 4\sqrt{34} = 4\sqrt{16}\sqrt{2.125} = 4 \times 4\sqrt{2.125} = \sqrt{16}\sqrt{2.125}$

   or use a calculator to find the answer with these keystrokes: $\boxed{544}$, $\boxed{\sqrt{\ }}$ or $\boxed{\sqrt{\ }}$, $\boxed{544}$ depending on the calculator you use.

13 Proof for this relationship uses the ability to measure area, and is given in Unit 3, Topic 2 – Area: Square Units.

Unit 2 – Measuring Shapes

Topic 3 – Practice Exam Questions

Question 1

A right triangle has sides of 2" and 5". What is the length of the hypotenuse?

a) $\sqrt{10}$ inches
b) 25^2 inches
c) $\sqrt{29}$ inches
d) $\sqrt{27}$ inches

Answer: c

Question 2

What is the length of the hypotenuse of a triangle if the sides are 10 and 20?

a) $5\sqrt{10}$
b) $10\sqrt{5}$
c) $\sqrt{120}$
d) $\sqrt{10}$

Answer: b

Explanation
10 squared is 100, and 20 squared is 400. The square root of their sum, 500, can be simplified to $10\sqrt{5}$.

Question 3

The length of the longest leg of a right triangle is 10 feet. One of the other legs measures 3 feet. What is the length of the third leg?

a) can't tell from this information
b) $\sqrt{91}$
c) 13
d) $\sqrt{109}$

Answer: b

Explanation
We can conclude that the 10 foot side must be the hypotenuse because it is the longest side. The hypotenuse is always the longest side in any right triangle because its square is equal to the sum of the squares of the other sides, which means each side by itself is smaller than the hypotenuse. Now we can solve the formula using c = 10 feet and one of the other sides (it doesn't matter which we choose) = 3. Substituting we get:

$100 = 9 + (\text{remaining side})^2$

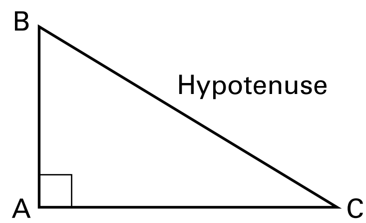
$100 - 9 = (\text{remaining side})^2$

$\sqrt{91}$ = length of remaining side

Unit 2 – Measuring Shapes

Topic 3.C – Ratios in Right Triangles: Trigonometry

Many trades, particularly construction, glass cutters, carpet layers, work with the relationships between the angles and sides of right triangles. Each of the two acute angles in every right triangle has an opposite side that it opens out towards, and an adjacent side that is not the hypotenuse.



AB is the side opposite angle BCA, and adjacent to angle ABC. AC is the side opposite angle ABC, and adjacent to angle BCA.

BC is the hypotenuse.

Locate any angle in a diagram of a right triangle and then you will be able to tell which side is opposite and which is adjacent to that angle. For any angle in a right triangle use the following definitions:

Sine of the angle = $\frac{\text{the opposite side}}{\text{hypotenuse}}$ remember S = O/H

Cosine of the angle = $\frac{\text{the adjacent side}}{\text{hypotenuse}}$ remember C = A/H

Tangent of the angle = $\frac{\text{the opposite side}}{\text{the adjacent side}}$ remember T = O/A

> People have memorized the meaning of **SOHCAHTOA** for more than two thousand years…

These ratios can be very helpful when the length of two sides of a right triangle are given and you want to know the size of the angle between them. A calculator, or tables, will show the angle that equals the value calculated for any of these ratios. A trigonometric equation can be solved for each of the three variables. When you have input values for any two of them, you can find the value of the third.[14]

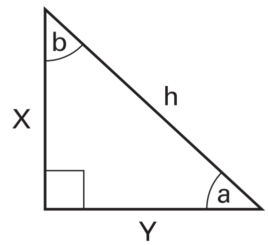
[14] Review Math – Module 3, Unit 1 – Understanding Equations for a discussion on rearranging and solving equations.

Unit 2 – Measuring Shapes

Topic 3.C – Ratios in Right Triangles: Trigonometry

In the following examples we solve for the relationships that express the three trigonometric ratios. Math level five has an extended unit on trigonometry that develops these ideas further.

Examples



If we know that the right triangle in the diagram has angle a = 45 degrees, and side y = 10 feet, we can find the length of side x by using tan a = x/y instead of directly measuring side x. The tangent of 45° = 1, because x = y in a 90, 45, 45 triangle.

We write this: tan 45 = 1. By substituting the values we know into the formula for tan a we get: 1 = x/10, and then x = 10.

1. Find the formula for the tangent of angle b

   Tangent = opposite/adjacent

   Tan b = y/x

2. Find the sine of angle b

   sine = opposite/hypotenuse

   sine of b (or sin b) = y/h

3. Find the cosine of angle a

   cosine = adjacent/hypotenuse

   cos a = y/h (notice that the cosine of a = the sine of b)

4. Find the tangent of angle a

   tan a = x/y

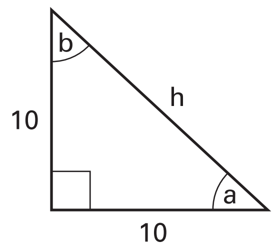
5. Find the cosine of angle b

   cos b = x/h

6. Find the sine of angle a

   sin a = x/h (notice that sin a = cos b)

Topic 3 – Triangles



7) What is the cosine of angle b?

The cosine is the adjacent divided by the hypotenuse. Angle b has an adjacent side = 10, and we have to find h by using the Pythagorean relationship:

$h^2 = 10^2 + 10^2$

$h = \sqrt{200}$

$h = 14.14$

Now express the cosine of angle b = 10/14.14 = .707.

We also know that angle a = 45° and angle b = 45°.

8) What is the hypotenuse if side a = 12 feet?

Use the Pythagorean formula to set sides a and b equal to 12.

$h^2 = 12^2 + 12^2$

$h = \sqrt{288} = 16.97$

Notice that the angles are still 45°, and the sine = cosine = .707.

Unit 2 – Measuring Shapes

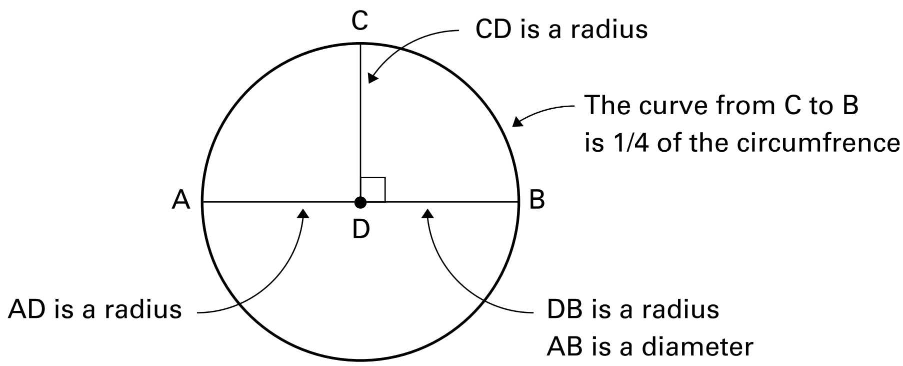
Topic 4 – Circles

Background

A circle is a curved line drawn at a distance from a fixed point called the centre. You can make a circle by spinning a compass around a point. There are 360 degrees in a circle. When a line drawn from the center to the edge of the circle goes around the circle once it goes through 360 degrees and traces the distance around the circle which is called the **circumference**. A circumference is also a line around a ball or sphere. The equator traces the circumference of the earth if we choose to picture the earth as a perfect sphere.

Why 360?
Early geometres thought there were 360 days in the circle of a year. A circle can be divided into any number of units but 360 has stuck.

Any line drawn across the circle through the center is called a **diameter**. Any line drawn from the center to the edge of the circle is called a radius. All **radius** lines equal each other, and all diameter lines equal each other. A diameter is twice the length of the radius, and the radius is equal to one half the length of the diameter.



For Greater Clarity:

90 degrees is the size of the right angle CDB formed by the lines that border $\frac{1}{4}$ of a circle's circumference.[15] The 90 degree angle opens out onto $\frac{1}{4}$ of the circumference. Any radius line meeting any diameter line at 90° will form two right triangles when the point of the radius on the circumference is joined by a line to each end of the diameter. Each side of the right angle is a radius, or one half diameter of a circle. A circle can have any line drawn through the center to show the diameter, but all of these lines will have the same length. The same holds true for a radius. Any line from the center to the circumference is a radius of the circle.[16]

15 The angles with a vertex at the center of a circle have the radius as legs. Because a radius has a fixed length, it is a line and does not extend indefinitely like a ray in an angle.

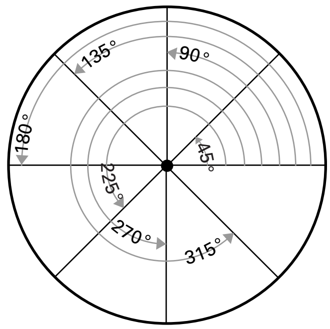
16 "The terms circumference and perimeter can refer either to the total distance around a circle, or to the edge of a circle at any point on its perimeter or circumference."

Unit 2 – Measuring Shapes

Topic 4 – Circles

Angles and Circles

A circle can be divided into angles between 0 degrees and 360 degrees. In the next diagram the circle has been divided into 45 degree sections. You may want to use your protractor to measure the angles that are shown.



Degrees in a circle are counted anti-clockwise starting from the horizontal diameter. 0° = 360° in the journey around a circle.

For Greater Clarity:

Picture a radius line in the diagram moving from 0 degrees up through 45 degrees = $\frac{1}{8}$ of the circumference to 90 degrees = $\frac{1}{4}$ of the circumference. As we continue anti-clockwise, the radius line goes through 90 + 45 degrees = 135 degrees, which is $\frac{3}{8}$ of the circumference. We continue through 180 degrees which is the diameter line on the left of the centre = $\frac{1}{2}$ of the circumference. The circle is completed as we rotate the radius through 225 degrees, 270 degrees, 315 degrees and finally 360 degrees which returns us to the diameter line. The journey around the circle can be divided into any stopping places we choose. Each angle will mark off a portion of the circumference and a portion of the area of the circle.

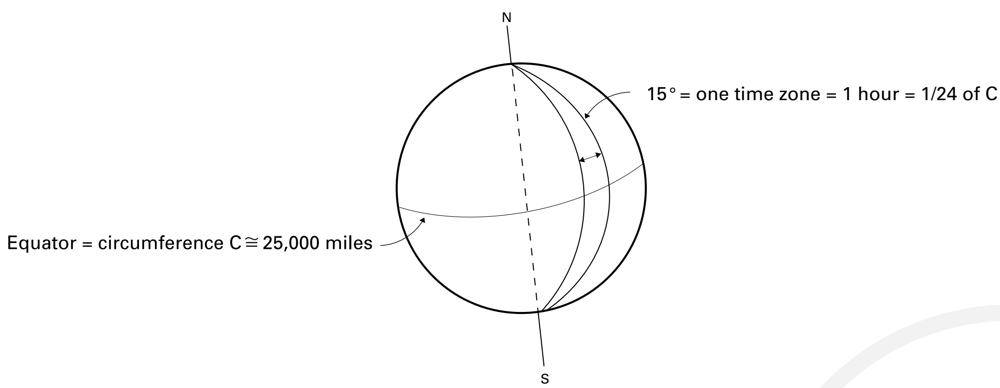
Special Topic: How Time Zones are Determined

Time zones were invented to organize the use of clocks around the world for daily living. For example, it is convenient to measure sunrise in any time zone between 5 a.m. to 8 a.m. However, as NT residents know, the further away from the equator one goes in a time zone, the more seasonal changes affect the amount of daylight, and when the sun rises. The geometry of the circle and the fact that the earth rotates on its axis once every 24 hours from east to west, determines how time zones are measured.

The earth is almost a round ball that rotates once each day on its north to south axis. When viewed from "above" the north pole, and looking "down", the direction of rotation is anticlockwise, or from east to west. In Canada, the sun rises first in the maritimes, and then 3 hours later in Yellowknife.

An imaginary line called the equator makes a circle like a belt around the earth at the point halfway between the north and south poles.

The equator is parallel to lines of latitude that "slice" the earth in equal divisions from north to south. Time zones are the result of dividing the earth into 24 equal sized areas that correspond to the amount of circumference that passes a fixed point on the equator each hour as the earth rotates on its north to south axis. Lines of longitude are used to mark these divisions.



360 degrees divided by 24 equals 15 degrees of rotation per hour. Each time zone covers 15 degrees of the earth's daily rotation. Since we know the distance around the earth is about 25,000 miles, we can estimate the distance across each time zone at the equator as $\frac{1}{24}$ of 25,000 = 1041.6 miles.

Unit 2 – Measuring Shapes

Topic 4 – Practice Exam Questions

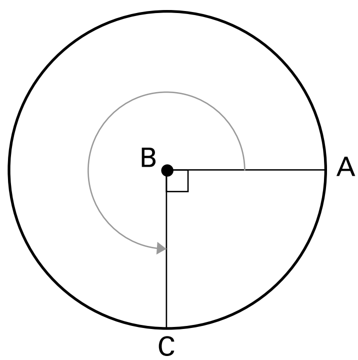
Question 1

What is the angle that is formed when $^3/_4$ of a circumference is bounded by the legs (sides) of an angle?

a) 90 degrees
b) 180 degrees
c) 270 degrees
d) 310 degrees

Answer: c

Explanation
Picture a line sweeping out three fourths of a circle and you reach the line perpendicular to the diameter you started from. This line makes a 270 degree angle measured from the starting point on the horizontal radius.



$\angle$ ABC = 270°

Question 2

How many degrees are in a complete circle?

a) 180
b) 360
c) 270
d) 90

Answer: b

Explanation
A circle has been divided into 360 degrees. Another number could be chosen and a conversion factor used to relate this number to the standard division into 360 degrees. 360 is based on the traditional connection between the approximate number of days in a year and one complete revolution.

Question 3

If you travel $\frac{1}{4}$ of the way around a circle and draw a line to the centre, how many degrees will be in the angle between this line and the diameter line you started from?

a) 180 degrees
b) 90 degrees
c) 360 degrees
d) 45 degrees

Answer: b

Explanation
Refer to the diagram used in Question 1 and see that one fourth of a circle is covered by a 90 degree angle.

Question 4

The circumference of a circle is:

a) greater than the diameter.
b) less than the diameter.
c) equal to the diameter.
d) unrelated to the diameter.

Answer: a

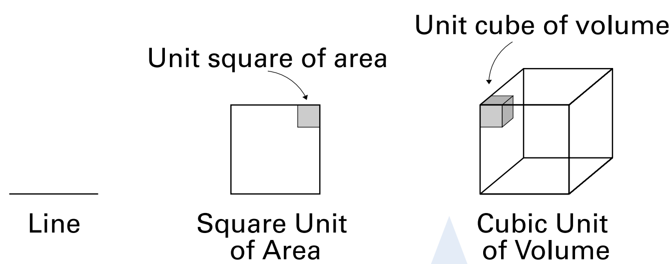
Explanation
The diameter is a straight angle and corresponds to (ie opens out onto) half of the circumference. Because the semicircle is curved and joins the same endpoints as the diameter we can conclude that it will be longer than the diameter, but we don't how much longer until we introduce the formula for circumference = π x diameter. You can also reach this conclusion by inspecting the sketch of a circle.

Unit 3

Measuring Spaces: Lines, Areas, Volumes

Background

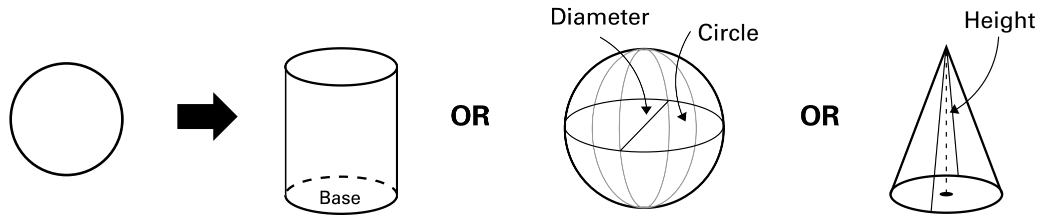
Each 2 dimensional shape can be a base for a 3D, or three dimensional, right angled extension: a circle becomes a sphere, cylinder, or cone, a triangle becomes a pyramid, and a square becomes a cube. In each case, a perpendicular, called the height or **altitude**, can be drawn from the top of the solid figure to the base. Each shape will have a formula for its area and for the volume of the solid that corresponds to it. The change from a line, to a square, and then to a cube, shows the changes from a dimension measurement to an area measurement to a volume measurement.



Here you can see the relationship between length, perimeter, area and volume. The next diagram shows the way that a circle can become the base for **cones**, **cylinders** and **spheres**, a rectangle the base for **rectangular solids** (including cubes when the sides are equal), and a triangle the base for **triangular pyramids**.

Unit 3 – Measuring Spaces

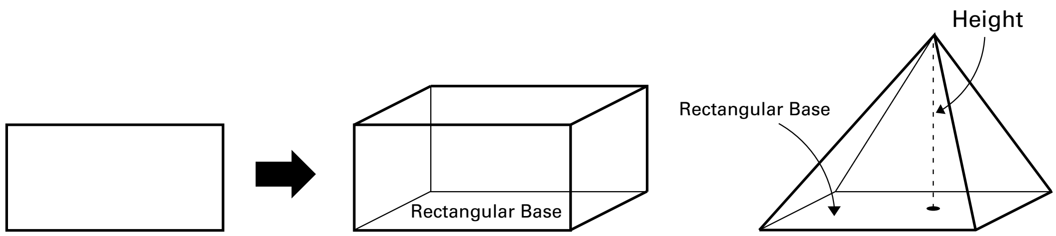
Topic 1 – Perimeter: Linear Units



Cylinder Circular Base

Sphere Circular Cross Section

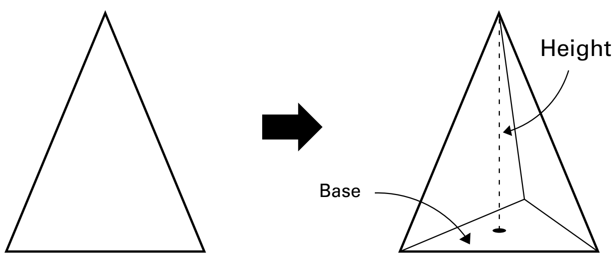
Cone Circular Base



Rectangle

Rectangular Solid

Pyramid



Triangle

Pyramid Triangular Base

Here you can see the way that a circle can become the base for **cones, cylinders and spheres,** a rectangle the base for **rectangular solids** (including cubes when the sides are equal), and a triangle the base for **triangular pyramids**. Each solid has a perpendicular from the top (apex) to the base that is called the height or altitude. **The area of the base and the size of the altitude are used as inputs for volume Formulas.** Formulas for the areas and volume of each shape are provided below.

Unit 3 – Measuring Spaces

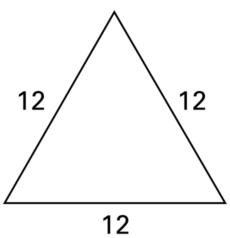
Topic 1 – Perimeter: Linear Units

The distance around a figure bounded by straight lines is called the perimeter, or "measurement that surrounds". If you walk around a house and count your paces, you are measuring the perimeter of the house. The distance around a circle is a curved line and the length of this curved line measures the circumference, or distance around the circle. If you count the paces around a circular water tower, you are measuring the circumference of the circular base of the cylindrical tower. The solution to perimeter problems with shapes and circles uses the following definitions:

Perimeter: the sum of the lengths of the sides of a figure on a flat, or two dimensional surface. The perimeter of a rectangle is the sum of the lengths of its sides. The perimeter of a triangle is the sum of the lengths of its sides. The perimeter of an irregular shape is the sum of its sides. In every case, the perimeter is the distance traveled by going along the outside of a figure or shape.

Examples

Calculate the perimeter by adding up the lengths of the sides.

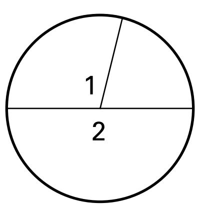


The perimeter = 12 + 12 + 12 = 36

Radius: the distance from the centre of a circle to its circumference. The radius = $\frac{1}{2}$ of the diameter, and the diameter = 2 x the radius.

Diameter: the length of a line through the centre of a circle from one side to the other. The diameter = 2 x the radius, the radius= $\frac{1}{2}$ x diameter

Radius = 1, diameter = 2



π = 3.14

The ratio of the circumference over the diameter of any circle.

$\pi = \frac{c}{d}$

Unit 3 – Measuring Spaces

Topic 1 – Perimeter: Linear Units

The diameter (and therefore the radius too) can be found from the relationship between circumference and π.

> **Circumference:** The circumference is the distance around a circle.
> The circumference can be found by using this relationship:
>
> **Circumference** = π times the diameter. $C = \pi D$
>
> **Diameter** = Circumference $\div \pi$
>
> **Radius** = $\frac{C}{2\pi} = C \div 2\pi$
>
> $\pi = 3.1416\ldots$
>
> We always know the value of π. Therefore, if we know the circumference we can find the diameter, and if we know the diameter we can find the radius, and vice versa.

The circumference of any circle = π times the diameter. The diameter is the distance across the centre of the circle from one side to the other. π is a Greek letter that stands for the number 3.1416… This number is a constant. 3.14… is the answer you get whenever you divide the distance around a circle by its diameter.[17] This relationship doesn't change no matter how big or small the circle is. A close approximation to pi is the fraction 22/7.

Examples

1. What is the circumference of a circle with a diameter of 12 feet?

   Use the formula that relates the circumference to the diameter: $C = \pi D$

   We know $\pi = 3.14$, and $D = 12$ feet

   Substituting:

   $C = 3.14 \times 12$ feet $= 37.68$ feet

2. The radius of a circle is 4 feet. Find the circumference.

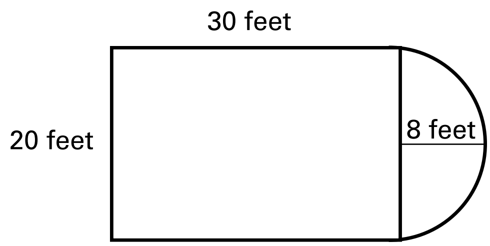
   Use the same relationship as in #1 and the fact that the radius = $\frac{1}{2}$ of the diameter, therefore the diameter is 2x the radius.

   $D = 2 \times \text{radius} = 2 \times 4 = 8$ feet

   $C = 3.14 \times 8 = 25.12$ feet

[17] Pi is an irrational number. Its decimal form neither repeats a pattern, nor ends. It can be rounded to any desired place value. It continues to be calculated by powerful computers to many thousands of places. Review Math – Module 1, Unit 1, Topic 4 – Rational Numbers and Signed Numbers.

3. What is the perimeter of this school yard?



This is a composite figure, made of a rectangle joined to a half circle.
To find the perimeter, we add up the lengths of each part of the schoolyard. The perimeter is the sum of 20' + 30' + 30' + $\frac{1}{2}$ of the circumference of a circle with radius = 8 feet.

$\frac{1}{2}$ of the circumference = $\frac{1}{2}\pi D$ and this equals π x the radius when we simplify D/2.

Since $C = \pi D$, $\frac{1}{2}C = \frac{1}{2}\pi D$

$1/2C = \pi r$ (because $\pi d/2 = \pi r$)

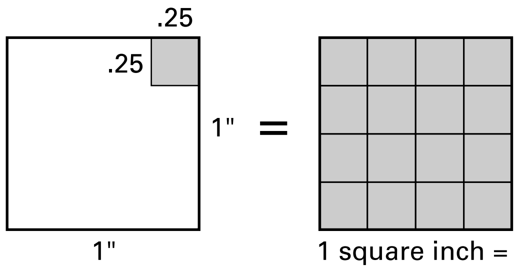
1/2C = 25.12 feet

The perimeter of the schoolyard = 25.12 + 20 + 30 + 30 = 105.12 feet

Unit 3 – Measuring Spaces

Topic 2 – Area: Square Units

Area is the amount of space covered by a figure. We measure area in square units. One way to find the area of a two dimensional figure is to place small squares inside its perimeter. The number of squares it takes to cover the space inside with no overlapping squares is the area. The simplest example of this process is the square:



1 square inch =
16 x .25 square inches

This is a drawing of a square that measures one inch on a side, and that has an area of one square inch. The scale in the drawing is 1:1. You can see that sixteen smaller squares measuring .25 inches on a side will exactly cover the space contained by the perimeter of this one inch square. This is the area of the one inch square. If we choose to measure the area in quarter inch squares, the area equals 16 quarter inch squares. 16 is a total that can be found by multiplying the number of units on one side by another side. 4 x 4 = 16. Because the sides are equal, the formula for area becomes s^2 = **area of a square**.

For Greater Clarity:

16 squares measuring a quarter inch on a side is equal to one square measuring one inch on a side. A person placing tiles on the floor of a square kitchen would measure the number of tiles that can fit along one side and then multiply this number by itself to find the total number of tiles needed to cover the floor.

In the trades units for area can be chosen for convenience

You can also see that this diagram can be scaled to any size. For example, we could set one side of the square to 10 feet instead of one inch. In this case, the area would be 100 square feet. If we wanted to measure the area by dividing each side into four equal squares, as in the diagram, each square would measure 10 ÷ 4 = 2.5 feet on a side, and once again 16 of these squares would be needed to cover the surface. People who install tiles of different sizes use measuring units that suit their materials. If the tiles measured 2.5 feet on a side, this would be a convenient unit for calculating area because we would know how many tiles we need for a job.

Unit 3 – Measuring Spaces

UNIT 3

Topic 2 – Area: Square Units

What you need to know about perimeter and area:

1. **Perimeter** is the distance around a two dimensional shape. These shapes are also called **plane figures** because they can be drawn on a flat surface, or plane. The perimeter of a circle is called the **circumference.** The perimeter of a many sided figure is the sum of the lengths of each side, no matter how many sides there are.
2. A four sided figure is called a **quadrilateral.** Squares, trapezoids, and parallelograms are quadrilaterals. A shape with more than four sides is called a **polygon**. When the sides are equal in length, the shape is called a regular polygon.
3. **Area** is the number of unit squares, that will cover a plane figure. You will need to know the formulas that give the area for the following plane figures:
   a) Area of four sided figures (quadrilaterals)
      1. **Area of a square = s2** (multiply a side by itself)
      2. **Area of a parallelogram = bh** (the product of any base and its corresponding height, aka altitude. A parallelogram is a quadrilateral with both pairs of opposite sides parallel. The opposite sides are also equal.)
      3. **Area of a trapezoid = $\frac{1}{2}$h(a + b)** (one half the product of the altitude and the sum of the bases. A trapezoid is a quadrilateral with one pair of parallel sides)

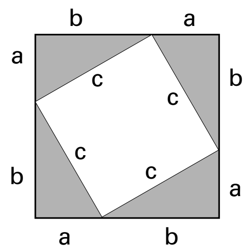
   b) **Area of a triangle = $\frac{1}{2}$ bh** (one half of the base times the height)

   c) **Area of a circle = πr2** (pi is a constant 3.14, r = radius)

Topic 2 – Area: Square Units

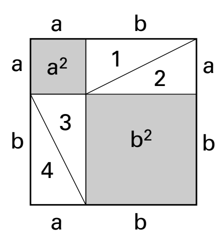
Special Topic: Using Area Relationships to Prove That $a^2 + b^2 = c^2$ in a Right Triangle.

1. Take any right triangle with sides a, b, and c, and construct a square whose sides have lengths a + b:



The four triangles in the diagram with right angles at each corner of the square, are congruent, and each hypotenuse is a side of the smaller square with side = c. Notice that the area of the contained square = c^2, and will be less than the area of the containing square whose area will equal $(a+b)^2$. Notice also that the area of each triangle abc = $\frac{1}{2}ab$. You can conclude that $(a+b)^2$ (the area of the outer square) = $4(\frac{1}{2}ab) + c^2$ because the area of the four right triangles plus the area of the inner square equals the area of the larger, outer, square with sides = a + b.

2. Copy the same larger square with sides equal to a + b and shade in the square areas that equal a^2 and b^2.



3. Now draw diagonals in the rectangular areas with sides b and a. This creates four congruent triangles, labelled 1, 2, 3, 4.

4. $a^2 + b^2 = c^2$ can now be proven by adding up the areas of the four triangles and combining the result with the areas for the two squares, a^2 and b^2. This gives the total area of the square whose sides are a + b in the second diagram.

We know that the total area in both diagrams is equal; we have simply expressed two ways of adding it up. This allows us to set the formula for the total area in the first diagram equal to the formula for the total area in the second diagram:

$4(\frac{1}{2}ab) + c^2 = 4(\frac{1}{2}ab) + a^2 + b^2$
solve for c^2 by subtracting $4(\frac{1}{2}ab)$ from both sides.

$c^2 = a^2 + b^2$

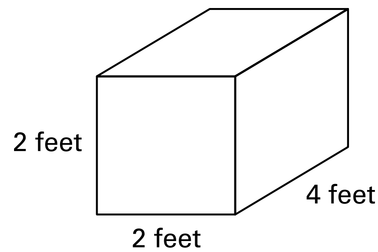
Unit 3 – Measuring Spaces

Topic 3 – Volume: Cubic Units

A figure that has surfaces on more than one plane is called a three dimensional solid. A cube of sugar has squares as sides, but each of the eight sides is on a different plane. Each pair of opposite sides occupies parallel planes. The world we live in contains three dimensional solid objects that take up space. Cubes, cylinders, cones, and pyramids are examples. Measuring the space occupied by something calculates its volume.

Example

1) What is the volume of this box?

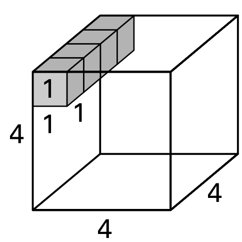


This is a solid object with three edges, or side lengths. The side lengths are not all equal, so we know it is not a cube. Length, width, and height will be equal to each other in a cube and we can use s^3 to find volume in that case. Here we use **Volume of a rectangular solid = L x W x H.**

Volume = 2 feet x 2 feet x 4 feet = 16 cubic feet

From Area to Volume

Like area, **volume** is calculated by a similar process of adding up the cubic units required to fill up the space contained within the boundaries of a three dimensional object.



The volume of this cube equals 64 cubes measuring one unit on a side. It takes 64 of these unit cubes to fill the space enclosed by the cube measuring four units on a side. This total can be found by multiplying the length of each of the three dimensions of the cube. 4 x 4 x 4 = 64. This is the same as s^3, if we think of each edge as a "side".

Unit 3 – Measuring Spaces

UNIT 3

Topic 3 – Volume: Cubic Units

What You Need to Know About Area and Volume

Area measures the number of squares with sides = 1 unit in some measurement system that will cover a two dimensional plane figure, for example a square, rectangle, triangle, or circle.

Formulas for the area of a shape use inputs that measure the length, width, height, thickness, radius, or diameter of a shape.

Examples:

Area of a circle = πr^2, and r is measured in units of length

Area of a triangle = $\frac{1}{2}bh$, and the base and height are measured in units of length

Volume measures the number of cubes with edges = 1 unit in a measurement system that will fill the space contained by a three dimensional solid. **Volume can be calculated from area**. Formulas for volume use the area of the corresponding plane figure as an input. Area is a variable in formulas for volume.

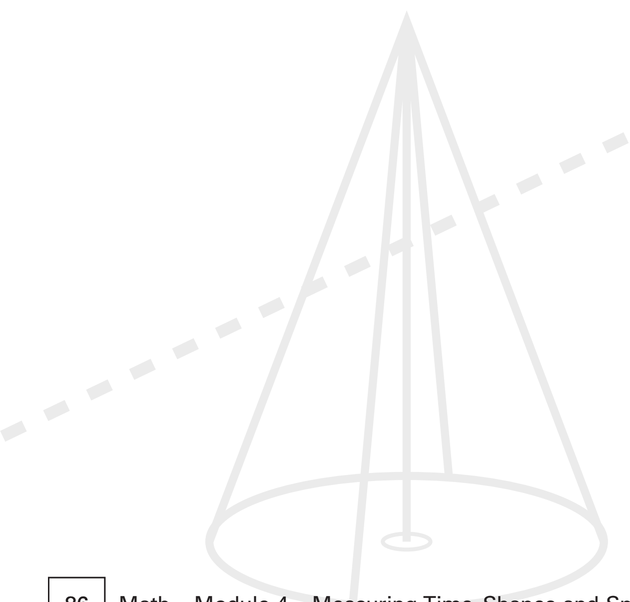
Formulas:

Volume of a sphere = (Area of a circle)$\left(\frac{4r}{3}\right)$ = $4/3\pi r^3$

Volume of a cylinder = (Area of circular base)(h) = $\pi r^2 h$

Volume pyramid = $\frac{1}{3}$ (Area of rectangular or square base)(h) = $\frac{l \times w \times h}{3}$

Volume of a cone = $\frac{1}{3}$ (Area of circular base)(h) = $\frac{\pi r^2 h}{3}$

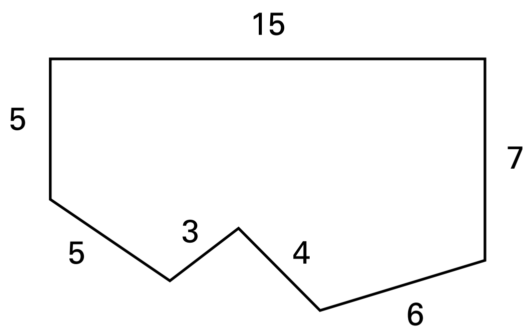


Unit 3 – Measuring Spaces

Topic 3 – Volume: Cubic Units

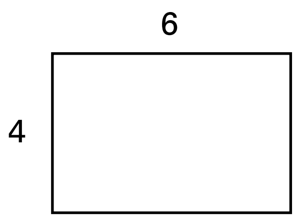
Examples: perimeter, area, and volume

1. Find the perimeter of this shape.



The perimeter = 5 + 15 + 7 + 6 + 4 + 3 + 5 = 45
This shape is a polygon. The length of the sides can be added in any order to find the distance around the entire figure.

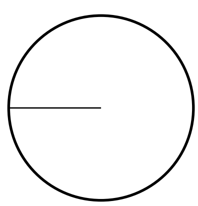
2. Find the area of this figure:



This is a four sided figure with unequal sides. It is a rectangle.
Area of rectangle = the product of two sides joined at a corner (adjacent sides).
Area = 4 x 6 = 24 square feet

3. Find the area of this circle:

r = 1 foot

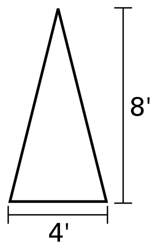


> **Radius** = distance from the centre to the circumference.

Area of a circle = πr^2 (π = 3.14, r = length of radius)
If r = 1 foot, then
Area = 3.14 x 12 = 3.14 square feet

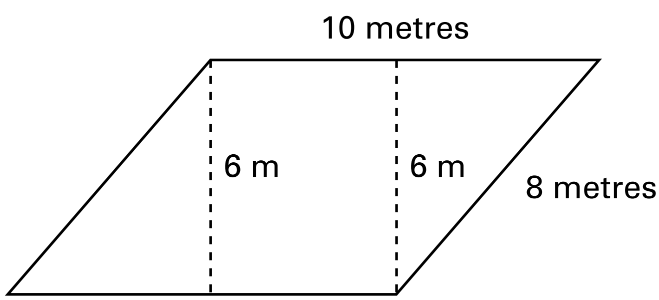
Topic 3 – Volume: Cubic Units

4. Find the area of this figure



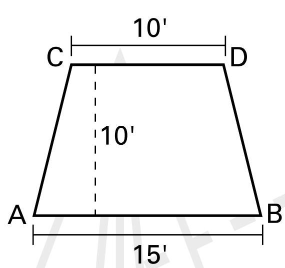
Base = 4′, h = 8′
Area of a triangle = $\frac{1}{2}$ base x height
Area = $\frac{1}{2}$ x 4 x 8 = 16 square feet

5. Find the area of this figure



Area of a parallelogram = bh
The height is 6 meters, and one base is given as 10 meters.
6 x 10 = 60 square meters.

6. Find the area of this figure



Area of a trapezoid = $\frac{1}{2}$h(a+b)
The height is 10, and the bases equal 10′ and 15′. Substituting into the formula we get:

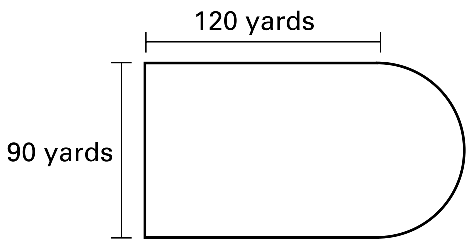
$\frac{1}{2}$(10) (10+15) = A

A = 5(25) = 125

125 square feet is the answer.

Topic 3 – Volume: Cubic Units

7. Find the area of this dump yard:



This is a two step problem because the area is the sum of a rectangle and a semicircle.

The area of the rectangle = L x W = 90 x 120 = 10,800 yards

The area of the semicircle is one half the area of the circle with diameter of 90 yards. Notice that the diameter is equal to the width of the yard, so the radius will be 45 yards.

Area of half circle = $\frac{1}{2}\pi r^2 = \frac{3.14}{2}(45^2)$ = 3,179.25 square yards

The total area = 3,179.25 + 10,800 = 13,979.25 square yards.

8. What is the volume of a cone if the area of the base is 10 square meters, and the height is 12 meters?

Use the formula for **volume of a cone** = :

$\frac{1}{3}$(Area of circular base)(h) = $\frac{\pi r^2 h}{3}$

Since we are given the area of the circular base for an input into the formula, we solve A x h = 10m^2 (ten square meters) x 12 meters to get 120m^3, or 120 cubic meters of volume, which gives 40 m^3 by completing the formula and dividing 120 m^3 by 3. Notice that the product of meters and meters squared is meters cubed, or m^3.

9. If the volume of a cylinder is 120 cubic liters, and the cylinder is 1 meter long, what is the area of the base?

Volume of a cylinder = Area of circular base x h= $\pi r^2 h$

In this problem we are given the volume, and the height, and have to rearrange the formula to solve for area of the base.

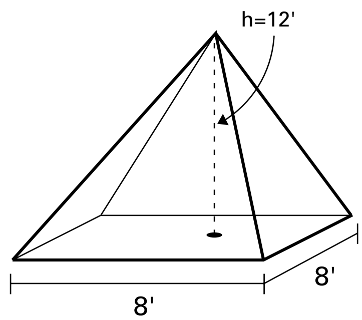
Area of base = volume ÷ height

Area of base = 120 cubic meters ÷ 1 metre= 120m^2, or 120 square metres. Notice that $m^3 \div m = m^2$.

Unit 3 – Measuring Spaces

Topic 3 – Volume: Cubic Units

10. Find the volume of this pyramid:



This is a pyramid on a square base. To find volume we use the area of the base as an input to the volume formula. The area of the base is 64 square feet.

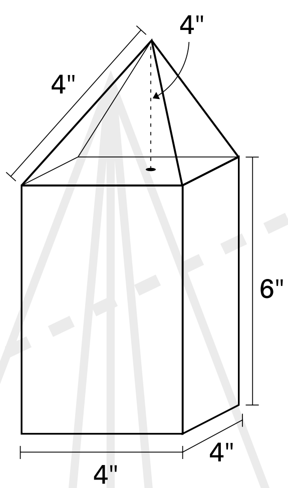
Volume pyramid = $\frac{1}{3}$(Area of rectangular or square base) x h = $\frac{l \times w \times h}{3}$

Volume = $\frac{1}{3}(64)(12)$ = 256 cubic feet

11. A carpenter wants to calculate how much plywood he needs to cover all surfaces of a cubic storage box that measures 4 feet by 4 feet by 4 feet. How many square feet of plywood will he need?

    This problem describes a cube with edge = 4. We can find the area of one surface and then multiply by the number of surfaces that have to be covered. The area of one surface = 4 x 4 = 16 square feet. There are 3 pairs of parallel surfaces in a cube, or 6 surfaces. Count the surfaces on a die to prove this to yourself. The answer will be 6 x 16 = 96 square feet.

12. What is the total surface area of this shape, including the base:

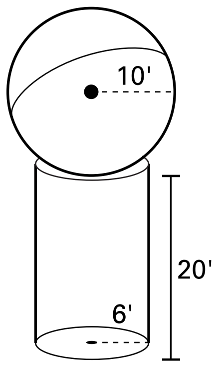


We have to add the area for each surface in a rectangle and in a pyramid that is placed on top of it. The problem can be done in steps:

1. Area of four sides of rectangle with sides = 4 x 4 x 6 = 4x 24 = 96 square inches.
2. Area of base = 4 x 4 = 16
3. Area of four surfaces of pyramid = 4 x triangular area on each surface = 4 x 1/2bh = 4 x 1/2(4)(4) = 32 square inches
4. Add the contributions to total surface area from steps 1 - 3: 96 + 16 + 32 = 144 square inches

13. An arctic airport control tower is being built with a complete climate control system. The engineer wants to know the total volume of air that will circulate in the structure. Use the information on this diagram to calculate the total volume.

We have to add the volume of the sphere to the volume of the supporting



cylinder to answer this question. We need these formulas:

Volume of a sphere = (Area of a circle) x $\frac{4}{3}r = \frac{4}{3}\pi r^3$

Volume of a cylinder = (Area of circular base) x h = $\pi r^2 h$

1. **Volume of sphere** = $\frac{4}{3}(3.14)(10)^3 = \frac{4}{3}(3140)$ = 4186.66 cubic feet
2. **Volume of cylinder** = $(3.146)(6)^2(20)$ = 2260.8 cubic feet
3. Add the contributions of each to get the total volume = 6447.46 cubic feet.

Topic 3 – Practice Exam Questions

Question 1

Jim wants to cut a carpet to cover a circular floor. The width of the room is 13 feet. How many square feet of carpeting will he need?

a) 1300
b) 132.665
c) 109.33
d) 91.5

Answer: b

Explanation
We need to find the area (square footage) of the circular floor. We know the area of any circle = πr^2

Since the room has a diameter of 13 feet, the radius is $^{13}/_{2}$ = 6.5 feet.

Now we use our formula for area:

$A = \pi(6.5)^2$

$A = (3.14)(6.5)(6.5) = 132.665$ square feet

Question 2

What is the area of a triangle with a base of 12 feet and a height of 15 feet?

a) 110 square feet
b) 27 square feet
c) 159 square feet
d) 90 square feet

Answer: d

Explanation
$A = {}^{1}/_{2}bh$

$A = {}^{1}/_{2} \times 12 \times 15 = 90$ square feet

Topic 3 – Practice Exam Questions

Question 3

A town pays $1.05 for each 100 cubic feet of water. How much will the town pay to fill the civic pool if the pool measures 12 feet x 20 feet x 8 feet?

a) $20.16
b) $11.05
c) $110.50
d) $15.05

Answer: a

Explanation
The volume expressed in cubic feet will allow us to calculate the cost of the water.

Volume = L x H x W the width can also be understood as the depth of the pool. There are three dimensions and we multiply them to get:

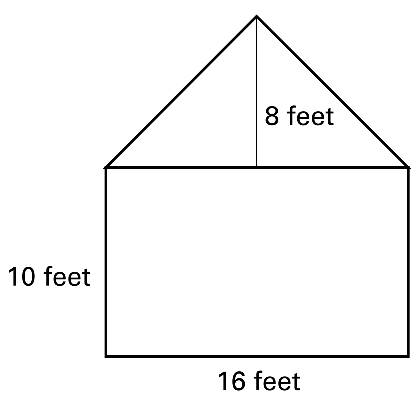
V = 12 x 20 x 8 = 1920 cubic feet

Now divide by 100 cubic feet to see how much the town will pay.

1920/100 = 19.2 The town will pay 19.2 x $1.05 cents = $20.16

Question 4

What is the area of the front of Frank's house if it has the following measurements:



a) 168 feet
b) 96 square feet
c) 224 square feet
d) 328 square feet

Answer: c

Topic 3 – Practice Exam Questions

Explanation
Rectangle = 16′x 10′

Triangle base = 16′, h = 8′

The area is the sum of the area of a rectangle and a triangle. Calculate the area of the rectangle = 16′x10′ = 160 square feet

The area of the triangle = ½ x 16′ x 8′ = 64 square feet

The combined areas = 224 square feet

Question 5

A rectangular tank holds 1872 cubic feet of sludge. The sides measure 12′ by 13′. How deep is the tank?

a) 32 feet
b) 16 feet
c) 12 feet
d) 15 feet

Answer: c

Explanation
We can find out without lowering a tape measure or climbing in. We know that the volume, 1872 cubic feet = 12′ x 13′ x depth. Let D = the depth

1872 cubic feet = 12 feet x 13 feet x D feet

To find the depth we divide both sides by the product of 12 x 13 = 156

Depth = 1872/156 = 12 feet.

Unit 4

Measurement and Estimation Problems

Topic 1 — Problems Involving Linear Estimation of Raw Materials for a Job

Estimates for a job involve rounding up so as not be short of materials for the job. Estimation of linear quantities requires that we know the relationship between the length needed for a job, and a material's measurement. For example, how many pieces of 12 foot moulding will it take to do a door frame? Problems like this are really **conversion problems using an exchange factor.** Review the section on converting from units to subunits, Unit 1, Topic 2 – Units and Subunits.

Sample Problems

1. How many bricks will be needed to line one side of a walkway if the walkway is 10 feet long and one brick measures 8 inches?

   We need to know the length of the walkway in inches, or the length of the brick in feet before we can find the relationship between the two lengths. If we go from "big to small", we convert 10′ into 10′x12 = 120".

   We can now divide 120" by the length of 1 brick to see how many bricks are required to equal 10 feet.

   120" ÷ 8" = 15

   We are in luck, we need exactly 15 bricks to line each side of the walkway.

2. A floor measures 12′ along one wall. We will put a stud every 16.5". How many studs will we need for this wall?

   We need to divide 12′ by 16.5" Convert 12′ into inches. 12 x 12 = 144 inches. 144" ÷ 16.5" = 8.72

> **Tip:**
>
> The units (inches, feet, metres, etc.) in the numerator and denominator of the division problem cancel out to give a "scalar" answer – the number of pieces is a quantity, not a dimension of something.

Topic 1 — Problems Involving Linear Estimation of Raw Materials for a Job

We cannot use part of a stud, so we can either use 8 studs or 9 studs. 8.72 is closer to 9 than to 8, so we will estimate 9 studs for this wall.

3. How many bolts will we need to put a bolt every 6 inches through an I beam that measures 32 feet?

   We divide 32′ by 6" to find how many bolts are needed. Here it is easier to convert inches to feet. We can convert 6" to $\frac{1}{2}$ foot and divide into 32 feet.

   $32 \div \frac{1}{2} = 64$

   We will need 64 bolts in order to place one every 6" on a 32′ beam.

4. A house needs 30′ of 110 wire for each room. There are 11 rooms in the house. Estimate the wire needed for the house.

   Multiply the number of rooms times the amount of wire needed in each to get the total required.

   11 x 30′ = 330 feet of wire

5. Amy wants to put metal roofing on her cabin. Each sheet will cover 2.5 feet of the length of the roof. The roof is 16 feet long. How many sheets will she need to cover the edge on one side?

   We divide 16 feet by 2.5 feet to see how many sheets are needed.

   16′ ÷ 2.5′ = 6.4 sheets. We will estimate 7 sheets and expect to cut one to fit.
   6 would leave us short, therefore we round up the next number of sheets.

Topic 2 – Problems Involving Estimation of Raw Materials for a Job Area

Review:

Area is expressed in square units. The area of a floor tile that measures 1 foot on each side is 1′x1′ = 1 square foot. The area of a tile measuring 2′ by 1′ is 2 square feet. A square metre is the area covered by a tile that measures 1 metre on a side. A rectangle and a square have the same formula for area. Multiply one side times the other side.

Think of coverage when you think of area. Remember to round up when estimating materials so that you have enough to finish the job. Every area problem involves square units, and converting from units to subunits and vice versa. Review the section on converting units.

Sample Problems

1. A floor measures 12′ by 14′. How much carpet material will be required for a wall to wall carpet?

> **Review Tip:**
>
> Units of area (square inches, square feet, square metres, etc.) in the numerator and denominator of a division problem may cancel out to give a "scalar" answer – a number of pieces or times is a quantity, not a dimension of something
>
> Example: 10 square feet ÷ 2 square feet = 5, not five square feet or five feet.
> 5 means "five times" when units "cancel out".

We need to know the area of the floor. The floor is a rectangle. Multiply 12′ x 14′ = 168 square feet. Since the carpet is wall to wall, we will need 168 square feet of material.

2. Amy wants to roof her cabin with metal sheets. Each sheet is 2.5′ wide by 8 feet long. Her roof measures 16′ by 20′ on each side. How many sheets will she need for the job?

We can calculate the total area of the roof and then compare this with the area of one sheet.

The total area is 2 x 16′ x 20′ = 640 square feet (because the roof has two sides)

Each sheet has an area = 2.5′ x 8′ = 20 square feet. We divide 640 square feet by 20 square feet to see how many sheets are needed. 640 square feet ÷ 20 square feet = 32 sheets. (the units, square feet, cancel out)

Topic 2 – Problems Involving Estimation of Raw Materials for a Job Area

3. Bill wants to paint his house. Each side of the house is 150 square metres in area. The paint can says that 1 litre will cover 4 square metres of surface area. How many litres will he need for his job?

   Divide the total area to be painted by the area 1 litre will cover.
   150 square feet ÷ 4 square feet = 37.5 litres. In order not to be short, Bill orders 38 litres of paint.

4. The Aklavik golf tournament requires astro turf to cover the course. The golf course measures 250 metres by 500 metres. What is the area of astro turf required?

   The area of the course is the product of the lengths of the sides.
   250 metres x 500 metres = 125,000 square metres.

5. The pool cover at the Ruth Inch Memorial Pool is a perfect square and has an area of 400 square feet. The edges of the cover need to be replaced with new material. How much material will be needed to re-edge the entire cover?

   We need to know the length of one side of the cover. Because we know the area of the cover, and that the cover is a square, we can take the square root of the area to learn the length of the side

   $\sqrt{400}$ square feet = 20 feet

   The cover has four sides, so we will need 4 x 20′ = 80 feet of edging material.

Topic 3 – Problems Involving Estimate of Raw Materials for a Job Volume

Review:

Volume is expressed in cubic units. A cubic unit is an output of a volume formula. A cube is a special case of a rectangular solid because each of its three edges are equal. The volume of a cube is the edge times itself three times. Length, width (or depth), and height are equal to each other in a cube. The volume of a cube = e^3 (where e = the length of an edge). The cube root of the volume of a cube is the length of its edge.

A cubic unit can be expressed in any system of measurement. A cubic foot describes a cube measuring one foot on each of its three edges, a cubic metre describes a cube measuring one metre on each of its three edges. A cubic metre = 1 metre x 1 metre x 1 metre. A cubic foot = 1 foot x 1 foot x 1 foot. Cubic feet must be divided by cubic feet, cubic metres by cubic metres and so on.

Be sure to convert all measurements into the same units before calculating volume, and express your answer in cubic units.

Sample Problems

1. A concrete foundation requires a slab measuring 16 feet by 12 feet and 6 inches thick. How much concrete will be needed?

   We need to know the number of cubic feet that will be poured into a form like this. First change all of the units into feet. 6" = .5′. The volume of the slab = 16′ x 12′ x .5′ = 96 cubic feet.

2. Water must be trucked into a community once a week to fill a tank that measures 12′ by 8′ by 6′. How may cubic feet of water must be trucked in?

   The volume of the tank = 12′ x 8′ x 6′ = 576 cubic feet.

3. The insulation required for a house consists of fiberglass batts measuring 2′ by 3′ by 1′. What will be the volume of fiberglass in a wall measuring 12′ x 16′ x 1′ thick? How many batts will be needed?

   The volume of the wall = 12′ x 16′ x 1′ = 192 cubic feet. One batt has a volume of 2′ x 3′ x 1′ = 6 cubic feet. Divide 6 cubic feet into 192 cubic feet to see how many batts are needed = 32 batts.

4. A landfill is being dug that will measure 1000 feet x 2000 feet x 20 feet. What is the volume of earth that will be hauled away?

   The volume of the earth being removed equals the volume of the hole being dug. Volume = 1000′ x 2000′ x 20′ = 40,000,000 or 4.0×10^7 cubic feet. This number could be simplified by dividing it by the number of cubic feet in one cubic yard.

Topic 3 – Problems Involving Estimate of Raw Materials for a Job Volume

5. A truck can carry 2000 cubic feet of milled lumber. A beam is milled near Fort Simpson that measures 6 inches x 10 inches x 20 feet. How many of these beams could the truck carry?

   We need the volume in cubic feet of one beam in order to divide into the total of 2000 cubic feet that the truck can carry. Change the dimensions into feet and multiply.

   Volume of one beam in cubic feet = .5′ x .83′ x 20′ = 8.3 cubic feet.

   2000 cubic feet ÷ 8.3 cubic feet = 241 pieces (rounded to nearest whole piece)

Tip:

The units "cubic feet" in the numerator and denominator ‘cancel out’ to give a "scalar" answer – the number of pieces is a quantity, not a dimension of something.

Unit 4 – Measurement and Estimation Problems

Topic 4 – Problems Involving Spacing and Dimensions

Spacing refers to how often something is installed, placed, located, or moved. Dimensions refer to the measurement of quantities that affect spacing, for example length, height, width, surface area.

Sample Problems

1. The GNWT is opening a new office building in Yellowknife. It will be 12 stories tall and each story will be 10 feet tall. Each side of the building will be 66 feet long. Two thirds of the way up from the base of each story there will be row of windows. Calculate the following:

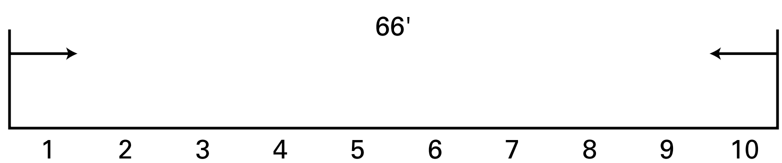
   a) Two 6 foot wide openings for doors will be at street level on the west side. Where will the centre of each opening be placed if they are equally distant from each other and from the corners?

   Two points on a line that are equally distant from each other and from the corners will divide the line into thirds. Draw a line to see that this is so.
   66′ ÷ 3 = 22′

   The centre of each door will be 22 feet from the corners, and 22 feet from each other.

   b) Each story will have a row of windows on each side. Each window is 2 feet 6 inches wide. How much space should be between the windows and the windows and corners in order to evenly space 10 windows per side?

   Think of the side as a line. Draw ten windows on the line approximately an equal distance apart. You will see that we want to divide the line into eleven equal spaces. You might think ten spacings will do, but if you make a drawing, the two spaces required between the end windows and the corners make 11 intervals (spaces) necessary.



66′ ÷ 11= 6′. The midpoints of the windows will be 6′ apart from each other and from the corners. You can check this answer by multiplying 6′ by 11= 66′. These points would be the midpoints for ten windows of any width on a 66′ line. The fact that each one is 2′6" allows a builder to mark the edges for framing each window at 1′3" from the centre of each window.

Topic 4 – Problems Involving Spacing and Dimensions

2. A carpenter is nailing boards onto a stud wall for sheathing. The wall is 8′ high and 16′ long. He has thirteen 1" x 6" x 16′ pieces to use up before starting on a pile of 1" x 8" x 16′ pieces. How many 1" x 8" x 16′ pieces will he need to finish the job?

   This problem can be solved by looking at the area that the thirteen 1" x 6" pieces will cover and subtracting this from the total area. The remaining area will then be covered by the 1" x 8" pieces.

   First calculate the total area to be covered: 8′ x 16′ = 128 square feet.

   Calculate the area that thirteen 1" x 6" x 16′ will cover. Convert to feet and multiply:

   .5′ x16′ = 8 square feet x 13 pieces = 104 square feet

   Now calculate the remaining area to cover = total area minus the part covered by the 13 pieces of 1" x 6". 128 square feet – 104 square feet = 24 square feet.

   Calculate what one piece of 1" x 8" x 16 feet will cover. Convert to feet and multiply. 8" = .75 foot. .75′ x 16′ = 12 square feet. Two pieces will cover 24 square feet.

   We will need two pieces of 1" x 8" x 16′ to finish the job.

3. A carpenter wants to put up a wall with a 2" x 4" stud every 16.5". The wall is 13 feet long. If he starts on one end of the floor and marks where to put a stud every 16.5", how many studs will fit, and what will be the space remaining between the last stud and the corner?

   This is a layout problem on a ruled line. We divide 13′ by 16.5 inches to find the number of studs that will fit. First convert feet to inches.

   13′ = 13 x 12" = 156". Now divide: 16.5" into 156" = 9.45 times. This means we can space 9 studs 16.5" inches apart with some space left over. .45 of 16.5" will be left over. .45 x 16.5" = 7.425" inches will be left between the last stud and the corner.

Unit 4 – Measurement and Estimation Problems

Topic 5 — Problems Involving the Addition, Subtraction, Multiplication and Division of Time (hrs and min)

Review the section on converting units to subunits and subunits to units. The exchange factors for time include 1 day = 24 hours, 1 hour = 60 minutes, 60 seconds = 1 hour.

Sample Problems

1. Bill worked $3\frac{1}{2}$ hours, Jane worked 7 hours and 20 minutes, Brian worked 13 hours and 10 minutes. How many hours were worked by all three people?

   This is an addition problem that requires hours to be added to hours and minutes or parts of an hour to be added to each other. First add the hours: 3 + 7 + 13 = 23 hours. Now add the remaining time: $\frac{1}{2}$ hour + 20 minutes + 10 minutes. We can't add $\frac{1}{2}$ an hour to minutes, so we change $\frac{1}{2}$ hour to the number of subunits, minutes, it equals. $\frac{1}{2}$ hour = 30 minutes.

   Now add all of the minutes: 20 + 10 + 30 = 60 minutes = 1 hour

   Our final answer is 23 hours + 1 hour = 24 hours = 1 day.

2. The sun will set in 3 hours and 45 minutes. It is now 11 p.m. What time will the sun set?

   We need to add 3 hours and 45 minutes to 11:00 pm. 11 pm + 3 = 2:00 a.m. Adding 45 more minutes gives 2:45 a.m. The sun will set at 2:45 am. It is probably July in Yellowknife.

3. Bill is willing to work the last 3 hours of my shift. If my 8 hour shift starts at 4 pm, when will Bill show up to replace me?

   The shift is 8 hours and Bill will work the last three. That means 8 – 3 = 5 hours that I will work before he comes to relieve me. Since I start at 4 pm, I will work 4:00 + 5 hours before he comes. This means he will come at 9:00 pm, after I work five hours.

4. Terry worked 3.5 hours per day for 8 days in a row. How many hours did he work
   in all?

   Multiply 3.5 x 8 = 28 hours in total

5. How many minutes are in 69.2 hours?

   We know there are 60 minutes in one hour, so 69.2 x 60 = the number of minutes in 69.2 hours = 4152 minutes.

6. How many hours are in 7198 minutes?

   We know there are 60 minutes in one hour. Divide the number of minutes by 60 to get the number of hours. 7198 ÷ 60 = 119.97 hours.

   .97 of one hour = .97 x 60 minutes = 58.2 minutes. The complete answer is 119 hours and 58.2 minutes.

Topic 5 — Problems Involving the Addition, Subtraction, Multiplication and Division of Time (hrs and min)

7. Five people want to share a computer that is available for 5.5 hours. How much time can each person have?

   We need to divide the available time by the number of people. 5.5 ÷ 5 = 1.1 hours, 0.1 hours x 60 minutes = 6 minutes, each gets 1 hour 6 minutes.

8. It took Bill 4 hours and 20 minutes to get from Rae Edzo to Yellowknife in bad weather in his Jeep. Jim took 2 hours and 10 minutes in his Ford on the same day. How much shorter was the trip for Jim than for Bill?

   We need to know the difference in the travel times. Subtract the shorter trip Jim had from the longer one that Bill had.

   4 hours 20 minutes – 2 hours 10 minutes = 2 hours 10 minutes. It took Bill twice as long to make the same trip that Jim made. Or put the other way, Jim got there twice as fast as Bill did.

9. It takes Clara 4 hours to sew the uppers on a pair of dress mukluks. She has 12 pairs to do. How long will she need?

   This is a multiplication problem. Multiply the time per mukluk (the unit time) times the number of mukluks (the number of units). 4 x 12 = 48 hours. Clara will need 48 hours or two days to get the job done.

10. The ice gets thicker by 1 mm for every 10 minutes that goes by at –30 degrees celsius. How thick will the ice be in 1 hour and 30 minutes?

    We need to know how many ten minute periods are in 1 hour and 30 minutes. This number times 1 mm will give the total increase in thickness after 1 hour and 30 minutes.

    Divide 1 hour and 30 minutes by 10. First convert hours to minutes.

    1 hour and 30 minutes = 90 minutes

    90 minutes divided by 10 = 9

    There are 9 periods of 10 minutes each, so the ice will thicken
    1 mm x 9 = 9 mm at the end of 1 hour and 30 minutes.

Unit 5

Practice Exam Questions for Math – Module 4 – Measuring Time, Shapes and Spaces

There are four modules in the common core required for all trades entrance math exams. Each module has a set of practice exam questions with an answer key. Each topic in the table of contents has sample questions to test your preparation for the trades entrance exam.

You should aim for 100%, and study the sections of the curriculum for any topics that you do not get right. After each answer the units and topics you should review are identified. Turn to the appropriate part of the curriculum whenever you need help.

The core math curriculum is based on "need to know" competencies that are important in all trades. You may want to use the following sample exam questions both as a way of assessing what you need to learn before you work on the curriculum, and as a test of what you know after you have completed your preparation for the exam.

Answer Key

The following questions are grouped in clusters of related items. You may want to randomly pick questions from different parts of the test for pre-test purposes, or you may want to home in on a particular area by using the answer key. Use the answer key to see which topics go with each question. For example, questions 1-5 are concerned with Unit 1 (measurement), Unit 3, Topic 1 (perimeter), and Unit 4, Topic 1 (problems with linear estimations for a job). Question 65 deals with Unit 2, Topic 3.A – Similar and Congruent Triangles. If you want to take a pre-test, select every fourth question, score yourself, and you will get an idea of what to study. After studying, try every third question and see how well you do and repeat the study cycle based on your results.

Unit 5 – Practice Exam Questions

Please feel free to refer to page 24 of this module for a table of formulas and conversions.

Question 1

How many posts will be needed for one side of a field that is $\frac{1}{2}$ mile long if a post is put in every 8 feet?

a) 500
b) 330
c) 1100
d) 264

Question 2

How many 8" bricks are needed to do a job across a window sill that is 4′6" wide?

a) 8
b) 10
c) 7
d) 9

Question 3

A spool of wire is 8760 feet long. Each wiring job requires 2330 feet. How many jobs will the spool do?

a) 2
b) 3
c) 4
d) 5

Question 4

A roof takes 600 nails. Each box of nails has 250 nails in it. How many boxes should you bring for the job?

a) 1
b) 2
c) 3
d) 4

Question 5

How many studs are needed for a wall that is 22 feet long if they are placed every 16.5 inches?

a) 12
b) 14
c) 15
d) 16

Unit 5 – Practice Exam Questions

Question 6

How many square metres of carpeting will be needed to cover a room that measures 6 metres by 8 metres?

a) 48 square feet
b) 86 square metres
c) 4.0 square metres
d) 48 square metres

Question 7

How many 4′ by 8′ sheets of plywood are required to cover a shed roof measuring 10′ by 12′?

a) 32
b) 5
c) 4
d) 3

Question 8

The ski club wants to paint their cabin. The walls add up to 2300 square feet of surface area. One gallon of paint covers 200 square feet. How many gallons will the club need?

a) 10
b) 11
c) 12
d) 13

Question 9

The pool committee wants to order a pool cover. The pool is 32 metres by 12 metres. How many square metres will be needed for the cover?

a) 2.54
b) 600
c) 384
d) 684

Question 10

A kitchen tile measures 4" by 3". A countertop measures 8 feet by 2 feet. How many tiles are needed to cover the countertop?

a) 109
b) 192
c) 96
d) 196

Unit 5 – Practice Exam Questions

Question 11

How much concrete is needed for a pour into a wall form measuring 10 feet x 25 feet x 8 inches?

a) 250 cubic feet
b) 8025 cubic inches
c) 167 cubic feet
d) 2000 cubic feet

Question 12

How much water will come out of a pool measuring 14′ x 12′ x 6′?

a) 10,000 gallons
b) 192 cubic feet
c) 90 cubic yards
d) 1008 cubic feet

Question 13

How much insulation will be available from 20 batts of fiberglass each measuring 2′ x 3′ x 6"?

a) 40 cubic feet
b) 36 cubic feet
c) 60 cubic feet
d) 600 cubic feet

Question 14

Will a pit measuring 8′ by 20′ by 15′ be filled by 2890 cubic feet of earth?

a) yes
b) no
c) almost
d) can’t tell from this information

Question 15

A cement job requires 890,000 cubic feet of cement mix. How many cubic yards should the cement company truck in for the job?

a) 5,000
b) 50,000
c) 26,090
d) 32,963

Unit 5 – Practice Exam Questions

Question 16

One sheet of drywall measures 4′ by 8′. How many sheets will be needed to cover a wall that measures 8′ by 58′?

a) 16
b) 16.5
c) 14
d) 14.5

Question 17

A 3′ wide door is being placed in the middle of a 14 foot wall. How far is the centre of the door from each end of the wall?

a) 6.5 feet
b) 7.5 feet
c) 7 feet
d) 4.5 feet

Question 18

How many studs will be needed to for a wall measuring 23 feet if they are placed on centre every 16.5 inches?

a) 17
b) 16.5
c) 15
d) 12

Question 19

How many pieces of 1" x 4" x 8′ siding will be needed to cover a wall measuring 8′ x 8′?

a) 32
b) 30
c) 24
d) 20

Question 20

It took 10 boards of 1" x 8" x 10′ nailed horizontally to cover a wall. How high is the wall?

a) 15″
b) 80″
c) 25″
d) 20″

Unit 5 – Practice Exam Questions

Question 21

What is $\frac{1}{3}$ of 16 feet?

a) $4\frac{1}{3}$ feet
b) $3\frac{1}{3}$ feet
c) $5\frac{1}{3}$ feet
d) 4 inches

Question 22

Multiply 3′2" by 6

a) 192.2 inches
b) 19 feet
c) 19 inches
d) 19.9 inches

Question 23

Divide 112 feet into 4 equal lengths. How long will each length be?

a) 44 feet
b) 144 inches
c) 28 feet and 4 inches
d) 28 feet

Question 24

Add 5′6", 7′2", and 6′11"

a) 12′6"
b) 17′8"
c) 19′4"
d) 19′7"

Question 25

Subtract 5′4" from 11′ 3"

a) 5′11"
b) 11′5"
c) 7′1"
d) 11′7"

Unit 5 – Practice Exam Questions

Question 26

How many seconds are in 4 hours?

a) 1,400
b) 4,000
c) 144,000
d) 14,400

Question 27

Convert 23 feet 3 inches to inches

a) 96 inches
b) 911.5 inches
c) 279 inches
d) 20 feet 39 inches

Question 28

What is the dollar value of 5500 pennies?

a) $155.00
b) $550.00
c) $55.55
d) $55.00

Question 29

How many miles are in 58,080 feet?

a) 12
b) 13
c) 11
d) 10

Question 30

How many ounces are in 52 lbs. 7 ounces?

a) 839
b) 527
c) 5839
d) 939

Question 31

How many metres are in 55 yards?

a) 55.292
b) 110
c) 46.47
d) 50.29

Unit 5 – Practice Exam Questions

Question 32

Change 13 gallons into an equivalent number of litres.

a) 39
b) 69.2
c) 59.1
d) 79.1

Question 33

How many kilometres are in 560 miles?

a) 1,001.23
b) 1,560
c) 915.63
d) 901.04

Question 34

How many pounds are equal to 43 kilograms?

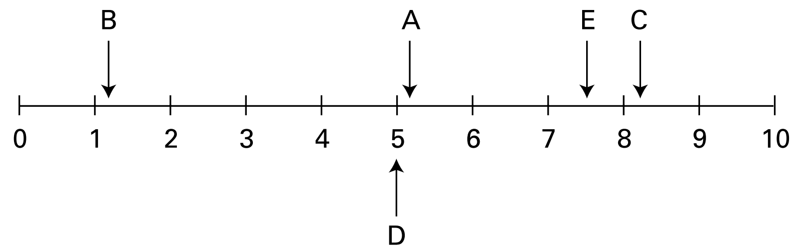
a) 220.15
b) 115.2
c) 230.2
d) 86

Question 35

How many litres are in 33 gallons?

a) 112.1
b) 150.2
c) 124.9
d) 132.2

Unit 5 – Practice Exam Questions



Question 36

The point labelled A measures:

a) 5 cm
b) 5.2 cm
c) 50 mm
d) 5 dm

Question 37

The point labelled B is closest to:

a) 1 dm
b) 10 mm
c) 10 cm
d) .7 cm

Question 38

The point labelled C equals:

a) 73 mm
b) 7 cm
c) 8.3 cm
d) 75 mm

Question 39

What does the point labelled D measure?

a) 11 cm
b) 111 mm
c) 11 dm
d) .5 dm

Question 40

What does the point labelled E measure?

a) 7.8 cm
b) 7 cm and 5 mm
c) 5 cm and 7 mm,
d) none of the above

Unit 5 – Practice Exam Questions

Question 41

What is the area of a gymnasium floor that measures 14 yards by 12 yards?

a) 368 square feet
b) 168 square yards
c) 144.12 square yards
d) 177 square feet

Question 42

Calculate the depth of a holding tank that measures 13′ x 12′ on top and that has a volume of 1248 cubic feet.

a) can't tell from this information
b) 10 feet
c) 8 feet
d) 1248 cubic feet

Question 43

The radius of a circle is 12′. What is the area of the circle?

a) 144 square feet
b) 12π
c) 576 square feet
d) 452.16 square feet

Question 44

A triangle has a height of 10 feet and a base of 3 feet. What is the area of the triangle?

a) 300 square inches
b) 30 square feet
c) 180 square inches
d) 15 square feet

Question 45

The area of a triangle is 32 square metres. The base is 8 metres.
What is the height?

a) 6 metres
b) 12 metres
c) 8 metres
d) 16 centimetres

Unit 5 – Practice Exam Questions

Question 46

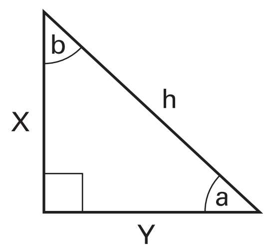
In a right triangle, the sine of angle a =

a) side adjacent to a/hypotenuse
b) the adjacent side/hypotenuse
c) the hypotenuse/the adjacent side
d) the side opposite/hypotenuse

Question 47

In a right triangle, the tangent of a =

a) side adjacent/hypotenuse
b) the hypotenuse/adjacent side
c) the opposite side/the adjacent side
d) the side opposite/hypotenuse



Question 48

What is the tangent of angle b?

a) h/y
b) x/y
c) y/x
d) x/h

Question 49

The cosine of angle a is equal to:

a) h/y
b) y/h
c) x/y
d) y/x

Question 50

If X = 6 and Y = 10, what is the tangent of angle a?

a) 1.4
b) .6
c) 6.0
d) .16

Unit 5 – Practice Exam Questions

Question 51

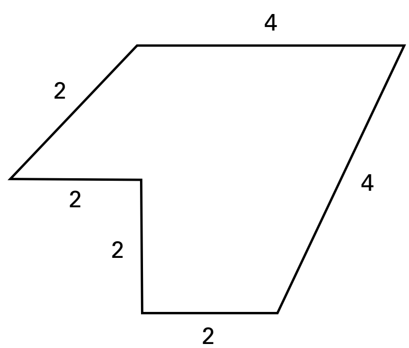
A circle has a radius of 5′. What is the circumference?

a) 10′
b) 15.14′
c) 31.4′
d) 25′

Question 52

What is the perimeter of this figure

a) 12′
b) 16′
c) 24"
d) 18′



Question 53

The circumference of a circle is 32 metres. What is the radius?

a) 16.6 metres
b) 5.09 metres
c) 64.09 metres
d) 8.09 metres

Question 54

The diameter of a circle is 3 when the circumference is:

a) 1.5
b) 9
c) 3.14
d) 9.42

Unit 5 – Practice Exam Questions

Question 55

How big should the diameter be to get a circular room that has a circumference of 35 feet?

a) 70 feet
b) 11.14 feet
c) 12.33 feet
d) Can’t tell from this information

Question 56

In a right triangle, $a^2 = 144$, and $b^2 = 144$, what is c?

a) $12\sqrt{2}$
b) 12
c) $\sqrt{28}$
d) $\sqrt{14}$

Question 57

What part of the circumference is bounded by an angle of 135 degrees?

a) 2/3
b) 4/5
c) 3/4
d) 3/8

Question 58

What angle is made with the horizontal diameter when a radius line is drawn at 4/5th of the circumference?

a) 1,500 degrees
b) 2,500 degrees
c) 288 degrees
d) 330 degrees

Question 59

The sum of two angles of triangle ABC = 75 degrees. What is the size of the third angle in the triangle?

a) 175 degrees
b) 90 degrees
c) 105 degrees
d) 115 degrees

Unit 5 – Practice Exam Questions

Question 60

Angle A in triangle ABC = 30 degrees. If we want to design ABC to have one of the remaining angles be twice the size of the other one, what will the two angles each equal?

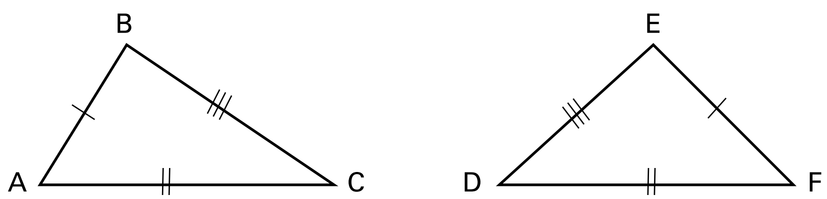
a) 25 degrees, 50 degrees
b) 75 degrees, 75 degrees
c) 75 degrees, 150 degrees
d) 50 degrees, 100 degrees

Question 61

If angle A = 45 degrees, and angle B = 60 degrees, find angle C in triangle ABC.

a) 90 degrees
b) 85 degrees
c) 75 degrees
d) 60 degrees

Question 62

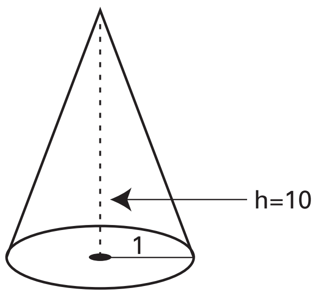


What has to be done to the triangle on the left to make it overlay the triangle on the right?

a) slide
b) flip
c) rotate
d) exchange places

Unit 5 – Practice Exam Questions

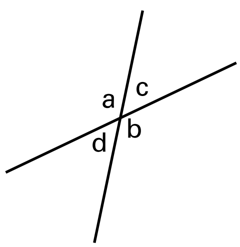
Question 63



What is the volume of this figure?

a) 3.14
b) $\frac{10\pi}{3}$
c) 100π
d) 10π

Question 64



Which angles are the same size?

a) a and d
b) c and d
c) a and c
d) none are equal

Question 65

Similar triangles have

a) equal angles and equal sides
b) equal angles and proportional sides
c) proportional angles and equal sides
d) identical sides

Question 66

What is the area of a triangle with a base of 10 inches and a height of 12 inches?

a) 22 square inches
b) 11 square inches
c) 60 square inches
d) 34 square inches

Unit 5 – Practice Exam Questions

Answers	Study Topics
1) b	Unit 3, Topic 1 and Unit 4, Topic 1
2) c	Unit 3, Topic 1 and Unit 4, Topic 1
3) b	Unit 3, Topic 1 and Unit 4, Topic 1
4) c	Unit 3, Topic 1 and Unit 4, Topic 1
5) d	Unit 3, Topic 1 and Unit 4, Topic 1
6) d	Unit 3, Topic 2 and Unit 4, Topic 2
7) c	Unit 3, Topic 2 and Unit 4, Topic 2
8) c	Unit 3, Topic 2 and Unit 4, Topic 2
9) c	Unit 3, Topic 2 and Unit 4, Topic 2
10) b	Unit 3, Topic 2 and Unit 4, Topic 2
11) c	Unit 3, Topic 3 and Unit 4, Topic 3
12) d	Unit 3, Topic 2 and Unit 4, Topic 2
13) c	Unit 3, Topic 2 and Unit 4, Topic 2
14) a	Unit 3, Topic 2 and Unit 4, Topic 2
15) d	Unit 3, Topic 2 and Unit 4, Topic 2
16) d	Unit 3, Topic 3 and Unit 4, Topic 3
17) c	Unit 3, Topic 1 and Unit 4, Topic 4
18) a	Unit 3, Topic 1 and Unit 4, Topic 4
19) c	Unit 3, Topic 1 and Unit 4, Topic 4
20) b	Unit 3, Topic 1 and Unit 4, Topic 4
21) c	Unit 1, Topics 1 to 4
22) b	Unit 1, Topics 1 to 3
23) d	Unit 1, Topics 1 to 3
24) d	Unit 1, Topics 1 to 3
25) a	Unit 1, Topics 1 to 3
26) d	Unit 1, Topic 2 and Unit 4, Topic 5
27) c	Unit 1, Topics 2 and 3
28) d	Unit 1, Topic 2
29) c	Unit 1, Topic 2

Unit 5 – Practice Exam Questions

Answers	Study Topics
30) a	Unit 1, Topics 2 and 3
31) d	Unit 1, Topics 2 and 3
32) c	Unit 1, Topics 2 and 3
33) d	Unit 1, Topics 2 and 3
34) b	Unit 1, Topics 2 and 3
35) b	Unit 1, Topics 2 and 3
36) b	Unit 1, Topic 4
37) b	Unit 1, Topic 4
38) c	Unit 1, Topic 4
39) d	Unit 1, Topic 4
40) b	Unit 1, Topic 4
41) b	Unit 3, Topic 2 and Unit 4, Topic 2
42) c	Unit 3, Topic 3 and Unit 4, Topic 3
43) d	Unit 2, Topic 4 and Unit 4, Topic 2
44) d	Unit 2, Topic 3 and Unit 4, Topic 2
45) c	Unit 2, Topic 3 and Unit 4, Topic 2
46) d	Unit 2, Topic 3.C
47) c	Unit 2, Topic 3.B
48) c	Unit 2, Topics 2.A, 2.B and 2.C
49) b	Unit 2, Topic 3.C
50) b	Unit 2, Topic 3.C
51) c	Unit 2, Topic 4 and Unit 3, Topic 1
52) b	Unit 3, Topic 1
53) b	Unit 2, Topic 4 and Unit 3, Topic 1
54) d	Unit 2, Topic 4 and Unit 3, Topic 1
55) b	Unit 2, Topic 4 and Unit 3, Topic 1
56) a	Unit 2, Topic 3.C
57) d	Unit 2, Topic 4 and Unit 3, Topic 1
58) c	Unit 2, Topic 4 and Unit 3, Topic 1

Unit 5 – Practice Exam Questions

Answers	Study Topics
59) c	Unit 2, Topic 3
60) d	Unit 2, Topic 3
61) c	Unit 2, Topic 3.A
62) b	Unit 2, Topic 3.A
63) b	Unit 3, Topic 3 and Unit 4, Topic 3
64) b	Unit 2, Topic 1
65) b	Unit 2, Topic 3.A
66) c	Unit 2, Topic 3 and Unit 3, Topic 2

Unit 6

Resources

Abbott, P. Algebra, Teach Yourself Books, NTC Publishing Group, 1999. (ISBN 0-340-65847-9)

Alberta Vocational College: Pre-Trades Math, 6 modules

B.C. Ministry of Education, ABE Math modules

Barry Maurice, Small Marian, et al, Constructing Mathematics, Nelson Learning, Thomson Publishing 2000. (ISBN 0-176-05983-0)

Cleaves, Cheryl, Hobbs, Margin, Dudenhefer, Paul, Introduction to Technical Mathematics, Prentice Hall 1988. (ISBN 0-13-498700-4)

Crown, A.W., The Language of Triangles, Pergamon Press, Macmillan, NY, 1964. (LOC# 63-22314)

Doran, Edward, Fundamental Mathematics For College and Technical Students, Finesse Publishing, Broomfield Colorado 1988. (ISBN 0-941775-02-X)

Elchuck, Larry et al., Interactions 7,8,9,10, Teacher's Resource Books, Prentice Hall Ginn Canada, 1996. (ISBN 0-13-273129-0)

Gage Canadian GED Study Guide Series: Mathematics GED, Gage publishing 1998, (ISBN 0-7715-5141-X)

Garfunkel, Solomon, project director, For All Practical Purposes: Introduction to Contemporary Mathematics, fourth Edition, Freeman and Co. NY, 1997. (ISBN 0-7167-2841-9)

Gelfand, I.M., Saul, M., Trigonometry, Birkhauser, Boston, 2001. (ISBN 0-8176-3914-4)

Hopkins, Nigel, et.al., The Numbers You Need, Gale Research, 1992. (ISBN 0-8103-8373-X)

Jacobs, Harold, Geometry, W.H. Freeman and Co., San Francisco, 1974.

(ISBN 0-7167-0456-0)

IBID, Mathematics: A Human Endeavor, 2nd edition, Freeman and Co, NY. (ISBN 0-7167-1326-8)

Unit 6 – Resources

McKeague, Charles P., Elementary Algebra, fifth edition, Harcourt Brace, 1995. (ISBN 0-03-097356-2)

Mathematics GED, Gage Canadian Study Guide Series, 1998, (ISBN 0-7715-5141-X)

O'daffer, Phares, Clemens Stanley, Geometry: An Investigative Approach, Addison-Wesley Publishing, 1977. (ISBN 0-201-05420-5)

Rosenberg, Robert, R., Business Mathematics, Canadian Edition, Book I, McGraw Hill Ryerson, 1977. (ISBN 0-07-082599-8)

Rudolf, Gordon, B., Introductory Trigonometry, Vancouver Community College Press, 1993. (ISBN 0-921218-54-0)

Shanks, Et.al, Pre-Calculus Mathematics, Addison Wesley, 1965. LC 65-19230)

Trivieri, Lawrence, Essential Mathematics, 2nd edition, McGraw Hill 1994. (ISBN 0-07-065229-5)

Washington, Allyn J., Basic Technical Mathematics with Calculus, Fourth Edition, metric version, Benjamin Cummings Publishing, 1985. (ISBN 0-8053-9545-8)

Appendix A

The following topics from the Alberta Entrance Level Competencies are covered in **Math – Module 4 – Measuring Time, Shapes and Spaces** of this curriculum.

SECTION SIX – MEASUREMENT OF SHAPE AND SPACE

A. Properties Of Circles, Angles And Time Zones

Outcome: Solve problems involving the properties of circles and their connections with angles and time zones. (1, 2, 3, 4, 5)

1. Measure the diameters, radii and circumferences of circles, and establish the relationships among them.
2. Solve problems involving the radii, diameters and circumferences of circles.
3. Explain how time zones are determined.
4. Research and report how measurement instruments are used in the community.

B. Metric And Imperial Measure

Outcome: Solve problems involving Metric and Imperial measure. (1, 2, 3, 4, 5)

1. Identify commonly used metric units of measurement.
2. Convert between units of measurement.
3. Convert imperial units:
   a. Feet to inches and vice versa.
   b. Square inches to square feet and vice versa.
   c. Cubic inches to cubic feet and vice versa.
   d. Cubic measures to gallons.

C. **Indirect Measurement Procedures**

Outcome: Apply indirect measurement procedures to solve problems. (1, 2, 3, 4, 5)

1. Use concrete materials and diagrams to develop the Pythagorean relationship.
2. Use the Pythagorean relationship to calculate the measure of the third side of a right triangle, given the other two sides in 2-D applications.

D. **Area, Perimeter, Surface Area And Volume**

Outcome: Generalize measurement patterns and procedures, and solve problems involving area, perimeter, surface area and volume. (1,2,3,4,5)

1. Describe patterns, and generalize the relationships by determining the areas and perimeters of quadrilaterals and the areas and circumferences of circles.
2. Estimate, measure and calculate the surface area and volume of any right prism or cylinder.
3. Estimate and calculate the area of composite figures.
4. Estimate, measure and calculate the surface area of composite 3-D objects.
5. Estimate, measure and calculate the volume of composite 3-D objects.

E. **Trigonometric Ratios**

Outcome: Use trigonometric ratios to solve problems involving a right triangle. (1, 2, 3, 4, 5)

1. Explain the meaning of sine, cosine and tangent ratios in right triangles.
2. Demonstrate the use of trigonometric ratios (sine, cosine and tangent) in solving right triangles.
3. Calculate an unknown side or an unknown angle in a right triangle, using appropriate technology.
4. Model and then solve given problem situations involving only one right triangle.

F. **Problem Solving Involving Dimension Changes In Two And Three Dimensional Objects**

Outcome: Describe the effects of dimension changes in related 2-D shapes and 3-D objects in solving problems involving area, perimeter, surface area and volume. (1, 2, 3, 4, 5)

1. Relate expressions for volumes of pyramids to volumes of prisms, and volumes of cones to volumes of cylinders.
2. Calculate and apply the rate of volume to surface area to solve design problems in three dimensions.
3. Calculate and apply the rate of area to perimeter to solve design problems in two dimensions.

Appendix A

SECTION SEVEN – 3-D OBJECTS AND 2-D SHAPES

A. Angle Measures And Properties Of Parallel Lines

Outcome: Link angle measures to the properties of parallel lines. (1, 2, 3, 4, 5)

1. Measure and classify pairs of angles as complementary or supplementary angles.
2. Investigate, identify and name pairs of angles pertaining to parallel lines and transversals, including: corresponding, vertically opposite, interior on the same side of the transversal, and exterior on the same side of the transversal.
3. Describe the relationships between the pairs of angles pertaining to parallel lines and transversals.
4. Explain, in more than one way, why the sum of the measures of the angles of a triangle is 180 degrees.
5. Use mathematical reasoning to determine the measures of angles in a diagram.
6. Construct angle bisectors and perpendicular bisectors.

B. Angle Measures, Properties of Parallel Lines and Properties of Quadrilaterals

Outcome: Link angle measures and the properties of parallel lines to the classification and properties of quadrilaterals. (1, 2, 3, 4, 5)

1. Identify, investigate and classify quadrilaterals, regular polygons and circles, according to their properties.
2. Build 3-D objects from a variety of representations (nets, skeletons).

C. Similar And Congruent Triangles

Outcome: Specify conditions under which triangles may be similar or congruent, and use these conditions to solve problems. (1, 2, 3, 4, 5)

1. Recognize when, and explain why, two triangles are similar, and use the properties of similar triangles to solve problems.
2. Recognize when, and explain why, two triangles are congruent, and use the properties of congruent triangles to solve problems.
3. Relate congruence to similarity in the context of triangles.

D. Describe And Analyze Geometric Shapes

Outcome: Use spatial problem solving in building, describing and analyzing geometric shapes. (1, 2, 3, 4, 5)

1. Draw the plan and elevations of a 3-D object from sketches and models.
2. Sketch or build a 3-D object, given its plan and elevation views.
3. Recognize and draw the locus of points in solving practical problems.

Appendix A

SECTION EIGHT – TRANSFORMATIONS

A. Create And Analyse Patterns And Designs

Outcome: Create and analyze patterns and designs, using congruence, symmetry, translation, rotation and reflection. (1, 2, 3, 4, 5)

1. Create, analyze and describe designs, using translations (slides), rotations (turns) and reflections (flips).
2. Use informal concepts of congruence to describe images after translations, rotations and reflections.
3. Draw designs, using ordered pairs, in all four quadrants of the coordinate grid, together with translation and reflection images.
4. Relate reflections to lines and planes of symmetry.

B. Architectural Patterns

Outcome: Create and analyze design problems and architectural patterns, using the properties of scaling, proportion and networks. (1, 2, 3, 4, 5)

1. Represent, analyze and describe enlargements and reductions.
2. Draw and interpret scale diagrams and colouring problems.
3. Describe, analyze and solve network problems.

C. Geometry And Pattern Recognition

Outcome: Apply coordinate geometry and pattern recognition to predict the effects of translations, rotations, reflections and dilatations on 1-D lines and 2-D shapes. (1, 2, 3, 4, 5)

1. Draw the image of a 2-D shape as a result of a single transformation, dilatation, combinations of translations, and/or reflections.
2. Identify the single transformation that connects a shape with its image.
3. Demonstrate that a triangle and its dilatation image are similar.
4. Demonstrate the congruence of a triangle with its translation image, rotation image, and reflection image.