NWT Apprenticeship Support Materials

Math

Reading Comprehension

Science

* *Module 1 – Foundations*
* *Module 2 – Patterns and Relations*
* *Module 3 – Variables and Equations*
* *Module 4 – Measuring Time, Shapes and Space*
* *Module 5 – Special Topics*

P A R T N E R S

Snap Lake Diamond Project De Beers

Aurora College

Northwest Territories Education, Culture and Employment

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The Apprenticeship Support Materials Project has been a true partnership. The challenge has been to develop a set of study materials, which covers every competency in the "Entrance Level Competencies for Apprenticeship Programs" developed by Alberta Advanced Education, Apprenticeship and Industry Training. These study materials although written for Northerners using northern examples, issues and problems, are appropriate for any individual entering the trades. The goal is to equip Northerners for the Trades Entrance Exam as well as to support them through the apprenticeship process.

The following partner organizations have all contributed to the development of these materials:

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Module One – Foundations: Number Concepts and Operations

Table of Contents

Introduction

This curriculum guide is designed for independent study. It will explain the math competencies required for the trades entrance examination. Guides for science and reading comprehension are also available. The trades entrance examination is based on competencies. This means that **what you know – not how you learned it,** will be assessed. It also means that only what you need to know for entrance into a trade will be assessed.

The same core math skills are required for all trades.

This math curriculum has four modules that go over the content that is required for every trade and that is common to all five exam levels. In addition to the four modules that make up this core, there is an additional special topics module for trades that need to pass a level five exam.

Don't Waste Time
Only learn what you need to know.

The exam tests what you know - not how you learned it.

How to Use This Guide

The core trades entrance math curriculum has been organized into four modules based on the Alberta trades entrance list of competencies[1]. Within each module, only the topics common to all five exam levels are included[2]. Modules 1 and 2 in the Alberta List have been combined into one foundations module titled Module 1 – Foundations – Number Concepts and Operations in this curriculum. Parts 4, 5 and 6 in the Alberta list have been combined into Part Four of the curriculum titled Measuring Time, Shapes, and Spaces. Topics in each module that are only required for Level 5 are not included in the foundations section, and can be found in the separate level five curriculum guide.[3]

The competency areas overlap, and on the examination more than one may be tested in the same problem. For example, if you are asked in a problem to find the percentage of workers who have been sick more than three days a year from a table or graph, you will need to know how to read the table or graph, how to convert numbers into fractions, and how to convert fractions into percentages. Within each section the topics build on each other and later topics rely on understanding the earlier ones.

Pre-Test Yourself

You can turn to Units 4 and 5 in this module and do some of the practice questions to see how much work you need to do. The Practice Exam Questions in Unit 4, and the Practice Exam Questions in Unit 5 are the same kinds of questions that you will find on your trades entrance exam. They are in multiple-choice format. If you have difficulty with any of the practice exam questions, you should go back and study the background and examples for the topics involved.

The Practice Exam Questions include a key to direct you to the competencies you need to review in the module for each question. This unit can be used as a pre-test to see what you need to study, and also as a post test to check how well you will do on the exam after you complete your study of the topics in this guide. Unit 4 includes an explanation for the correct answer to every problem.

1 The complete Alberta List of Competencies can be found in Appendix A.

2 An exam will always test a representative cross section of the total list of competencies. This curriculum is based on what could be asked on a trades entrance exam, not on what recent or actual exams have asked for. This curriculum assumes that exam designs can change, but that the competencies they test will not change.

3 Section Five (relations and functions) and Section Nine (probability and statistics) from the Alberta list are not included in the core because these competencies are only required for Math – Module 5 – Special Topics.

How to Use This Guide

Entrance Level Competencies in Mathematics:

Module 1 – Foundations: Number Concepts and Operations
Module 2 – Pattern and Relations
Module 3 – Variables and Equations
Module 4 – Measuring Time, Shapes & Spaces

Organization of Topics

The emphasis in trades is on using mathematics to solve practical problems quickly and correctly. Each topic in this curriculum guide includes:

Background and Theory

Examples with Explanations

Practice Exam Questions with Answers and Explanations

This guide outlines competencies, but does not provide the detailed lessons given in a textbook. If you need more instruction on a particular competency, or if you want to build up speed as well as accuracy in a competency, you will find additional study materials useful.

Examples are the Focus

In this curriculum guide, examples with explanations are the primary tool used for review. Background for each competency is also given with a brief overview of what you need to know. Before any examples are given, the main ideas in each topic are explained and "need to know" information is summarized in rules and definitions.

You may want to skip the background given on a topic and go right to the examples to see how well you do. You can always go back to the theory if you find you need it. For example, Unit 1 – Number Concepts, is based on pure math and may not be needed for those ready to work immediately on operations with numbers in Unit 2 – Number Operations. Refer to Unit 1 – Number Concepts only as you need it.

Key concepts and guidelines for success are placed in text boxes and put in bold type. Some text boxes include optional topics or supplementary details. Review the key points in bold type before taking the exam.

How to Use This Guide

When you study an example, the right approach is to work slowly:

After giving your answer, think carefully through each explanation. If you get this part right, you will be able to solve similar problems in the practice exam questions.

The explanations given for right answers suggest how you might talk to yourself as you think through the questions. Even when you don't need an explanation, check to see if it describes how you found your answer. **Cover the answers as you read. You will not have to flip pages to find answers in this guide because they are given immediately below each example.**

If you only need this guide for a quick review, you may want to read the text boxes, skip the longer explanations, and do the practice exam questions at the end of each section.

Develop a Study Plan that Works for You

Adults have busy lives with many responsibilities. Finding study time will be a challenge. Here are a few suggestions to guide you.

Study in several short sessions rather than in one long period

This will help you remember more and learn faster. Make and use flash cards to test yourself at odd moments during the day. This is a good way to learn formulas, definitions, and new vocabulary.

Use more than one channel and you will learn more

Seeing, listening, writing, and speaking are different channels that can help to reinforce what you need to know for the exam. For example, try reading out loud, speaking a term as you write it down, sketching a problem before you solve it. Always write down what you are learning by taking notes, solving practice examples, and recording definitions and formulas. Writing adds to the "channels" you are using when you learn from reading.

Pick the best time of day for study

Many people find mornings are best, but you will know what times are best for yourself.

Find quiet times and places that are comfortable

Learning requires undivided and undisturbed time for concentration.

Write down your questions

One of the most important things you can do for yourself is identify what you need to know more about. Take your questions to someone who can help you learn how to answer them.

Develop a Study Plan that Works for You

Keep a pencil and paper handy as you work through the guide

There are many examples for you to repeat for yourself on paper and to use as self-test items. Stress the "aka" - "also known as" relationships that are so important in mathematics. These are relations of equivalence. For example, it is useful to understand that one fourth is also .25, one quarter, 25% and the answer to the subtraction problem $1 - \frac{3}{4}$.

Read the explanations as if they are conversations

The steps given to explain how to solve problems are written as if someone is talking to you. They show what thinking out loud looks like. Read every sentence carefully; there are no wasted words if you are learning a competency for the first time. The right approach is to start slow: think carefully through each explanation. If you get this part right, you will be able to solve all of the problems in the competency area. Then you will gather speed and many solutions will become routine. The problems chosen for the examples are all trades related and illustrate the kind of practical problems you can expect to see on the exam.

Consider working with a partner

Two people can help each other by reading through the curriculum and working on the examples together. This process can also focus on good questions that can be brought to a tutor or instructor when extra help is needed.

Know your attitude

Many adults have mixed feelings about "going back to school" if their classroom experiences were unsuccessful. Identify your issues surrounding education and describe them to an instructor or counsellor. If your attitude towards learning is not helping you, ask for help to change it. Tutoring can include valuable support as well as instruction.

One size does not fit all…

This curriculum is not the answer for everyone seeking to enter a trade. For some people, trades entrance math requires only a quick review of what was learned in school. For others, including those who had unsuccessful learning experiences in school, this curriculum will give an opportunity to learn mathematics competencies used in trades for the first time. In both cases, independent study requires discipline, commitment, and motivation, as well as good literacy skills.

Develop a Study Plan That Works for You

Assess Your Learning Needs

If you have needs that cannot be met by this resource, there are programs available to help you. See your advisor, career counsellor, or educational institution for assistance. Ask them to guide you to a resource that will help you. You may decide that the best way for you is to work on this curriculum with a tutor or in a study group guided by a teacher. Alternatively, you may decide that your best option is to enrol in a college course in pre-trades math.

If you need more help, see your tutor or advisor for extra coaching and instruction. In many cases a supplementary textbook or GED Study Guide will provide needed individual lessons related to the topics in this curriculum. A complete list of competencies within each of the four modules of core trades entrance math based on the Alberta list of trades entrance math competencies is attached to this curriculum as an appendix.

If you turn to a competency required for your exam and don't understand the explanation, you can read the earlier units and topics that lead up to it. The competencies are covered in a logical order from simple to more difficult problems. **Each unit and topic builds on what came before**. Many people find it helpful to read through each competency area before going to the practice exam questions for that area.

Unit 1

Number Concepts

All trades use basic operations to find the answers to questions that involve numbers. In this section we review the kinds of numbers that are important for work in the trades. This section reviews the theory, or pure math, that is intended to support operations with numbers in Unit 2 – Number Operations.

Number concepts are ideas. “Counting”, “fraction”, “adding”, “rate”, “estimate”, and “percent” are number concepts. Ideas about numbers are expressed with definitions, and models that help us understand the meaning of the mathematical language used to solve problems.

Unit 1 – Number Concepts

Topic 1 – Whole Numbers (Integers)

Whole Numbers (Integers)

When you count the fingers on your hand you are counting with whole numbers. When you want to know how many nails are in a 2kg box of nails the answer will be a whole number. A whole number is also known as a positive integer or natural number. 1,2,3,4,etc. are known as positive integers, counting numbers, or as whole numbers.

Counting is labour intensive

It saves time when we don't have to count every nail. If we know that there are 100 nails in every pound we can figure out that a ten pound box has 10 x 100 = 1000 nails. As you will see in this section, the concept of whole numbers is necessary for many calculations in practical situations.

Whole numbers are the numbers we count with. 1, 2, 3, 4, etc. They are also called integers. Whole numbers can be created by repeatedly adding one to itself until the whole number is reached. Example: three is the result of 1 + 1 + 1 = 3, 1 + 1 repeated fifty times = 50, etc. Integers can be divided evenly by 1 and equal themselves. Whole numbers can also be expressed as the product of 1 times the number. Taking the number one time doesn't change its value.

You Need to Know…

Dividing or multiplying by one doesn't change a number

Any number times one gives itself

1 x 50 = 50; 89 x 1 = 89; 767,234 x 1 = 767,234

Any number divided by one gives itself

50 / 1 = 50; 89 / 1 = 89; 767,234 / 1 = 767,234

AKA "also known as"

This happens a lot in mathematics

" = " can mean AKA.

Unit 1 – Number Concepts

Topic 1 – Whole Numbers (Integers)

How to read whole numbers

Whole numbers are read from left to right according to their place value.

Example

987,450,201

Millions		**Thousands**		**Units**	
9	Hundred millions	**4**	Hundred thousands	**2**	hundreds
8	Ten millions	**5**	Ten thousands	**0**	tens
7	Millions	**0**	thousands	**1**	ones
987,		**450,**		**201**	

is read,

"Nine hundred eighty seven million, four hundred fifty thousand, two hundred one." Each column can hold any number from 0 to 9, but the place tells us what power of ten it represents. Read the value of each individual digit from the place it occupies in the number.

Our system for numbers is based on powers of ten

10, 100, 1000, 10,000… Etc.

$10 = 10 \times 1$; $100 = 10 \times 10 = 10^2$; $1000 = 10 \times 10 \times 10 = 10^3$.

These are called "powers of ten"

The small number, called an **exponent**, tells how many times to multiply a number by itself. For completeness, $10^0 = 1$ and $10^1 = 10$

See Unit 3, Topic 6 – Bases, Exponents and Square Roots for more on exponents.

We can compare any two integers and decide which one is larger by placing them on a number line.

1 2 3 4 5 6 7 8 9 10 11 12 13

Each number is bigger than all of the numbers to its left on a number line.

You Need to Know…

= **Means two numbers are the same;** 12 = 12, 2 = 2 etc.

> **Means "greater than";** 12 > 4; 5 > 2 etc.
(bigger number goes on open side of >)

< **Means "less than";** 4 < 12; 2 < 5; 12 < 23 etc.

Unit 1 – Number Concepts

Topic 1 – Practice Questions

Question 1
72,320 expressed in words is

a) seven hundred two hundred and thirty two thousand
b) seventy two thousand three hundred and two
c) seventy two thousand three hundred and twenty
d) seven hundred twenty three thousand and twenty

Answer: c

Explanation
Start by identifying the place value of the left-most digit, 7. It may help to write seven followed by the number of zeros that correspond to the number of integers in the number. In this case there are four places to the right of the 7 and we would get 7 followed by four zeros: 70000. A comma is used to separate groups of three places counting from the right to the left. Now you can read the number easily by putting the digits in their places and reading 72,320 in thousands, tens and ones.

Question 2
How is 170,321 spoken in words?

a) “one hundred seventy three thousand and twenty one”
b) “one million, seventy thousand, three hundred and twenty one”
c) “one hundred seventy thousand three hundred and twenty one”
d) “seventeen thousand, three hundred and twenty one”

Answer: c

Explanation
Start by reading from the left. Identify the place value of one as one hundred thousand. Now add to this 70 thousands, three hundreds, and twenty-one units to get the answer.

Question 3
Which statement is true?

a) $10 < 5$
b) $6 > 4$
c) $7 = 6$
d) $999 < 909$

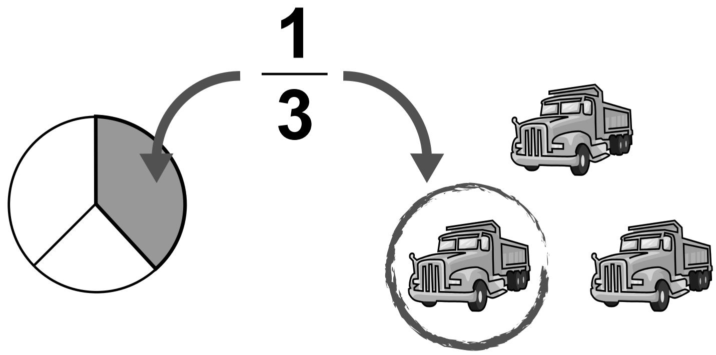
Answer: b

Explanation
Six is greater than four. The other choices are false. Choice a says 10 is less than 5, (false), choice c says 7 equals six, (false), and choice d says nine hundred and ninety nine is less than nine hundred and nine (false).

Unit 1 – Number Concepts

Topic 2 – Fractions

Fractions are parts of something. The "something" can be a single object, like a pie, or a whole collection of objects, like the number of trucks used to supply a mine. We can use the fraction "$^1/_3$" to refer to a third of a pie or to a third of the trucking fleet. By itself, a fraction can mean either of these things. In practical situations we will know whether an object or a collection is being divided into equal sized parts.



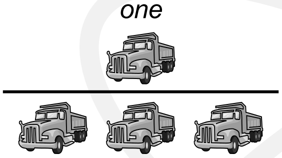
We will also know that the way we divide a pie into equal parts and the way we divide a trucking fleet into equal parts are different. However, in both cases the size of each part will equal the size of every other part. The pieces of the pie will be equal sized, and the pieces of the trucking fleet will consist of the same number of individual trucks. Even though the trucks are not identical, we can talk about $^1/_3$ or 1/4 of the total. For example, one fourth of one hundred trucks is 25 trucks whether or not they are a mixture of Fords or GMC's, new and old vehicles, etc.

$$\frac{1}{3} = \frac{\textbf{Part}}{\textbf{Whole}} = \frac{\textbf{Top \#}}{\textbf{Bottom \#}} = \frac{\textbf{Numerator}}{\textbf{Denominator}} = \textbf{Numerator} \div \textbf{Denominator}$$

A fraction is written as a **numerator** (the top number) over a **denominator** (the bottom number). Every fraction has a top number over a bottom number. **This is also known as a ratio.** The bottom number (denominator) tells us how many equal sized pieces something has been divided into. The top number tells us how many of these pieces we are considering, selecting, or using. In a fraction the denominator is always divided into the numerator, **a fraction is always a division problem as well as a description of how part of something is related to a whole object or complete collection.**

$$\frac{1}{3} = \frac{\textbf{How Many}}{\textbf{Of How Many In Total}} =$$

one



of three

$$= \frac{\textbf{Part}}{\textbf{Whole}}$$

Topic 2 – Practice Questions

Question 1

A telephone company finds that 5 of every 7 calls are long distance calls in the Northwest Territories. What is the part to whole relationship?

a) 7/7

b) 7/5

c) 5/12

d) 5/7

Answer: d

Explanation

The whole in this case is the total number of calls, 7, of which a part, 5, are long distance. The ratio of long distance calls to total calls (part to whole) is 5 to 7, which is written as the fraction 5/7.

Topic 2.A – Three Kinds of Fractions

There are three kinds of fractions

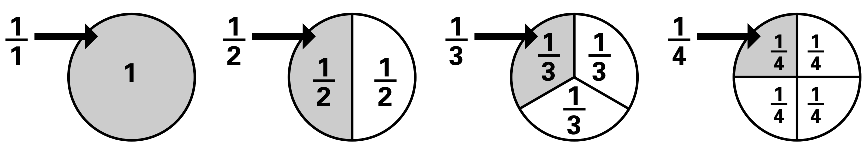
1. $^1/_3$, $^3/_4$, $^7/_9$, are examples of **proper fractions**. When the top is smaller than the bottom, the fraction is "proper" and the fraction is less than one whole. A proper fraction is always less than 1 in size. A proper fraction is "bottom heavy" in that the denominator is larger than the numerator. When the top is equal to the bottom the fraction equals one.

$$\frac{1}{4}, \frac{3}{4}, \frac{7}{12} \text{ are proper fractions} \longrightarrow \frac{\text{smaller \#}}{\text{BIGGER \#}}$$

2. Some fractions are equal to one $^3/_3$, $^4/_4$, $^{340}/_{340}$ etc., all equal one. You can picture this by applying the description of a fraction: $^3/_3$, or three thirds, equals one whole because you have divided something into three equal sized pieces (the denominator) and then selected all three of them (the numerator). This is the same thing as taking the entire whole, for example a pie, to start with. The same process is involved whenever top and bottom numbers are the same in a fraction.

$$\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \text{ all} = 1$$

Proper fractions: as the denominator gets bigger the fraction gets smaller



$1 = \frac{1}{1}$ $1 = \frac{2}{2}$ $1 = \frac{3}{3}$ $1 = \frac{4}{4}$

3. An improper fraction has a numerator that is larger than its denominator. The top is bigger than the bottom. These "top heavy" fractions are larger than 1. They can be expressed as a whole number with a fraction part added on. These expressions are equivalent to improper fractions and are called "**mixed numbers**" because they mix a fraction with a whole number.

$$\frac{5}{4}, \frac{4}{3}, \frac{12}{7} \text{ are improper fractions} \longrightarrow \frac{\text{BIGGER \#}}{\text{smaller \#}}$$

Topic 2.A – Three Kinds of Fractions

A **mixed number** is a whole number with a fraction part added on

Examples:

$3\frac{1}{2}$, $32\frac{5}{8}$, $100\frac{3}{4}$

Three **and** one half, thirty two **and** five eighths, one hundred **and** three quarters **"and" means "plus" and "plus" means add (+).**

Examples

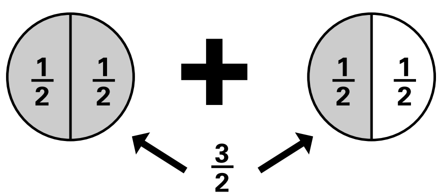
Improper fraction $\frac{3}{2}$ aka mixed number 1 and $\frac{1}{2}$

Improper fraction $\frac{5}{2}$ aka mixed number 2 and $\frac{1}{2}$

(and the reverse is also true, $1\frac{1}{2} = \frac{3}{2}$, $2\frac{1}{2} = \frac{5}{2}$)

More on Improper Fractions

$\frac{3}{2}$ is read as "three halves" and is an improper fraction. This means that something has been divided into two equal sized parts, and we have selected three of them- suggesting that we have a problem about where to find the third part. The solution is to take another whole, divide it into two equal sized pieces, and take one of them to add to our selection so that now we have three equal sized pieces. $\frac{3}{2}$ can be spoken as "three halves" or as "one and one half". Both expressions mean the same thing.



$$\frac{3}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

Topic 2.A – Practice Questions

Question 1
Which of these fractions is greater than one: $^1/_2$, $^{11}/_{12}$, $^9/_8$, $^{100}/_{101}$?

a) $^1/_2$

b) $^{11}/_{12}$

c) $^9/_8$

d) $^{100}/_{101}$

Answer: c

Explanation
Only choice c has a numerator larger than the denominator. This "top heavy" fraction is greater than one. It is an improper fraction that can also be expressed as 1 and $^1/_8$, which is the same as $^8/_8 + ^1/_8$. Remember that any fraction with a numerator that is greater than the denominator will be larger than 1.

Question 2
Which choice goes from smallest to largest for the following fractions: $^1/_2$, $^1/_5$, $^1/_3$, $^1/_7$?

a) $^1/_2$, $^1/_5$, $^1/_3$, $^1/_7$

b) $^1/_2$, $^1/_3$, $^1/_7$, $^1/_5$

c) $^1/_3$, $^1/_2$, $^1/_5$, $^1/_7$

d) $^1/_7$, $^1/_5$, $^1/_3$, $^1/_2$

Answer: d

Explanation
All of the choices have proper fractions with a numerator of one. The smallest fraction in the list will have the largest denominator. In choice d, $^1/_7$ is less than $^1/_5$, and $^1/_5$ is less than $^1/_3$, and $^1/_3$ is less than $^1/_2$. The other choices fail because a fraction with a smaller denominator is placed before one with a larger denominator, for example in choice a where $^1/_2$ is not smaller than $^1/_5$.

Topic 2 – Fractions

A Second Look at Proper Fractions

$^{2}/_{3}$ = two thirds, $^{5}/_{16}$ = five sixteenths

$^{2}/_{3}$ means that something has been divided into three equal sized parts and that we have selected two of the parts. $^{5}/_{16}$ means that something has been divided into sixteen equal parts and we have selected five of them. Think of a pie cut into three pieces. If we eat $^{2}/_{3}$ of the pie, $^{1}/_{3}$ (one piece) will be left. Think of a pie cut into sixteen pieces. If we eat $^{5}/_{16}$ of the pie, $^{11}/_{16}$ will be left.

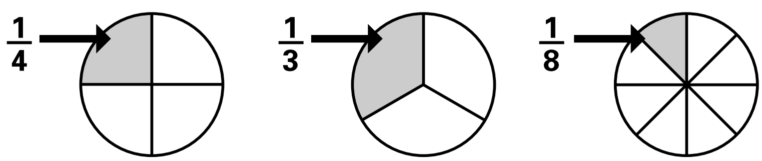
The same rules apply if we think of groups of objects instead of pieces of a pie. $^{5}/_{16}$ can also mean that we have divided a collection of objects into sixteen equal sized piles, and that we are thinking of five of the piles. The key point is that the denominator by itself doesn't tell us what has been divided up or how many objects are in each sixteenth, we only know the number of divisions that have taken place on something when we look at a denominator.

More on the Concept of Fractions

The following discussion is for those desiring a more detailed explanation on how to think about fractions.

1. **Parts of single objects**

   A pie cut into four equal pieces is cut into fourths. A pie cut into three equal sized pieces is cut into thirds, and so on. The top number (numerator) tells how many pieces we have selected. $^{1}/_{4}$ means we are thinking of one of four pieces, $^{1}/_{3}$ means we are thinking of one of three pieces, and $^{5}/_{8}$ means we are thinking of five of eight pieces.



   When we measure something in a trade we often look at fractions of something. $^{1}/_{4}$ of an inch, $^{2}/_{3}$ of a foot, $^{3}/_{4}$ of a cup, $^{1}/_{2}$ of an expense, etc. Any fraction that is less than one whole is called a **proper fraction**. When the denominator is smaller than the numerator, the fraction is greater than one, and is called an **improper fraction**. An improper fraction can be changed into a mixed number by dividing the denominator into the numerator. This process is discussed in Unit 2, Topic 2.D – Dividing Fractions.

Unit 1 – Number Concepts

Topic 2 – Fractions

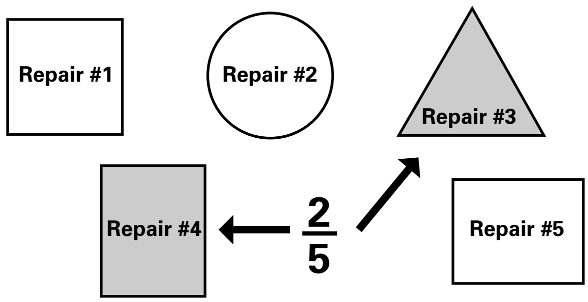
2. Parts of a collection

Fractions can also refer to the total number of members in a group of things, events, or people. The denominator refers to this total number of objects.

Examples

a) Only 25 of every 100 adults over 40 years of age maintain their proper weight. This means that $^{25}/_{100}$, also known as $^{1}/_{4}$ or 25% of this group maintain their proper weight.

b) 5 repair orders were completed in the shop today. Of these 2 took twice as long as the customer was told they would take. This means that 2/5, or 2 out of a total of 5 repairs took twice as long as the customer expected they would.



Notice that any two of the five shapes could be shaded in to represent $^{2}/_{5}$. In practice, the people in the shop would know which repairs were the ones that took twice as long, but the fraction relationship would be the same no matter which two of the five repairs took twice as long.

Unit 1 – Number Concepts

Topic 2 – Practice Questions

Question 1

An expediter asks you to divide 150 gallons of bottled water into three equal truckloads. What fraction will describe the ratio of gallons carried in each load to the total number of gallons?

a) $^{50}/_{50}$

b) $^{3}/_{150}$

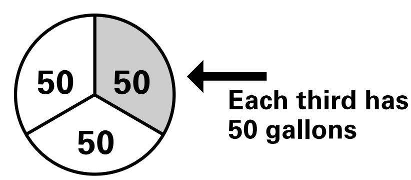
c) $^{50}/_{150}$

d) $^{15}/_{150}$

Answer: c

Explanation

A ratio is a fraction. You know that the top number is the part from a collection that is being selected. The collection has 150 bottles in it. If you divide this into three loads, each load will have 50 gallons because 50 + 50 + 50 = 150. See later sections on the multiplication and division of integers (Unit 2, Topics 1.D and 1.E) if you need help understanding this. The question asks for a fraction that describes the relationship between a part (50 gallons) and the whole (150 gallons). $^{50}/_{150}$ expresses this relationship. This fraction could be reduced to $^{1}/_{3}$, as you will see in later sections on reducing fractions. Each of the three trucks will carry 50 gallons, or one third of the total. Saying this is equivalent to saying that one third of the total is carried on each truckload, or that one out of every three gallons will be on each of the three truckloads. See Unit 3, Topic 8 – Ratios, Rates and Proportions for more problems with ratios.



Unit 1 – Number Concepts

Topic 2 – Practice Questions

Question 2

A survey shows that 25 of the 85 people working in a mining camp watch television for more than half of their time off work. What number will be the denominator in a fraction that describes how many people watch television more than half of their off-duty time?

a) 50

b) 25

c) 115

d) 85

Answer: d

Explanation
The denominator in a fraction describes the total in a collection that we select a part of. In this problem 25 have been identified as the part from the total by the survey. $^{25}/_{85}$ is the fraction that tells how many of the 85 people in camp watch television for more than half of their off-work time. The denominator is the total number of workers in camp, which is 85.

Question 3

If 19 people out of a total camp population of 85 watch television more than half of their off duty time, what fraction will express this fact?

a) $^{19}/_{76}$

b) $^{19}/_{85}$

c) $^{85}/_{19}$

d) $^{19}/_{100}$

Answer: b

Explanation
The fact we want to describe with a fraction is that 19 out of a total of 85 people watch television more than half of their free time in camp. The denominator of this fraction will be the total population of 85, and the numerator will be the number within that total that fit the description: 19. $^{19}/_{85}$ tells us that 19 of the 85 in camp fit the fact that is given in the question.

Topic 2 – Practice Questions

Question 4

If 23 of the 85 people working in the camp watch television more than half of their off-work time, what fraction will describe those who do not watch television more than half of their off-work time?

a) $^{85}/_{1}$

b) $^{23}/_{85}$

c) $^{62}/_{85}$

d) $^{85}/_{107}$

Answer: c

Explanation
We know that there are 85 people in the total and that 85-23 will equal the number of that total who do not watch television more than half of their off-work time. The fraction that describes this fact is $^{62}/_{85}$. You can also see that the sum of $^{62}/_{85}$ + $^{23}/_{85}$ will equal the total camp population of $^{85}/_{85}$ = 1. More on adding fractions with like denominators can be studied in Unit 2, Topic 2.A – Adding Fractions.

Question 5

Which of the following is not a proper fraction?

a) $^{25}/_{32}$

b) $^{500}/_{1000}$

c) $^{52}/_{14}$

d) $^{2}/_{3}$

Answer: c

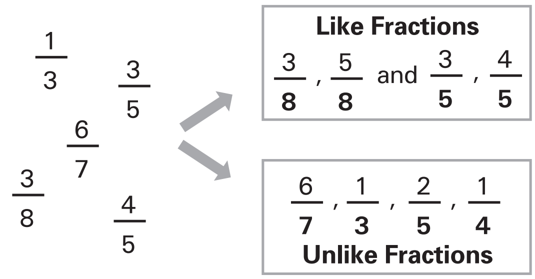
Explanation
Only choice c has a numerator that is larger than the denominator. This means the fraction is greater than 1 because the denominator goes into the numerator more than once. $^{52}/_{14}$ = 3 with a remainder of $^{10}/_{14}$.

Topic 2.B – Comparing Fractions

Comparing the relative size of fractions

1) Comparing like fractions

$\frac{4}{5}$ is larger than $\frac{3}{5}$ by one fifth, $\frac{7}{3}$ (an improper fraction) is larger than $\frac{2}{3}$, and $\frac{3}{7}$ is less than $\frac{5}{7}$. These are pairs of like fractions.



When fractions have the same denominator they are called like fractions, and can be compared by simply looking at the size of the numerators. The larger the numerator the larger the fraction will be.

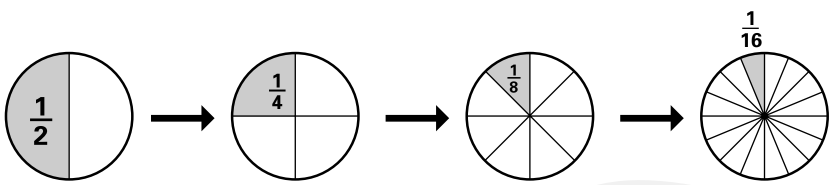
Example

Is $\frac{6}{7}$ larger than $\frac{5}{7}$?

Yes, these are like fractions and can be compared in size on the basis of their numerators. Six sevenths is one seventh more than five sevenths, 6 is larger than 5, $\frac{6}{7}$ is larger than $\frac{5}{7}$.

2) Comparing unlike fractions

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ – These pieces are getting smaller. **These are also unlike fractions because they have different denominators.** Each piece is only half as big as the one that came before it. Each fraction was divided by 2 to get this result. For example, $\frac{1}{8}$ is half as much as $\frac{1}{4}$, and $\frac{1}{16}$ is half as big as $\frac{1}{8}$.



If we keep going this way, the fractions yet to come on the right will get close to becoming nothing, or zero.

Example

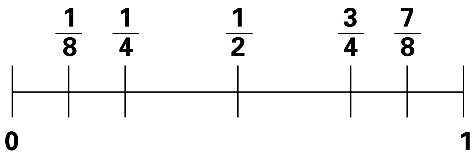
Think about this series of fractions: …$\frac{1}{32}$, $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1: these pieces (unlike fractions) got larger by making each one twice as big as the one before it (on the left) until one (a whole) was reached. Each proper fraction was multiplied by two to get this result. We can see that $1 = \frac{2}{2}$, $\frac{1}{2} = \frac{2}{4}$, $\frac{1}{4} = \frac{2}{8}$, $\frac{1}{8} = \frac{2}{16}$ and so on. You can see that $\frac{1}{4}$ is bigger than (twice as big) as $\frac{1}{8}$, and $\frac{1}{8}$ is bigger than $\frac{1}{16}$.

A quarter is also known as a fourth.

Unit 1 – Number Concepts

Topic 2.B – Comparing Fractions

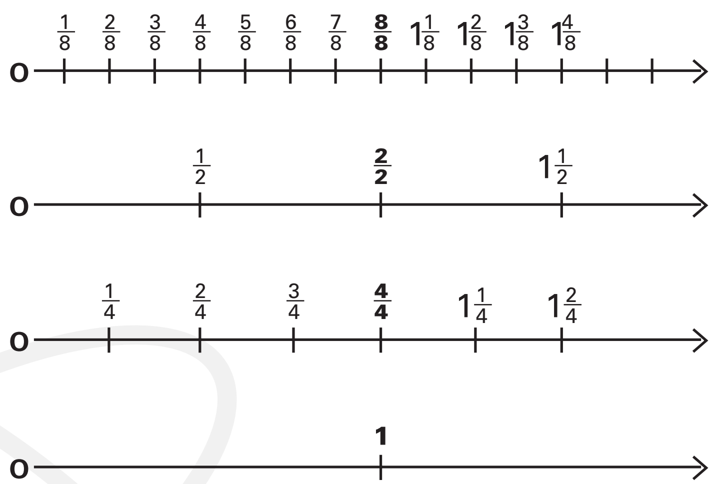
All fractions can be put on a number line, and **all proper fractions are on the line between 0 and one**. A carpenter's tape is an example of a number line. It is important to recognize that all numbers, both very small and very large, can be found on a complete number line. We "blow up" or expand our view of the line between 0 and 1 to locate any proper fraction.



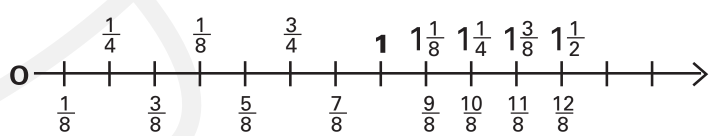
Notice that the closer a fraction is to zero, the smaller it is and vice versa. By putting two fractions on the line where they belong, we can tell which one is larger. In symbols we could arrange this selection of fractions in ascending (increasing) order using the symbol < "is less than":

$0 < {}^{1}\!/_{8} < {}^{1}\!/_{4} < {}^{1}\!/_{2} < {}^{3}\!/_{4} < {}^{7}\!/_{8} < 1$

Many fractions can name the same point on the number line as the following diagram shows. **Two fractions that name the same point are called equivalent fractions.** Unlike fractions are sometimes equivalent fractions. The unlike fractions in the diagram can be compared vertically to see that they name the same points. for example ${}^{4}\!/_{8} = {}^{1}\!/_{2} = {}^{2}\!/_{4}$. **They express the same value and can be exchanged for one another.** A ruler divided into eighths shows these fractions on one line. The fractions that line up vertically name the same point on the line and are equivalent to each other.



A ruler shows all of these subdivisions



Topic 2.B – Comparing Fractions

Know what you are comparing when you use fractions

Notice that when we say an eighth is smaller than a quarter, we mean that an eighth of something is always smaller than a quarter **of the same thing.** If we didn't understand this, we could be wrong. For example an eighth of an acre has more area than a quarter of a square meter. Acres and meters are different kinds of wholes, and can't be compared directly by fractions. This problem is discussed in Module 4 – Measuring Time, Shapes and Spaces on measurement and conversions between measurement systems.

For greater clarity

A pie would have to be very large for $^1/_{16}$ to amount to a satisfying serving. This observation allows us to see that the size of a whole object (or collection) will determine how big a fraction turns out to be in a real world situation. For example, $^1/_{100}$ of a hundred thousand dollars is \$1000 , but $^1/_{100}$ of \$10.00 is only a dime. The fraction number is the same, $^1/_{100}$, but the meaning changes depending on the size of the collection (dollars here) that we start with.

$$\frac{1}{4} \text{ of } \$1 = 25¢$$

BUT:

$$\frac{1}{4} \text{ of } 25¢ = 6.25 \text{ cents}$$

Unit 1 – Number Concepts

Topic 2 – Practice Questions

Question 1

Which fractions are equivalent?

a) $^1/_2$ and $^3/_3$

b) $^3/_8$ and $^1/_4$

c) $^2/_4$ and $^1/_2$

d) $^3/_2$ and $^5/_4$

Answer: c

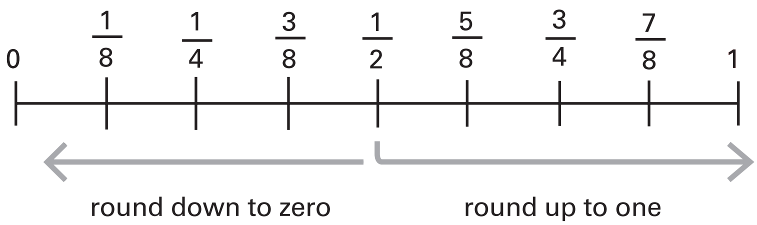
Explanation
$^2/_4$ and $^1/_2$ name the same point on the number line. They have the same value. You can see from the number line that $^1/_4 + ^1/_4 = ^2/_4$ and this is also equal to $^1/_2$.

Topic 2 – Fractions

Rounding and estimation with fractions

When a mixed number has a fraction part equal to $\frac{1}{2}$ or more, we can round up to the nearest whole number. If the fraction part is less than one half we round down. 4 and $\frac{5}{8}$ rounds up to five, and 3 and $\frac{1}{3}$ rounds down to 3. Rounding helps to estimate answers to problems involving mixed numbers and fractions. A good estimate will not be exact, but will be close to the exact answer. An estimate rounded to the nearest whole number will not be off from the exact answer by more than $\frac{1}{2}$.

In the case of proper fractions, we can round to the nearest whole number, which will be either 0 for fractions less than $\frac{1}{2}$, or 1 for fractions between $\frac{1}{2}$ and one. $\frac{3}{5}$ rounds to 0, and $\frac{7}{8}$ rounds to one using this rule. More commonly, however, situations require that we round to the nearest half, quarter or third.



Rounding off proper fractions to another fraction

To do this you need to work with common denominators. This is explained later in Unit 2, Topic 2.C – Multiplying Fractions. You can get the idea by thinking of a carpenter's tape measure. $\frac{3}{8}$ inch is very close to $\frac{1}{4}$ inch. If you know that $\frac{1}{4} = \frac{2}{8}$, you can see that $\frac{3}{8}$ is $\frac{1}{8}$ bigger than $\frac{1}{4}$. If you know that $\frac{1}{2} = \frac{4}{8}$, you can see that $\frac{3}{8}$ is $\frac{1}{8}$ smaller than $\frac{1}{2}$. $\frac{3}{8}$ is in the middle of the line between$\frac{1}{4}$ and $\frac{1}{2}$ In this situation we can decide whether to round up to $\frac{1}{2}$ or down to$\frac{1}{4}$. Our work will guide this decision. A carpenter who knows that wood will shrink prefers to round up, a retailer packaging candy by the ounce may prefer to round down.

Example

Work group A worked 5 and $\frac{3}{5}$ overtime shifts, work group B worked 5 and $\frac{3}{8}$ overtime shifts. Which group worked more overtime?

Round each number of shifts to the nearest whole number. 5 3/5 rounds up to 6, and 5 $\frac{3}{8}$ rounds down to five. We can use this information to estimate that group A worked more overtime than group B. Without knowing exactly how much more, we can be confident that group A did put in more overtime than group B because the fraction part of the mixed number 5 and $\frac{3}{5}$, is $\frac{3}{5}$, and $\frac{3}{5}$ is greater than $\frac{1}{2}$. We know this because $\frac{1}{2} = \frac{2.5}{5}$. See Unit 2, Topic 2 – Fractions on how to compare unlike fractions by converting them into equivalent fractions with common denominators.

Topic 2 – Practice Questions

Question 1

Sally worked 2 $^{1}/_{3}$ hours, Jim worked 2 $^{5}/_{8}$ hours, and Martha worked 2$^{3}/_{4}$ hours. If work time is rounded to the nearest hour to determine pay, who will be able to claim pay for 3 hours of work?

a) Sally and Jim

b) Jim and Martha

c) Martha and Sally

d) Jim, Martha and Sally

Answer: b

Explanation
Each mixed number has a fraction part that we must round to the nearest whole number. For Sally, $^{1}/_{3}$ rounds to 0 because it is less than $^{1}/_{2}$, but $^{5}/_{8}$ and $^{3}/_{4}$ for Martha and Jim round up to 1 because they are each greater than $^{1}/_{2}$. After rounding, Sally will only be able to claim pay for 2 hours (2 + 0), but Jim will claim 3 hours (2 + 1) and Martha will claim 3 hours (2 + 1).

Question 2

A diesel fuel drum holds 50 gallons. 13 gallons have been pumped out to fill a pick up truck's fuel tank. What fraction describes how much fuel has been taken out?

a) $^{13}/_{1}$

b) $^{37}/_{50}$ gallons

c) $^{13}/_{37}$

d) $^{13}/_{50}$

Answer: d

Explanation
This question describes a part to whole relationship. The denominator will be the total number of gallons (the whole), and the numerator will be the number of gallons that have been pumped out (the part).

Topic 2 – Practice Questions

Question 3

What is the correct order from largest to smallest for $^1/_8$, $^1/_4$, $^1/_3$?

a) $^1/_8$, $^1/_4$, $^1/_3$

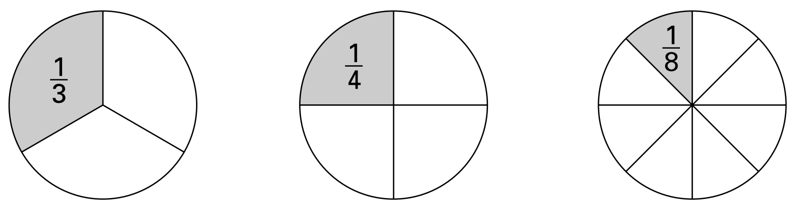
b) $^1/_3$, $^1/_4$, $^1/_8$

c) $^1/_4$, $^1/_8$, $^1/_3$

d) $^1/_8$, $^1/_3$, $^1/_4$

Answer: b

Explanation
The numerators in the problem are all one, so relative size will depend only on the size of the denominator. The smaller the denominator the larger the size of the piece we are taking from a whole. A picture using the symbol > (greater than) can also show this:



$$\frac{1}{3} > \frac{1}{4} > \frac{1}{8}$$

Question 4

Which improper fraction is equal to 7 $^4/_5$?

a) $^{28}/_5$

b) $^{27}/_5$

c) $^{39}/_5$

d) $^{33}/_5$

Answer: c

Explanation
We know the numerator will be larger than the denominator in an improper fraction. Because the question is about a mixed number with a fraction part in fifths, we will change the whole number part of the mixed number, 7, into the number of fifths it equals. There are five fifths in each 1 in 7. 7 wholes (ones), each divided into five parts, means 35 fifths in all (7 x 5). We add $^{35}/_5$ to $^4/_5$ to get $^{39}/_5$ for the answer.

Unit 1 – Number Concepts

Topic 2 – Practice Questions

Question 5

5 snow machines were delivered in damaged condition to the Hunters' Coop in Inuvik. 40 machines were delivered in the total shipment. What is the fraction of machines that were damaged?

a) $^{5}/_{40}$

b) $^{35}/_{40}$

c) $^{45}/_{5}$

d) $^{10}/_{40}$

Answer: a

Explanation
Here a collection of objects, snow machines, has forty members. Of these, five were damaged. The fraction that expresses this part to whole relationship is $^{5}/_{40}$. Choice b describes the number that was not damaged. As you will see below, but can understand now, when we add $^{5}/_{40}$ to $^{35}/_{40}$ we get $^{40}/_{40} = 1$. This is because the damaged machines (5) combined with the undamaged machines (35) makes up the total order of 40 machines.

Question 6

Which number can be rounded up to 7?

a) $6\ ^{7}/_{8}$

b) $6\ ^{3}/_{8}$

c) 7

d) $6\ ^{3}/_{7}$

Answer: a

Explanation
In order to round up to 7 we need a mixed number whose fraction part is $^{1}/_{2}$ or greater. Only $^{7}/_{8}$ is greater than $^{1}/_{2}$ in these choices. You can see that whatever the denominator is, you divide it by two to find the equivalent of $^{1}/_{2}$. For example, $8 \div 2 = 4$, and $^{4}/_{8} = {}^{1}/_{2}$. $7 \div 2 = 3.5$, and $^{3.5}/_{7} = {}^{1}/_{2}$, $16 \div 2 = 8$, and $^{8}/_{16} = {}^{1}/_{2}$.
This information will allow you to quickly compare a proper fraction to $^{1}/_{2}$ to decide if it is closer to 0 or to 1. For example, if you know that $^{8}/_{16}$ is equal to $^{1}/_{2}$, then you can see that $^{11}/_{16}$ is greater than $^{1}/_{2}$ and rounds up to 1.

Unit 1 – Number Concepts

Topic 3 – Decimals

Decimals

Everything you know about fractions will also apply to decimal numbers because decimals are a kind of fraction.

Decimals are aka fractions with a power of 10 as the denominator. They are written to the right of a "decimal point".

Powers of 10 are discussed in Unit 3, Topic 6 – Bases, Exponents and Square Roots.

Examples

Decimal:

from the Greek "deka" meaning ten

.2 = two **tenths** = $^2/_{10}$ (a proper fraction)

2.2 = two and two **tenths** = 2 $^2/_{10}$ (a mixed number)

.02 = two **hundredths** (a proper fraction)

2.02 = two and two **hundredths** = 2 $^2/_{100}$ (a mixed number)

.002 = two **thousandths** = $^2/_{1000}$ (a proper fraction)

2.002 = two and two **thousandths** = 2 $^2/_{1000}$ (a mixed number)

.002 (aka $^2/_{1000}$) is ten times smaller than .02, (aka $^2/_{100}$) and .02 is ten times smaller than .2, (aka $^2/_{10}$). Two is the numerator and 1000, 100, 10 are the denominators of these increasingly smaller decimal numbers.

This means a decimal number is always some number (the numerator) "over" or divided by, either 10, 100 1000 or a larger power of ten.

As with any fraction, when denominators grow larger, the equivalent decimal number gets smaller. $^2/_{1000}$ is ten times smaller than $^2/_{100}$, and $^2/_{100}$ is ten times smaller than $^2/_{10}$.

Topic 3 – Decimals

How to convert fractions and decimals

The number of zeros minus one equals the number of zeros to put after the decimal point and before the numerator of the decimal number when changing to a fraction form. The reverse is also true.

Example

$^{2}/_{10,000}$ = .0002 “two ten thousandths”

4 zeros in the denominator - 1 = 3 zeros in the decimal form of the same number.

Example

.00005 = $^{5}/_{100,000}$ “five one hundred thousandths”

4 zeros in the decimal number + 1 = 5 zeros in the fraction form of the same number.

A decimal point is a “dot” put to the right of a number to create columns that get smaller by a power of ten each time we move another step to the right.

Decimal numbers can be read by saying “point” or “and” for the decimal point. For example, 39.4 could be read as “thirty nine point four”, or as “thirty nine and four tenths.”

Place values get smaller to the right…

Each column is worth ten times less than its neighbor to the left.

thousands,	**hundreds,**	**tens,**	**units**	•	**tenths,**	**hundredths,**	**thousandths**
9 x 1000	**0** x 100	**6** x 10	**3**x1	•	**1**x1/10	**2**x1/100	**5**x1/1000

9,063.125 = 9 thousands, 0 hundreds, 6 tens, 3 units, 1 tenth, 2 hundreths, and 5 thousandths

“nine thousand sixty three **and** one hundred twenty five thousandths”

Numbers to the right of the decimal are read in terms of the rightmost place value.

.12 is read “twelve hundredths” .130 is read “one hundred and thirty thousandths”.

Topic 3 – Decimals

Rounding and estimating with decimals

Round a decimal number up or down using the same principle as for fractions. If the decimal number is 5 or greater in a column, you round up to the next largest column (always to the left) because 5 is the $\frac{1}{2}$ point for each column. For example, .7 rounds up to 1 because 7 tenths is closer to 10 tenths = 1 than to 0 tenths, .09 rounds up to .1 because 9 hundredths is closer to 100 hundredths = 1 tenth, than to 0 hundredths and so on.

Applying this idea, you can see that .77 rounds up to 1 for the nearest whole number, and to .8 for the nearest tenth. In order to round, you must know what place value you want to round up or down to. As with proper fractions, rounding down applies for decimal numbers less than .5. .3 rounds down to 0, .7 rounds up to 1. 1.2 rounds down to 1.0, and .73 rounds down to .70 for the nearest hundredth but up to 1 for the nearest whole number.

Rounding up or down is also called rounding off. **The place value in the column you are rounding to will determine whether you go up or down when you round off.**

Example

Instructions can be given about which place to round off to. For example, if you are asked to round 45.678 to the nearest tenth, you look in the tenths place to find 6. Now look to the rightmost column where you will find 8 thousandths. This is greater than 4 and can be rounded up to the next hundredth to the left giving 45. 68. Now round the hundredths, which has become 8, up to the nearest tenth to get 45.7.

Be Careful…

In this example a shortcut would result from seeing that we don't need to start at the rightmost column, because 7 in the hundredths place is already greater than four and can be rounded up to give 45.7. However, in a different example this might not be true. For example, round 45.446 to the nearest tenth. Here the rightmost column has a 6 and can be rounded up to give 45.45. This tips the scale in favour of rounding up to 45.5. The shortcut, however, would be inaccurate here and we would incorrectly round to 45.4 by only inspecting the hundredths column. **You must inspect the rightmost column and work left to round up to each column on the way to your desired rounding off place.**

Practical problems involving rounding and estimating can be found in Math – Module 4, Unit 4 – Measurement And Estimation Problems.

Topic 3 – Practice Questions

Question 1

What is the correct order from largest to smallest of .101, 1.101, and 11.101?

a) 1.101, .101, 11.101

b) 11.101, 1.101, .101

c) .101, 11.101, 1.101

d) 1.101, .101, 11.101

Answer: b

Explanation
The largest number will always be the number with the largest left-most digit. Choice b starts with a 1 in the tens place, followed by a number with no tens and one one, and finally by a number with no ones, and one tenth, no hundredths and one thousandth.

Question 2

How is 7.874 expressed in words?

a) "seventy eight hundredths and seventy four thousandths"

b) "seven point eighty seven four"

c) "seven and eight hundred seventy four thousandths"

d) "seven and eighty seven hundred and four thousandths"

Answer: c

Explanation
The decimal point is read as "point" or as "and". This decimal number is also a mixed number, with a whole number part, 7, and a fraction part of 874/1000. Always read decimal numbers in terms of the rightmost place value.

Topic 3 – Practice Questions

Question 3

Round off 0.8734 to the nearest hundredth.

a) 0.87

b) 1.1

c) 0.9

d) 0.8740

Answer: a

Explanation
7 is in the hundredths place. Starting from the right we round up if possible. 4 is less than five, and so we do not round up. 3, in the thousandths place, is also less than five and we do not round up. 7 is in the hundredths place and so the answer rounds to 0.87.

Question 4

Round 56.9456 to the nearest hundredth.

a) 56. 97

b) 57.05

c) 56.95

d) 56. 99

Answer: c

Explanation
4 is in the hundredths place so we look to the rightmost column and find 6 in the ten thousandths place which can be rounded up because it is greater than 5. This changes 5 to 6 in the thousandths place to give us 56.946 and we now can round up again to give 5 instead of 4 in the hundredths column for an answer of 56.95.

Unit 1 – Number Concepts

Topic 4 – Rational Numbers and Signed Numbers

Integers and fractions are also part of the larger class of rational numbers that can have either a positive or negative value. Rational numbers include everything discussed so far- integers, fractions, and decimals.

Signed Numbers

A number can be positive or negative

-6 is a negative number, 6 is a positive integer. **Numbers are assumed to be positive unless there is a minus sign in front of them**. -1, -2, -3 etc. are negative numbers, 1,2,3 etc. are positive. Negative numbers are also known as signed numbers.

Examples:

1. Jim owes Bill \$12.00. After Jim pays Bill he can record his payment as -\$12.00 and Bill can record his receipt of the money as + \$12.00.
2. The temperature drops to twenty degrees below zero Celsius. This temperature is recorded as -20°.

   Negative numbers can be represented to the left of zero on a number line.

   ...-5 -4 -3 -2 -1 0 1 2 3 4 5...

   See Unit 2, Topic 1 – Integers for more on adding and subtracting signed numbers.

$^1/_2$, $^5/_{16}$, $^{-3}/_2$, $-^1/_{16}$, .60, $^6/_{10}$, $^1/_3$, are all rational numbers. **A rational number can be expressed as a ratio of two whole numbers. A fraction is aka a ratio and vice versa.** Equivalent relationships mean "also known as" (aka). You have already seen that every counting number can be expressed as a ratio of itself divided by one. 6 = $^6/_1$, 2 = $^2/_1$, etc. This means that counting numbers (aka integers or whole numbers) are also rational numbers.

Exchanging Equals is a Useful Concept

When two things are equivalent one can be exchanged for the other. A warehouse attendant reads a report that says his supplies are 33% below what they were a month ago. He has instructions to reorder whenever supplies go below $^1/_3$ of full stock. He can substitute (or convert, or exchange) $^1/_3$ for 33% because they are equal. This knowledge alerts him to the need to reorder.

Topic 4 – Rational Numbers and Signed Numbers

Repeating Decimals

Some rational numbers, for example $^2/_3$, have repeating decimals when they are converted into a decimal by dividing the denominator into the numerator. $^2/_3$ = .666… The fact that the pattern, here a single digit, repeats no matter how many places we divide tells us that the number is rational. A repeating pattern is indicated by three dots after a desired number of places, or by a bar over the numbers that repeat. See Unit 3, Topic 5, question one for more examples.

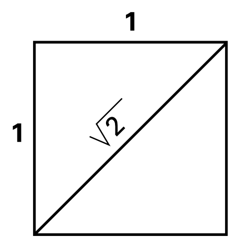
Example

$^2/_3$ = .66… = .6

Other repeating patterns can be found when fractions are converted into decimal numbers, for example $^4/_{33}$ = 0.121212… = 0.12. On the exam, you can identify such numbers as rational numbers when division produces a repeating pattern. **When division does not produce a repeating pattern, the number is irrational.** For example, $^1/_3$ = .33 is rational, but $\sqrt{2}$ = 1.4142… and is irrational. For a review on how to divide fractions and convert them to decimals see Unit 2, Topic 2.C – Multiplying Fractions and Topic 2.D – Dividing Fractions.

Irrational Numbers

Irrational numbers cannot be expressed as a ratio of integers and are called irrational for this reason. Irrational means not rational. **Irrational numbers are on the number line but they cannot be represented as the division of one integer by another.** You can find an irrational number when you draw a diagonal in a one-unit square.



From the Pythagorean theorem we can prove that the diagonal on a unit square is equal to the product of some number times itself that equals 2.[4] This number is the square root of two ($\sqrt{2}$). There is no integer that can be multiplied by itself to give 2, and yet the line has a definite length to be measured. This number is close to 1.4142, but it can be calculated to further degrees of accuracy. Although it won't be larger than 1.4142 because this is the result of rounding up the nearest ten thousandths place.

Notice that a fraction number can have any kind of number in its numerator or denominator. Rational, irrational, positive and negative integers may all appear in a fraction. It is possible to have a fraction with irrational parts, for example, $\sqrt{\,^2/_9}$ where the numerator is irrational and the denominator is rational.

[4] See Math – Module 4 – Measuring Time, Shapes and Spaces for a demonstration of the Pythagorean theorem that proves $\sqrt{2}$ is irrational.

Unit 1 – Number Concepts

Topic 4 – Rational Numbers and Signed Numbers

People in Practical Situations Use Irrational Numbers All of the Time.

For example, building contractors use diagonals across large foundations to make sure that the corners are at right angles and at the right distances from each other. Irrational numbers are used in measurement, trigonometry, and geometry and will be reviewed in Modules 3 and 4 of this curriculum.

There are many irrational numbers

Example:

≠ = 3.14… is irrational. This number is the same for the division of the circumference (distance around) of any circle by it’s diameter and comes close to $^{22}/_{7}$. However this ratio can be divided out further and shown to be slightly larger than 3.14. Many square roots are also irrational, for example no ratio of two whole numbers can be simplified by division to give the exact square root of two or of three.

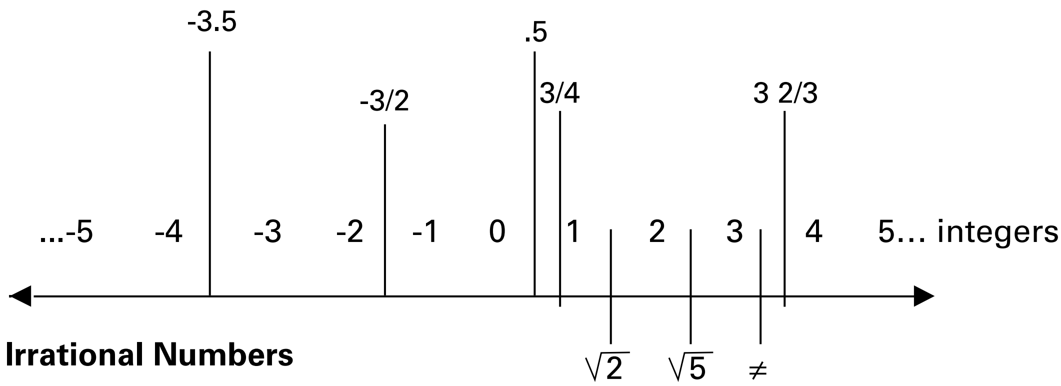
Exposing the number line

On the number line, whole numbers can be divided into fractions and decimals. For example, a bridge measuring 32 feet can be thought of as a number line 32 feet long. We may want to divide the span of this bridge into thirds of 10.66… feet, or into sections based on a percentage (decimal fraction) of the whole length, for example 45% of the bridge would be 14.4 feet. The closer we look into the number line the more numbers we will find there. We can divide the span of the bridge into any sized sections that are useful for our work. We can choose to measure it in tenths, thousandths, or hundredths. In practical situations we will round off irrational numbers and repeating decimal numbers.

As you will see, fractions can be expressed as decimals and vice versa.

You can now appreciate that the number line is like a ruler that helps us to locate each kind of number we have studied.

Topic 4 – Rational Numbers and Signed Numbers



Notice that any choice of units can be attached to any of these numbers, such as feet, degrees, metres, pounds, dollars, or hours.

Unit 1 – Number Concepts

Topic 4 – Practice Questions

Question 1

Which number is not a rational number?

a) 2.3

b) The square root of five

c) $^{5}/_{8}$

d) - $^{2}/_{3}$

Answer: b

Explanation
An irrational number cannot be expressed by the division of two integers. The square root of five is the number that can be multiplied by itself to give five as the answer. This number is somewhere between 2 , and 3, because 2 squared is 4, and 3 squared is 9. A calculator will estimate the number as 2.236… Because no repeating pattern of decimal numbers is involved, we conclude that the number is irrational. Choice a can be represented by the rational number $^{23}/_{10}$, choice c equals 6.25, or $^{625}/_{100}$ when you convert it to a decimal by dividing 8 into five. See Unit 2, Topic 3.D – Dividing Decimals for more on this topic. Finally, choice d is equal to -.66…, a repeating decimal and therefore a rational number.

Question 2

Which of the following is a negative integer?

a) $-\sqrt{2}$

b) $-^{2}/_{3}$

c) -5 $^{6}/_{8}$

d) -7

Answer: d

Explanation
An integer is aka a counting number or a whole number. Only seven is a whole number. $-\sqrt{2}$ is a negative irrational number, $-^{2}/_{3}$ is a negative rational fraction, and –5 $^{6}/_{8}$ is a negative rational mixed number. The minus sign in front tells us that it is located left of zero (aka the origin) of the number line. All integers left of the origin are negative.

Topic 4 – Practice Questions

Question 3

Arrange the following integers from largest to smallest: 32, 14, 67, 76, 2.

a) 67, 76, 32, 2, 14

b) 2, 14, 32, 67, 76

c) 76, 67, 14, 32, 2

d) 76, 67, 32, 14, 2

Answer: d

Explanation
The question asks us to put the numbers given in a list that starts with the largest and then arranges the remaining numbers in descending order as they get smaller. Notice that we are asked to put the numbers in the opposite order to the way they would appear on the number line. We start with the largest first, and first means on the left in the answer choices, but on the number line the largest number would be the one most to the right.

Question 4

Which decimal is a rational number?

a) .4534..

b) .4444..

c) .007603

d) .777664

Answer: b

Explanation
Only choice b has a pattern that repeats. .4444 = 4/9. A rational number is one that can be expressed as a ratio of two integers. Choice a tells us that there are more digits after .4534, but the absence of a bar line over this number means that there is no pattern that repeats.

Topic 4 – Practice Questions

Question 5

What fraction is equivalent to .66…?

a) $^{66}/_{10}$

b) $^{2}/_{3}$

c) $^{2}/_{6}$

d) $^{66}/_{666}$

Answer: b

Explanation
This answer can be found by a process of elimination, or from memorizing fractions that are commonly used as decimals, or by converting .66… to a fraction after rounding off. By process of elimination, choice a fails because ten divided into 66 is 6.6, not .66… Choice c fails because $^{2}/_{6} = ^{1}/_{3}$ and $2 \div 6 = .33\ldots$, not .66… Choice d fails because $66 \div 666 = .0990\ldots$.

$^{2}/_{3}$ should be memorized as a commonly occurring fraction with the decimal equivalent of .66…

Finally, .66… can be rounded off to $^{66}/_{100}$, and reduced to $^{33}/_{50}$. Because we are rounding off, we know this is only an approximation. A proportion can be set up to test how closely $^{33}/_{50}$ is to $^{2}/_{3}$. Cross multiplying gives 99 = 100 which is false because we rounded off, but it is very close, - only off by 1%. This reinforces choice b as the correct answer. This last method involves setting up a proportion which is discussed in Unit 3, Topic 8 – Ratios, Rates and Proportions.

Question 6

What is the decimal equivalent of $^{1}/_{3}$?

a) .323

b) .30

c) .13

d) .33..

Answer: d

Explanation
We find the decimal equivalent by treating the fraction as a division problem and dividing 3 into 1. The pattern repeats, here the single digit 3, which is to be expected for all rational numbers that do not have a denominator that goes evenly (e.g. without a remainder) into a dividend (the numerator).

Unit 2

Number Operations

Number operations

In all trades four basic operations (adding, subtracting, multiplying and dividing) are used on three kinds of numbers: integers, fractions, and decimals. All four operations and all three kinds of numbers are required in every trade. If you have difficulty working on the problems in this unit, review Unit 1 – Number Concepts, for the theory you will need.

Unit 2 – Number Operations

Topic 1.A – Adding Integers

Adding Integers

Addition is the work of putting numbers together to find their total or sum. You can add by counting out loud, or on your fingers, but this is too slow. That's why we memorize all the ways that the numbers from 1 to 10 can be combined. This gives us a basis for adding bigger numbers.

Review the place value of whole numbers. Each whole number (also known as a digit, or integer) is in a place or column that tells its value.

Learn how to add any pair of numbers from 1 to 10 and you will be ready to add larger numbers as well.

> 7 + 2 = 9 and 2 + 7 = 9 and 1 + 3 + 4 + 1 = 9
>
> There are several ways to make the same total.

Examples

We add by first combining the units, then the tens, then the hundreds, then the thousandths, and so on.

9 + 7 = 16 = 6 units + 1 ten

9 + 70 = 79 = 9 units + 7 tens

9 + 700 = 709 = 9 units, no tens, and 7 hundreds

16 + 70 = 86 = 6 units + 8 tens

> " = " can connect more than two things, as long as they are equal
>
> 15 = 7 + 8 = 5 + 10 = 14 + 1 = 12 + 3

> **Example: Place values for 9000**
>
> Each column is worth ten times more than its neighbor to the right.
>
> 9000 = 9 thousands, 0 hundreds, 0 tens, and 0 units.

....thousands,		**hundreds,**		**tens,**		**units**
9	=	90x100	=	900x10	=	9000x1

Adding is like travelling

You can also think of addition as travel on a number line. Picture a ruler and find one of the numbers being added. Add the next number by traveling along the line. It doesn't matter which number you pick first. The destination will be the same.

> "=" works both ways if 3 = 2 = 1 then 2 = 1 = 3

Unit 2 – Number Operations

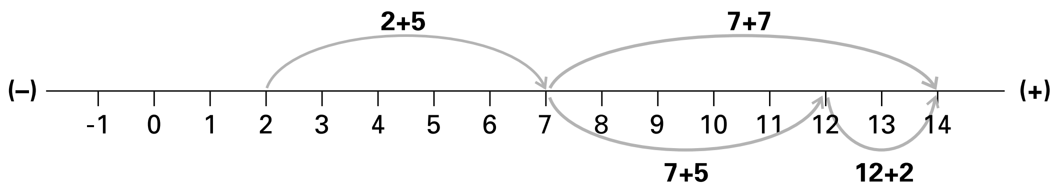
Topic 1.A – Adding Integers

Example

2 + 5 + 7 = 14 or

$$\begin{array}{r} 2 \\ 5 \\ +7 \\ \hline 14 \end{array}$$

> Adding, subtracting and multiplying can be written horizontally or vertically.



Start at two and travel five spaces to arrive at seven. Now travel seven more spaces to arrive at 14. Try starting at seven, and then travelling five spaces to arrive at twelve. Complete the journey by travelling two spaces to arrive at fourteen. With addition, “it all works” if you follow the directions given by each number in the list. Any number in the list of numbers to be added can be chosen as the starting point on the number line.

Carrying in addition

Adding numbers in the one’s column can result in an “overage”, in which case we add one to the next column and leave the remainder. This is sometimes called “carrying” the number.

> **Suggestion:** Imagine that there are columns on both sides of a number **even when no integers appear in them when you start to add.**

Example

$$\begin{array}{r} 5 \\ +7 \\ \hline 12 \end{array}$$ can be thought of and written as: $$\begin{array}{r} \mathbf{1} \\ 05 \\ +07 \\ \hline 12 \end{array}$$

> Units are aka “ones”.

Example

Twelve is “too big” to fit in the one’s column, so we put 1 ten in the ten’s column and leave two behind in the one’s column. The same is true for any “overage” in a column. The number 12 can be broken down into one ten and two units.

Unit 2 – Number Operations

Topic 1.A – Adding Integers

Whole numbers are a sum of ones + tens + hundreds + thousands etc.

1340	= 1 thousand plus 3 hundreds + 4 tens + zero units
+ 97	= 9 hundreds + 7 units
1437	= 1 thousand + 4 hundreds + 3 tens + 7 units

Example

Talk your way through it this way:

34	1. 4 + 8 = 12, and 12 is 10 + 2, we have one ten and two ones
+ 98	2. Put one in the tens column and leave two behind to get this:
???	

10	3. Now add the ten's column figures to get 1 + 3 + 9 = **13 tens**
34	4. 13 tens = 130 = 1 hundred and three tens
+ **9**8	
??2	

100	5. Put **1 in the hundreds column** and leave three tens behind,
34	6. Writing the zeros can help keep columns straight. The answer is 132. Don't bother writing the zeros unless they help you keep track of place value.
+ 98	
132	

You can check your work by adding the numbers in the reverse order to see if you get the same answer. In the last example, you would put 98 above 34 and add to get 132.

Unit 2 – Number Operations

Topic 1 – Practice Questions

Question 1

John wants to add the tonnage from three dump trucks that delivered loads of gravel so he can bill for the total. The first truck brought 5 tons, the second 7, and the third 4. What is the total?

a) 16 tons

b) 25 tons

c) 17 tons

d) 15 tons

Answer: a

Explanation
The numbers can be added in any order to find the total. 5 + 7 = 12 and 12 + 4 = 16, or 5 + 4 = 9 and 9 + 7 = 16, or 7 + 4 = 11 and 11 + 5 = 16.

Question 2

Pete and Bill each worked 40 hours of overtime, but Sally and Jim only worked 25 hours overtime each. How much overtime was put in by all four people?

a) 130 hours

b) 105 hours

c) 80 hours

d) 50 hours

Answer: a

Explanation
The questions asks us to add four overtime amounts, one for each worker. Pete + Bill = 40 + 40 = 80, and Sally + Jim = 25 + 25 = 50. When both of these totals (called subtotals) are combined we get a grand total of 80 + 50 = 130 hours. When combining 8 tens and 5 tens, we get 13 tens, 3 tens stay in the tens column, and 10 tens are exchanged for 1 one hundred and added to the hundreds column to give 130 as the answer.

Topic 1.B – Subtracting Integers

Subtracting Integers

Subtraction is the work of taking one number away from another one. Subtraction makes something smaller by removing part of what was there to start with. We can also think of subtraction as the distance between two numbers on a number line. The larger number will always be to the right of the smaller number, and the difference between them will be the result of measuring how far apart they are. Subtraction is also referred to as finding the difference between numbers.

$$\begin{array}{r} 9 \\ -7 \\ \hline 2 \end{array} \qquad \begin{array}{r} 12 \\ -7 \\ \hline 5 \end{array} \qquad \begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array}$$

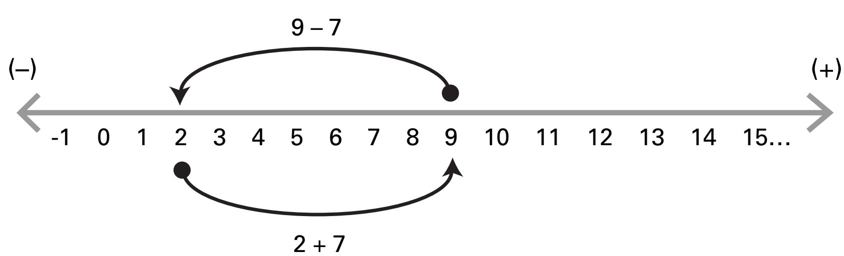
> Always place the larger number on top in a subtraction problem.

nine minus 7 equals 2, the difference between 9 and seven is two

twelve minus 7 equals 5, the difference between 12 and seven is 5

5 minus three equals 2, the difference between 5 and 3 is two.

If you picture a ruler, you can also see that nine minus seven is like going from where nine is on the ruler to where two is. You had to move to the left seven spaces to reach 2. A ruler is one kind of number line. The answer to a subtraction problem is the destination you arrive at when you travel from a starting point a given number of places to the left.



Unit 2 – Number Operations

Topic 1.B – Subtracting Integers

Notice that a "round trip" can be found for any subtraction problem by adding the number taken away to the answer. 9 – 7 = 2, and 2 + 7 = 9. Subtraction problems can be checked this way to see if the round trip works by giving back the original number you subtracted from. Notice that 7 units are between 2 and 9 on the line, but that subtraction (–) tells us that the direction is to the left, and addition (+) that the direction is to the right. This is the principle behind all adding and subtracting using a tape measure in many trades.

Example:

Check your subtraction with addition

13 – 7 = 5 (?)

check: 5 + 7 = 12

correction: 13 - 7 = 6

check: 6 + 7 = 13

Exchanges

Exchanging is sometimes called **"borrowing"** in subtraction problems

We begin by subtracting numbers in the one's column. This can result in a "shortage", when the lower number is larger than the top number. In this case we look to next column to the left and exchange one ten for the ten ones it is equal to. We now have a larger number to subtract from and come out with a result that will fit in the one's column. Then, when we subtract from the ten's column the same process can be repeated if we need to, by exchanging one of the hundreds for ten tens, and so on.

$$\begin{array}{r} 12 \\ -7 \\ \hline 5 \end{array}$$ 12 minus 7 is five

$$\begin{array}{r} 112 \\ -77 \\ \hline ?? \end{array}$$ one hundred and twelve minus seventy seven = ?

Topic 1.B – Subtracting Integers

Examples

Seven is too large to subtract from two in the ones column, so we exchange one ten for ten ones and subtract seven from twelve to get **five ones**. This leaves a situation like this:

$$\begin{array}{rr} 10 & 12 \\ -7 & -7 \\ \hline ? & \mathbf{5} \end{array}$$

Separate the columns to show 1 hundred = 10 tens, and 12 = 12 units.

We have ten tens, or one hundred to work with after the exchange that took one ten away from the ten's column. Now we subtract 7 tens from 10 tens to give **3 tens.**

$$\begin{array}{rr} 10 & 12 \\ -7 & -7 \\ \hline \mathbf{3} & 5 \end{array}$$

The final answer is 35. You will find your own "shorthand" to keep track of exchanges. Many people cross out the number being exchanged in a column, reduce it by one and write the new number in. It is very important to make neat columns and not get mixed up in the exchange process.

Here is another example taken step by step through necessary exchanges:

$$\begin{array}{r} 457 \\ -389 \\ \hline ??? \end{array} \qquad \begin{array}{rrr} 4 & 4 & 17 \\ -3 & 8 & 9 \\ \hline ? & ? & 8 \end{array} \qquad \begin{array}{rrr} 3 & 14 & 17 \\ -3 & 8 & 9 \\ \hline 0 & 6 & 8 \end{array}$$

The answer is 68 and can be checked by adding it back to 389 to see if we get 457 as the sum.

$$\begin{array}{r} 457 \\ +\ 68 \\ \hline 457 \end{array}$$

Tip:
On the exam it is a good idea to check your work this way if there is time.

Unit 2 – Number Operations

Topic 1 – Practice Questions

Question 1

What is the difference between 773 and 981?

a) 123

b) 208

c) 254

d) 321

Answer: b

Explanation
Put the larger number on top and subtract. 981 is larger, but you can't subtract 3 from 1 in the ones column without exchanging one ten for ten ones. This gives you 11 – 3 = 8 for the ones column in the answer. The tens column now has 7 – 7 = 0, and for the hundred column 9 – 7 = 2. The answer is 208. You can check this answer by adding it to 773 and seeing that it gives 981.

Question 2

John will be off work 13 of the next 60 days, and Amos will be off work 17 of the next 60 days. How many days will both men be at work in the next sixty days?

a) 30 days

b) can't tell from this information

c) 60 days

d) 90 days

Answer: d

Explanation
This problem has two parts. We need to know how many days each man will work and then add to get the total number of days worked by both. Subtract 13 from 60 to find that John will work 47 days in the next sixty, and subtract 17 from sixty to find that Amos will work 43 days. Adding the days worked gives 90 in total. You can check this answer by adding days off and days worked to make sure the "round trip" works out to 60 days for each man.

47 + 13 = 60

43 + 17 = 60

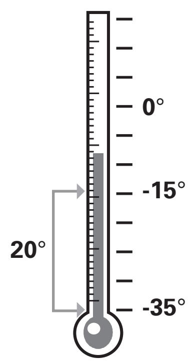
Topic 1.C – Signed Integers

Signed Numbers

Signed numbers carry their own instructions about whether to move left (subtract, –) or right (add, +). Signed numbers have a + or – sign in front and can be combined by following these instructions. Recall that an integer is always positive unless a - appears in front of it. **Combine is another word for addition, but with an emphasis on the fact that the numbers being added can include negative, or signed numbers.**

For example, -3 + 2 = -1 because we move 3 places left of zero and then two places right to arrive at -1. Because we are combining the numbers, it doesn't matter which one we start from. Check this by reversing the order. 2 – 3 = -1 also. Start at 2 and then go three units left and you arrive at -1. This can also be written as 2 + (-3) = -1. It can be read as "two plus minus three equals minus one" or as "two combined with minus three equals minus one". Think of + as an arrow pointing to the right on the number line, and a - as an arrow pointing left on the number line.

You can picture combining signed numbers by thinking of a thermometer or a bank book. If the temperature is -35° and goes up to -15° by noon, this can be expressed as -35° + 20° = -15°. The temperature increased by 20° degrees. If you are overdrawn \$35.00 in your chequing account and put \$20.00 in, you will still be overdrawn by \$15.00 but your account increased in value by \$20.00. This can be expressed as -\$35.00 + \$20.00 = -\$15.00. You are still in a negative position, but it is smaller than before.



Bank of the NWT	
Oct. 30	<$35.00>
Nov. 1 Deposti $20.00	<-$15.00>
Nov. 10 Deposit $50.00	$35.00

Topic 1 – Practice Questions

Question 1

What is the total of -6 + 2 - 4?

a) -4

b) -9

c) -8

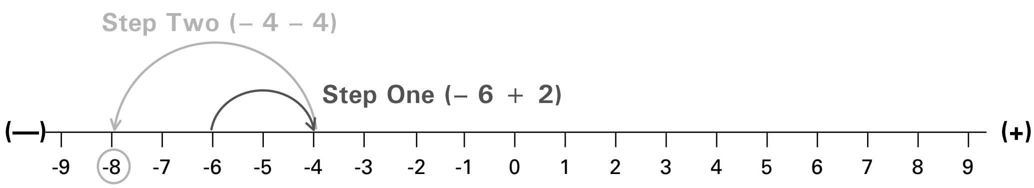
d) +4

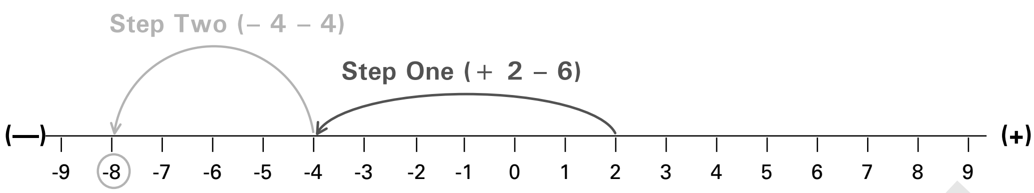
Answer: c

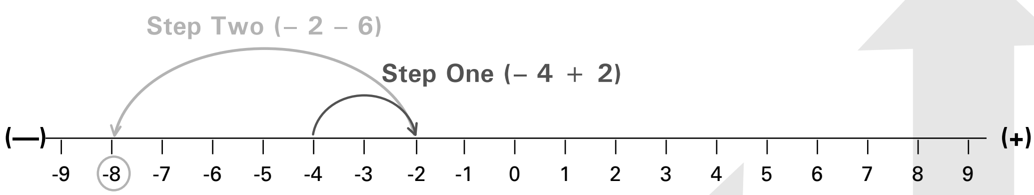
Explanation

Use a number line to prove that the answer is -8. Any of the starting points can be chosen, and any order of combination will give -8 as the answer.

Here are three of the six possible routes on the number line:







Unit 2 – Number Operations

Topic 1 – Integers

Combining Operations: Adding and Subtracting

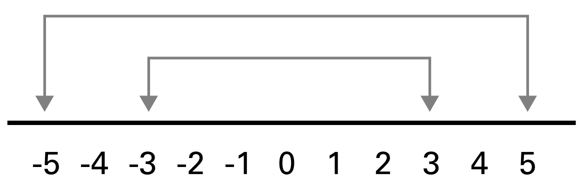
A number line can show that adding and subtracting are both ways of travelling on the line, with negative numbers to the left of zero, and positive numbers to the right. The numbers to the left of zero are called "negative numbers". These numbers can be compared to a debt, or debit, and the positive numbers to the right of zero can be compared to money in the bank, or to a credit. Because these numbers are identified by a sign that places them on the number line, they are also known as signed numbers. We will deal with the subtraction of a negative number from a negative number in a later section.

"Balancing the books" involves combining both negative and positive numbers. Any list of integers, fractions, or decimals can be combined by noticing that minus means travelling left, and plus means travelling right. The answer is the total remaining after the journey is completed. Sometimes this will be a negative number. Many trades problems involve negative numbers.

Example:
5 - 3 + 2 - 1 = 3 Means start at five, go left three, right two, and then left one. You will arrive at 3. You will arrive at 3 no matter what order you choose to complete the combination of operations. (**Note:** when there is no "–" sign in front of a number, it is positive and is located to the right of zero on the number line).

Every Number Has an Opposite

The opposite of 3 is –3, the opposite of $-\frac{3}{2}$ is $\frac{3}{2}$. **When opposites are added they produce zero.** Opposites are the same distance from the origin, (zero), but in opposite directions. Here is a picture showing the relationship between the opposites -5 and 5, and –3 and 3.



Unit 2 – Number Operations

Topic 1 – Practice Questions

Question 1

What is the opposite number to 15?

a) 51

b) 30k

c) –15

d) –51

Answer: c

Explanation

The opposite of a number is defined as the number that will give zero when they are added together. If you reverse the sign of a number you get its opposite.

Question 2

What is the total of –6 + 2 and –14?

a) -22

b) -20

c) -18

d) -16

Answer: c

Explanation

-18 is the result of combining these numbers. You can check this on a number line or calculate by starting with any of the numbers given:

-6 + 2 = -4 (find the difference and keep the sign of the larger number, here 6. This is also called the absolute value of the number. The absolute value of -6 is "6", and 6 is larger than 2 so we assign – to the difference.)

Now combine this result, -4, with -14 to get -18. When the signs are the same for two numbers, we keep that sign for the answer. Again, the number line shows why this is so.

Alternatively, start with +2 – 14. Here the signs are different, 14 is bigger than two (has a larger absolute value) therefore the difference will be negative.
+2 – 14 = -12. Now complete with -12 – 6 = -18. All combinations produce -18 for the answer.

Topic 1 – Practice Questions

Question 3

Which pair of numbers will produce zero when they are added together?

a) 6 and 0

b) 5 and $^1/_5$

c) 12 and $^{60}/_{12}$

d) 520 and –520

Answer: d

Explanation

When a positive number is added to its negative value on the number line the result is zero. Any number combined with its opposite is zero.

Unit 2 – Number Operations

Topic 1.D – Multiplying Integers

Multiplying Integers

Multiplication is a fast way to add. Learn the multiplication tables (aka "times tables") for the integers (whole numbers) 1 to 10. This will give you the basis for multiplying large numbers. Remember that 0 times anything is still 0 and 1 times anything doesn't change it.

0 x 5 = 0

0 x any number = 0

1 x 5 = 5

1 x any number = the number

Multiplication can also be represented by a "." (dot) between the factors, or by placing the factors inside brackets next to each other. (16)(2) = 32

Numbers can be multiplied in any order and still give the same answer
2 x 4 = 8, and 4 x 2 = 8

Examples

a) 2 + 2 + 2 + 2 = 8, is the same as 2 x 4 = 8 = 4 + 4

Four groups of two gives the same total as two groups of four. Every multiplication problem can be expressed as a longer addition problem.

b) 3 x 4 x 5 = 60, is the same as (4 + 4 + 4) + (4 + 4 + 4) + (4 + 4 + 4) + (4 + 4 + 4) + (4 + 4 + 4) = 60

The brackets show how five groups of three fours make sixty. We put three fours inside each of the brackets to make sense out of 3 x 4 in the first part of "three times four times five" or "3 x 4 x 5". Multiplication saves work and groups numbers for us. Notice also that 3 x 4 x 5 is the same as 4 x 3 x 5 and 5 x 3 x 4, and 5 x 4 x 3. Also notice that 12 x 5 = 60 and 15 x 3 = 60. Finally, we could have started with 4 x 5 and then multiplied that answer times 3. Three groups of five fours also equal 60. In this approach we could group this way and get the same answer: (4 + 4 + 4 + 4 + 4) + (4 + 4 + 4 + 4 + 4) + (4 + 4 + 4 + 4 + 4) = 60. Every multiplication problem can be expressed as a long addition problem.

Time is money: Multiplication saves work.

Unit 2 – Number Operations

Topic 1.D – Multiplying Integers

Carrying in multiplication

Review the multiplication tables for 1 to 9 and memorize them. Multiplying numbers will create “overages” in columns that require you to carry them into the next larger column. A multiplication problem can be broken down into a multiplication problem combined with an adding problem.

Example

89	(factor)
x6	(factor)
54	(product of 6 x 9)
+480	(product of 6 x 80)
534	(product of 6 x 89)

> The answer to a multiplication problem is called the **product**, the numbers multiplied are called **factors**. A number multiplied by a variable is called a **coefficient**. 6 is a coefficient in the expression 6k.

A shorthand called “carrying” saves the step of writing out the addition of the units product (6 x 9) to the tens product (6 x 80).

$$\begin{array}{r} \mathbf{5} \\ 89 \\ \times\ 6 \\ \hline 4 \end{array}$$

Step One: 6 x 9 = 54

Carry the five tens in 54 to the tens column

$$\begin{array}{r} \mathbf{5} \\ 89 \\ \times\ 6 \\ \hline 534 \end{array}$$

Step Two: multiply 6 x 80 = 480 and add 5 tens to get 530. Put five in the hundreds column and three in the tens column.

> You can check the answer by reversing the order of the factors (numbers being multiplied) and see if you get the same product (answer).

Unit 2 – Number Operations

Topic 1.D – Multiplying Integers

You can multiply large numbers by carrying and lining up the columns to keep track of place values.

Example

$$\begin{array}{r} 236 \\ \underline{\times\ 342} \end{array}$$

Step 1:

$$\begin{array}{r} \mathbf{1} \\ 236 \\ \underline{\times\ 34\mathbf{2}} \\ 472 \end{array}$$

Multiply 236 by 2 ones. 472 is the product with a one carried to the tens column and added to 2 x 30 to give 70, or 7 tens.

Step 2:

$$\begin{array}{r} 236 \\ \underline{\times\ 3\mathbf{4}2} \\ 472 \\ 944 \end{array}$$

Multiply 236 by 4 tens. The answer (944) is lined up from the right under the tens column because we are multiplying by 4 **tens**.

Step 3:

$$\begin{array}{r} 236 \\ \underline{\times\ \mathbf{3}42} \\ 472 \\ 944 \\ 708 \end{array}$$

Multiply 236 by 3 hundreds. The answer (708) is lined up from the right under the hundreds column because we are multiply by 3 **hundreds**. The unoccupied paces could show 0's.

Step 4:

$$\begin{array}{r} 236 \\ \underline{\times\ \mathbf{3}42} \\ 472 \\ 944 \\ \underline{708} \\ 80712 \end{array}$$

Add the products from the past three steps keeping place value columns in line.

80 thousands, seven hundreds, one ten, two units

Unit 2 – Number Operations

Topic 1 – Practice Questions

Question 1

What is the product of 45 and 236?

a) 10,620

b) 281

c) 12,744

d) 8, 064

Answer: a

Explanation

Use a calculator to verify this multiplication problem. Work it on paper by putting 45 under the 36 in 236.

Consult Arithmetic textbooks for more practice multiplying integers as needed.

Unit 2 – Number Operations

Topic 1.D – Multiplying Integers

Multiplying Signed Numbers

You Need to Know…

+ x + = +

- x - = + **(multiplication with like signs gives a positive product)**

-15 x -3 = 45

- x + = - **(multiplication with unlike signs give a negative product)**

-3 x 5 = -15

Unit 2 – Number Operations

Topic 1 – Practice Questions

Question 1

Evaluate -2 x 6 x -3.

a) -36

b) 36

c) 0

d) -18

Answer: b

Explanation

The product of -2 x 6 is -12, because multiplying unlike signs gives a negative product. Next -3 x -12 gives 36 because the product of like signs is positive. You can get the same answer by multiplying in any order using the same rules for signed numbers and multiplication.

Topic 1.E – Dividing Integers

Dividing Integers

Division takes something apart into equal sized pieces. If we divide 20 by 2, the answer is 10. Notice that dividing by 2 is the same as breaking something in half, or sharing equally.

Division can be expressed in three ways:

$10 \div 5 = 2$ $\quad 5\overline{)10} = 2$ $\quad {}^{10}/_{5} = 2$

Remember we can't divide anything by 0, division by zero is undefined- but anything divided into zero is zero.

Each can be read as "Ten divided by five equals two", aka "5 goes into ten two times". In each version, the answer (the quotient, 2) multiplied by five (the divisor) equals ten (the dividend). The divisor is also the denominator of the improper fraction ${}^{10}/_{5}$, and this shows that every fraction can also be thought of as a division problem.

What About "Goes Into"?

"5 goes into 10 two times",

The number of times (2) is called the quotient, the number "going in" (5) is the divisor, and the number being "gone into" (10) is the dividend.

If you know two of these numbers you can find the third.

$3\overline{)18} = 6$ and $3 \times 6 = 18$

Check a division problem by multiplying the divisor times the quotient.

Example

How many groups of 6 are in 18?

$6\overline{)18} = 3$ There are 3 sixes in 18, six goes into 18 three times
check: $3 \times 6 = 18$

Review the multiplication tables as division tables

$3 \times 6 = 18$ and $18 \div 6 = 3$ and $18 \div 3 = 6$

Unit 2 – Number Operations

Topic 1.E – Dividing Integers

Example

Remember these steps in long division problems[6]:

D 1. Divide

M 2. Multiply

B 3. Bring down

S 4. Subtract

5. Go back to step one and repeat until no more division is possible.

Divide 283 by 6.

Step 1:

$$\begin{array}{r} \mathbf{4} \\ 6\overline{)283} \end{array}$$

Divide
There are forty groups of 6 in 280 with 4 tens left. Four is placed above 8 to show that 4 "goes into" 280 40 times. The 4 shows 4 tens. Note that 5 would be too large.

Step 2:

$$\begin{array}{r} 4 \\ 6\overline{)283} \\ \underline{\mathbf{240}} \end{array}$$

Multiply
4 tens x 6 ones = 240. This is what forty groups of six are worth.

Step 3:

$$\begin{array}{r} 4 \\ 6\overline{)283} \\ \underline{-240} \\ \mathbf{43} \end{array}$$

Subtract
Subtract 240 from 283 to get the difference we still need to divide 43 by 6.

Step 4:

$$\begin{array}{r} \mathbf{47} \\ 6\overline{)283} \\ \underline{-240} \\ 43 \end{array}$$

Bring Down and Divide
There are no more numbers in the dividend (283) to bring down. Go to step one and divide 6 into 43 to get 7.

Step 4:

$$\begin{array}{r} 47\ \text{R}\mathbf{1} \\ 6\overline{)283} \\ \underline{-240} \\ 43 \\ \underline{-42} \\ \mathbf{1} \end{array}$$

Multiply and Subtract the Result
7 times 6 = 42 subtract 42 from 43 = 1

6 This is one of several algorithms (procedures) that can be used to calculate quotients. If you prefer another way and get the right answers, use it.

Topic 1.E – Dividing Integers

The remainder of 1 describes what is left over after we have divided 283 into groups of 6. The answer tells us that if we had 283 objects, we could make 47 piles of 6 each, but that one would be left over. The remainder can also be expressed as a fraction with the remainder as the numerator, and the denominator as the divisor. In this example the remainder is $^1/_6$. The answer can be expressed as 47 $^1/_6$, a mixed number.

Example 3: D, M, S, B

$$7\overline{)1421}\quad 203$$

- 14	(200 x 7 = 1400)
02	(7 goes into 2 zero times)
- 0	(0 x 7 = 0)
21	(subtract 0 from 2 and bring down 1)
- 21	(multiply 3 x 7 = 21)
0	remainder

Dividing signed numbers

You Need to Know…

+ ÷ + = +
- ÷ - = + **(like signs give a positive quotient)**
-15 ÷ -3 = 5

- ÷ + = - **(unlike signs give a negative quotient)**
-15 ÷ 5 = -3

Example

125 ÷ -5 = -25

Unit 2 – Number Operations

Topic 1 – Practice Questions

Question 1

What is the quotient of 54 divided by 9?

a) 6

b) $^{9}/_{54}$

c) 8 remainder 1

d) 12

Answer: a

Explanation

6 x 9 = 54 and 9 goes into 54 six times. This problem asks you divide 9 into 54. The question asks how many groups of 9 are in 54.

Question 2

Which rule tells us that 575 can be divided evenly by 5?

a) all numbers ending in an odd number are divisible by 5

b) all numbers ending in a 5 or a zero are divisible evenly by 5

c) a seven followed by a five is an example of a prime number

d) the tens column has more than five in it

Answer: b

Explanation

It is true that 5 will divide evenly into any number ending in 5 or 0. This is an example of a useful divisibility rule. You can count by 5 to any number ending in 5 or 0.

Unit 2 – Number Operations

Topic 2.A – Adding Fractions

Adding Fractions

Important:
Fractions can only be added or subtracted when they have the same denominator!

Fractions that have the same denominators are called like fractions. Unlike fractions have different denominators. The first step is to decide if the fractions being added are like, or unlike.

Adding Like Fractions

Do the fractions have the same denominators? If so, simply add the numerators and keep the common denominator in your answer. If the answer is improper (i.e. greater than 1), simplify it.

Examples

$$\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$$

(3 is the common denominator)

$$\frac{9}{16} + \frac{1}{16} + \frac{5}{16} = \frac{15}{16}$$

(16 is the common denominator)

$$\frac{3}{4} + \frac{2}{3} + \frac{1}{4} = \frac{4}{4} + \frac{2}{3} = 1\frac{2}{3}$$

(notice that $^2/_3$ is added to the result of combining 2 and $^1/_4$. We cannot combine thirds and fourths because the denominators are different. Here, a mixed number, 1 $^2/_3$, is the answer.)

Simplify the answer
Simplifying fractions means dividing the denominator into the numerator and expressing the remainder in lowest terms.

Unit 2 – Number Operations

Topic 2.A – Adding Fractions

Step one:

If the answer is an improper fraction, divide the denominator into the numerator to get the mixed number it is equal to.

Examples

$$\frac{1}{3} + \frac{2}{3} + \frac{2}{3} = \frac{5}{3}$$ (3 goes into 5 once with a remainder of $^2/_3$)

$$\frac{5}{12} + \frac{7}{12} + \frac{11}{12} = \frac{23}{12}$$

simplify $12\overline{)23} = 1\frac{11}{12}$ (12 goes into 23 once with a remainder of $^{11}/_{12}$)

Step two:

If the mixed number has a fraction part that contains a numerator and a denominator that can be divided evenly by another number, use that number to divide into the numerator and denominator and get the reduced proper fraction that it is equal to.

Example

$^5/_6 + {}^5/_6 = {}^{10}/_6 = 1\ {}^4/_6$

Ask: Can $^4/_6$ be reduced (i.e. simplified)? Is there a number that can divide evenly into both 4 and 6?

Answer: 2

Is this the **largest number** we can find that does this?

Answer: yes

Now use 2 as a divisor into both 4 and 6 to get 4 ÷ 2 = 2, and 6 ÷ 2 = 3.

We have **reduced** $^4/_6$ to $^2/_3$, by putting it in "**lowest terms**". We used the largest common divisor of 4 and 6 to do this. The largest common divisor of these two numbers is 2. Notice that four pieces selected from a pie divided into six pieces, is the same size as two pieces of another equal sized pie that is divided into three pieces. Fewer cut lines are needed to get two thirds than to get four sixths, but someone eating two thirds will get just as much as someone eating four sixths. This is the one way to get the idea behind "simplest" terms, or reducing a fraction.

Topic 2 – Fractions

Example

$^1/_4 + ^3/_4 + ^3/_4 + ^1/_4 = ^8/_4 = ^2/_1 = 2$

(notice that 4 is the **largest** number to divide both 8 and 4 evenly. Use it to divide both numerator and denominator)

> When we divide the numerator **and** denominator of a fraction by the **same** number, the result will be a reduced fraction of the same size.
>
> $^4/_8 = ^1/_2$ (divide with 4)
>
> $^6/_{12} = ^1/_2$ (divide with 6)
>
> $^{40}/_{100} = ^2/_5$ (divide with 20)

Unit 2 – Number Operations

Topic 2 – Practice Questions

Question 1

Add $^5/_8 + {}^3/_8 + {}^7/_8$ and simplify.

a) $^{15}/_8$

b) $1\ ^7/_8$

c) $2\ ^5/_8$

d) $5\ ^{10}/_8$

Answer: b

Explanation

These are like fractions so we can add the numerators to get 15 eighths, or $^{15}/_8$. To simplify this improper fraction divide the denominator into the numerator. There is one 8 in 15 with a remainder of 7. The answer is a mixed number, 1 and $^7/_8$. $^7/_8$ cannot be simplified further (reduced) because there is no integer that will go evenly into both seven and eight.

Unit 2 – Number Operations

Topic 2.A – Adding Fractions

Adding Unlike Fractions

Unlike fractions have different denominators. First convert them into like fractions and then add as explained above. When we add fractions that are unlike, we need to find a common denominator, express the fractions in terms of the common denominator, and then add the numerators of the new like fractions.

It can be hard to find the smallest common denominator. A few facts will point the way. The product of the denominators will be a number that each of them divides into evenly- but this number may be very large, a smaller number that does the job is preferred. This is why we search for a least, or **lowest common denominator**.

As a last resort, the product of the denominators can be used for a common denominator, but it won't always be the lowest one. This method will work, however, we will have to reduce our answer by finding the largest number that will divide numerator and denominator evenly and do the work of reducing the fraction in our answer to lowest terms.

Example

Using the product of denominators as our common denominator

$^{11}/_{18} + {}^{7}/_{24}$

Step 1:
Find a number that both 18 and 24 go into evenly. **18 x 24** will give such a number = 432. This is a common denominator, but it is not the smallest. That number is 72 – difficult to find without a more advanced method.[7] Trial and error is a good method that can help you discover the lowest common denominator.

Continuing with 432 as common denominator:

(432 can be divided into 24 groups with 18 in each group. This tells us that $^{1}/_{18} = {}^{24}/_{432}$, each 18th is equal to twenty four 432nds.)

and

(432 can also be divided into 18 groups with 24 in each group. This tells us that $^{1}/_{24} = {}^{18}/_{432}$, each 24th is equal to 18 four hundred thirty seconds.)

We can exchange eighteenths and twenty fourths for the number of four hundred thirty seconds each is equal to. Think of 432nds as the "common currency" into which 24ths and 32nds can be converted.

Step 2:
Convert $^{11}/_{18}$ and $^{7}/_{24}$ into equivalent numbers of 432nd sized pieces.

a) Divide 18 into 432 = 24. This means that each 18th sized piece is equal to $^{24}/_{432}$.
We have 11 eighteenths, so multiply 24 x 11 = 264 to get $^{11}/_{18}$ = 264/432

b) Divide 24 into 432 = 18. This means that each 24th is equal to $^{18}/_{432}$.
We have 7 twenty-fourths, so multiply 7 x 18 = 126 to get $^{7}/_{24} = {}^{126}/_{432}$.

Topic 2.A – Adding Fractions

Step 3:
Add the resulting like fractions.

$^{126}/_{432} + {}^{264}/_{432} = \mathbf{{}^{390}/_{432}}$

The answer is a proper fraction, but it must be simplified. We need to find a number that divides both numerator and denominator evenly. If we try using 2 we get: $^{390}/_{2}$ = 195, and $^{432}/_{2}$ = 216, now we have

$^{195}/_{216}$, which is simpler, but not simple enough. It can be further reduced by dividing with 3 to get

$^{195}/_{3}$ = 65, and $^{216}/_{3}$ = 72.

Now we have $^{195}/_{216} = {}^{65}/_{72}$, **a fraction in lowest terms because no number will divide top and bottom evenly**. The fact that we used 2 and then 3 suggests that 6, the product of 2 x 3 could have been used to simplify in one step. Check and you will see that this is true.

The key is to find the **LCM (least common multiple) of the denominators** in the problem. Here are the steps for adding unlike fractions:

Adding Unlike Fractions

Step One: find the LCM of the denominators

Step Two: change the fractions into equivalent fractions with the LCM as the denominator.

Step Three: Add the like fractions created in step two.

Step Four: If possible, simplify the answer.

The smallest number that all of the denominators can divide into evenly is their least common multiple **(LCM)**, this number is aka their lowest common denominator **(LCD)**.

Unit 2 – Number Operations

Topic 2.A – Adding Fractions

Example

Add $^{13}/_{15}$ and $^{5}/_{36}$

Step 1:
The LCM of 15 and 36 is 180 (a larger multiple of 540 results from multiplying the denominators, but we have chosen the smallest common denominator. 540 will work, but there is a lot of work to simplify the answer involved. 180 makes the job easier.)

Step 2:

$15\overline{)180} = 12$, and $36\overline{)180} = 5$,

so each 15^{th} is worth $^{12}/_{180}$, and each 36^{th} is worth $^{5}/_{180}$. We have thirteen 15^{ths} so
13x 12 = 156, and we have five 36^{ths}, so
5 x 5 = 25.

Step 3:
Our problem now involves adding the like fractions:

$^{156}/_{180} + {}^{25}/_{180} = {}^{181}/_{180}$

Step 4:
This is an improper fraction, simplify it by dividing denominator into numerator,
$180\overline{)181} = 1\ {}^{1}/_{180}$

Is $^{1}/_{180}$ in lowest terms?

Answer
Yes, no number will divide both numerator and denominator evenly except for 1, and that doesn't change anything.

Example

Add $^{1}/_{4} + {}^{2}/_{3} + {}^{3}/_{4} + {}^{1}/_{2}$

Step 1:
The LCM of the denominators 4, 3, 2 is 12.

Step 2:
Convert each fraction into the number of 12ths it is worth (or equal to)

$^{1}/_{4} = {}^{3}/_{12}$, $^{2}/_{3} = {}^{8}/_{12}$, $^{3}/_{4} = {}^{9}/_{12}$, $^{1}/_{2} = {}^{6}/_{12}$

Step 3:
Add like fractions.

$^{3}/_{12} + {}^{8}/_{12} + {}^{9}/_{12} + {}^{6}/_{12} = {}^{26}/_{12}$

Step 4:
Simplify.

$^{26}/_{12} = 12\overline{)26} = 2\ {}^{2}/_{12}$ reduce $^{2}/_{12} = {}^{1}/_{6}$

$2\ {}^{1}/_{6}$

Unit 2 – Number Operations

Topic 2 – Practice Questions

Question 1

A coop keeps track of how many moose hides are used to make mukluks. On Monday $^1/_3$ of a hide was used, on Tuesday, $^1/_4$ and on Friday $^5/_8$ of a hide was used. What is the total number of hides used?

a) $1\ ^5/_{12}$

b) $^{18}/_{24}$

c) $1\ ^7/_8$

d) $1\ ^5/_{24}$

Answer: d

Explanation

The problem gives us unlike fractions and we need a common denominator to change them into before we can add them. The product of the three denominators is 96, and this will work, but 24 is better because it is smaller and 3, 4, and 8 each divide evenly into 24. The next step is to convert and we get a new problem of adding the like fractions $^8/_{24} + {}^6/_{24} + {}^{15}/_{24} = {}^{29}/_{24}$. This improper fraction is simplified by dividing 24 into 29 = $1\ ^5/_{24}$, given in choice d. You can see that $^5/_{24}$ is very close to $^6/_{24}$, which is equal to $^1/_4$. This answer could have been rounded off to $1^1/_4$ moose hides in total.

Consult other math texts as needed for more practice in adding fractions.

Unit 2 – Number Operations

Topic 2.B – Subtracting Fractions

Subtracting Fractions

The same principle applies as for the addition of fractions: with like fractions simply subtract the numerators and keep the same denominator. For unlike fractions the LCM must be used to convert each fraction into a like fraction of equivalent value so that they can be subtracted. Review the overview on fractions and the rules for adding unlike fractions.

To subtract unlike fractions:

Step One:	Find the LCM of the fractions.
Step Two:	Change each fraction into an equivalent fraction using the LCM.
Step Three:	Subtract the smaller from the larger like fraction created in Step 2.
Step Four:	Simplify the answer if possible.

Examples

1. Subtract $^1/_4$ from $^3/_4$.

These are like fractions. $^3/_4$ is larger, and we are taking $^1/_4$ away

$^3/_4 - {}^1/_4 = {}^2/_4$

Simplify this proper fraction by dividing numerator and denominator by the largest common divisor = 2.

$^2/_4 = {}^1/_2$

2. Subtract $^{17}/_{35}$ - $^4/_{15}$

The LCM of 35 and 15 is 105

$35 \overline{)105}$ = 3, and 3 x 17 = 51, so $^{17}/_{35} = {}^{51}/_{105}$

$15 \overline{)105}$ = 7, and 4 x 7 = 28, so $^4/_{15} = {}^{28}/_{105}$

51/105 – 28/105 = $\frac{51 - 28}{105}$ = 23/105 (subtract like fractions)

23/105 is already a proper fraction in simplest form because no number will divide numerator and denominator evenly.

Unit 2 – Number Operations

Topic 2 – Practice Questions

Question 1

What is the difference between $^5/_8$ and $1\ ^2/_3$?

a) $1\ ^1/_{24}$

b) $^5/_8$

c) $1\ ^{11}/_{16}$

d) $1\ ^3/_{24}$

Answer: a

Explanation

First notice that $1\ ^2/_3$ is a mixed number and $^5/_8$ is a proper fraction. $1\ ^2/_3$ is larger. We need a common denominator that 8 and 3 go into evenly. The product, 24, is the smallest such number. Converting we get $1\ ^2/_3 = {}^{40}/_{24}$ and $^5/_8 = {}^{15}/_{24}$. The difference is $^{25}/_{24}$ which reduces to $1\ ^1/_{24}$.

Unit 2 – Number Operations

Topic 2.C – Multiplying Fractions

Multiplying Fractions

To multiply fractions simply multiply numerators by numerators and denominators by denominators and simplify the answer.

Remember that any whole number can also be expressed as a fraction over one. $5 = {}^{5}/_{1}$, $25 = {}^{25}/_{1}$ etc.

Top times top,
bottom times bottom.

"of" means "times"
= multiply

${}^{3}/_{4}$ of 12 =

${}^{3}/_{4} \times {}^{12}/_{1}$ =

${}^{36}/_{4} = 9$

Topic 2.C – Multiplying Fractions

Example:

1. $\frac{1}{3} \times \frac{2}{3} = \frac{1 \times 2}{3 \times 3} = \frac{2}{9}$

This can be read "one third of two thirds". This means that two thirds of something is being further subdivided into three equal parts and we are selecting one of them.

Examples:

1. A practical problem based on these figures might ask, "if you have a drum of oil that you know is $\frac{2}{3}$ full, and someone takes one third of this, what will be the volume of oil remaining in the drum"? We know that one third of two thirds is two ninths. ($\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$) We need to know what $\frac{2}{3} - \frac{2}{9}$ equals to find the oil that remains in this drum. By converting $\frac{2}{3}$ into ninths, we can see that $\frac{2}{3} = \frac{6}{9}$, and $\frac{6}{9} - \frac{2}{9}$ leaves the barrel $\frac{4}{9}$ full of oil. It is often useful in practical situations to round fractions off. $\frac{4}{9}$ is very close to $\frac{1}{2}$ or 4.5/9 because $\frac{9}{2}$ is 4.5.

2. $\frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$

   "One half of one half equals one quarter". Think of half of a pie that someone is taking a half share out of. They will wind up with $\frac{1}{4}$ of the original whole pie. In a garage, one half of a half litre of oil will equal 1/4th of a litre.

3. Multiply 4 and $\frac{6}{11}$

   First write 4 as a fraction number = $\frac{4}{1}$

   $\frac{4}{1} \times \frac{6}{11} = \frac{4 \times 6}{1 \times 11} \quad \frac{24}{11} = 11\overline{)24} = 2\frac{2}{11}$

4. Multiply $\frac{3}{8} \times \frac{4}{9}$

   $\frac{3}{8} \times \frac{4}{9} = \frac{3 \times 4}{8 \times 9} = \frac{12}{72}$ The largest common divisor is 12

   $\frac{12}{72} = \frac{1}{6}$

Topic 2 – Practice Questions

Question 1

What is $^2/_3$ of 67?

a) 57

b) $^{134}/_{201}$

c) 44.66

d) 1 $^{75}/_{204}$

Answer: c

Explanation

"Of" always means multiply when we are asked to find what a is of b, where a and b are two numbers. In this problem, top times top = 134, and the product of the denominators is 3 because 67 = $^{67}/_1$ and 3 x 1 = 3. Simplify by dividing the denominator into numerator to get $^{134}/_3$ = 44.66, which is equivalent to 44 $^2/_3$

Question 2

What is the product of $^5/_8$ and $^3/_4$?

a) 1$^4/_{20}$

b) $^{31}/_4$

c) $^{15}/_{32}$

d) 3 $^1/_2$

Answer: c

Explanation

The product is $^{15}/_{32}$, which cannot be reduced because no whole number goes evenly into 15 and 32. In some situations you could round this off to $^{16}/_{32}$, which equals $^1/_2$.

Unit 2 – Number Operations

Topic 2 – Practice Questions

Question 3

A cook has to prepare food for 35 people. Each serving include $^3/_8$ pound of hamburger. How much hamburger will be needed in total?

a) 48 lbs

b) $13^1/_8$ lbs

c) 105 lbs

d) $24^3/_8$ lbs

Answer: b

Explanation
This is a multiplication problem because we have to find out what 35 x $^3/_8$ pound equals. This will be the total when 35 servings of $^3/_8$ pound each are added together. Recall that multiplying is a quick way to add the same number to itself a specified number of times. To multiply put 35 over one times $^3/_8$ and you will get an improper fraction of $^{105}/_8$ to reduce for the answer. 105 ÷ 8 = 13 $^1/_8$.

Unit 2 – Number Operations

Topic 2.D – Dividing Fractions

Dividing Fractions

To divide fractions you need to understand what a reciprocal of a fraction is. Check the answers to every division problem in this topic by multiplying the answer times the divisor to see if it equals the dividend (i.e. the number you divide into).

Reciprocals
The reciprocal of $^1/_3$ is $^3/_1$

The reciprocal of $^2/_3$ is $^3/_2$

The reciprocal of $^4/_5$ is $^5/_4$

The reciprocal of a fraction can be found by turning it upside down (inverting) it.

To divide fractions, "invert and multiply" – this means turn the division problem into a multiplication problem using the **reciprocal** (upside down version) of the divisor.

The reciprocal of a fraction is also found by dividing it into one.

When we invert a fraction and multiply we get the reciprocal.

Any fraction divided into one produces its reciprocal. The product of reciprocals is always one.

Invert the divisor and multiply.

If Divisor = a/b

Reciprocal = b/a

Where a and b are any integer you choose.

Example
Find the reciprocal of $^2/_3$

$$1 \div {}^2/_3 = \frac{1}{{}^2/_3} = \frac{1}{1} \times \frac{3}{2} = {}^3/_2$$

$^3/_2$ and $^2/_3$ are reciprocals of each other. The product of reciprocals is one.

Topic 2.D – Dividing Fractions

Examples:

$^3/_2 \times {}^2/_3 = {}^6/_6 = 1$

$^1/_3 \times {}^3/_1 = {}^3/_3 = 1$

$^2/_3 \times {}^3/_2 = {}^6/_6 = 1$

$^4/_5 \times {}^5/_4 = {}^{20}/_{20} = 1$

You can see that the reciprocal of a fraction is the fraction with the numerator and denominator reversed. You have to have a fraction in mind before you can find its reciprocal.

Examples:

$1 \div {}^1/_3 = {}^3/_1$ and $^3/_1$ is the reciprocal of $^1/_3$

$1 \div {}^2/_3 = {}^3/_2$ and $^3/_2$ is the reciprocal of $^2/_3$

$1 \div {}^5/_6 = {}^6/_5$ and $^6/_5$ is the reciprocal of $^5/_6$

Unit 2 – Number Operations

Topic 2 – Practice Questions

Question 1

Divide $4/7$ by $2/5$.

a) $1\ 5/8$

b) $1\ 3/7$

c) $8/35$

d) $4/5$

Answer: b

Explanation
The divisor is $2/5$

Invert the divisor = $5/2$

Now multiply $5/2 \times 4/7 = 20/14$

Simplify: $20 \div 14 = 1\,6/14 = 1\,3/7$

Question 2

Divide 14 by $2/3$

a) 14

b) $3/4$

c) 21

d) 9

Answer: c

Explanation
The divisor is $2/3$

Invert the divisor = $3/2$

Now Multiply $14/1 \times 3/2 = 42/2$

Simplify: $2\overline{)42} = 21$

Topic 2 – Practice Questions

Question 3

Find $^{2}/_{3} \div {}^{1}/_{2}$

a) $1^{1}/_{3}$

b) $^{1}/_{6}$

c) $1^{1}/_{2}$

d) $1^{3}/_{4}$

Answer: a

Explanation
The divisor is $^{1}/_{2}$

Invert the divisor = $^{2}/_{1}$

Multiply: $^{2}/_{3} \times {}^{2}/_{1} = {}^{4}/_{3}$

Simplify: $3\overline{)\,4} = 1^{1}/_{3}$

Question 4

Which numbers are reciprocals of each other?

a) $^{2}/_{3}$ and $^{3}/_{5}$

b) 1 and $^{1}/_{2}$

c) $^{3}/_{5}$ and $^{5}/_{33}$

d) 200 and $^{1}/_{200}$

Answer: d

Explanation
200 is a whole number. Its reciprocal is $^{1}/_{200}$. You can check your answer by seeing that the product of $200 \times {}^{1}/_{200} = 1$.

Topic 2 – Practice Questions

Question 5

Find the reciprocal of $6\frac{5}{8}$

a) $\frac{8}{65}$

b) $\frac{53}{8}$

c) $\frac{8}{53}$

d) $\frac{8}{11}$

Answer: c

Explanation
First convert this mixed number into an improper fraction. $6\frac{5}{8} = \frac{53}{8}$. The reciprocal is $\frac{8}{53}$, and the product of $\frac{53}{8}$ and $\frac{8}{53}$ will be one as expected.

Question 6

Which numbers will give one as their product?

a) $\frac{1}{3}$ and $\frac{3}{2}$

b) $\frac{5}{8}$ and $\frac{8}{5}$

c) $1\frac{2}{3}$ and $1\frac{3}{2}$

d) $\frac{1}{4}$ and $\frac{1}{2}$

Answer: b

Explanation
Only choice b has a pair of reciprocals, and we know that reciprocals multiplied give one. Recall that a decimal is a fraction with a denominator that is a power of 10. The decimal point tells how to line up the columns in working with decimal problems.

Topic 3.A – Adding Decimals

Adding Decimals

Recall that a decimal is a fraction with a denominator that is power of 10. The decimal point tells us how to line up the columns in working with decimal problems.

To Add Decimals:

Step One: Organize the decimal numbers with the decimal points lined up below each other. Adding zeros in empty columns can help.

Step Two: Add the digits starting from the right and use the rules for adding whole numbers, (carrying forward).

Step Three: Put the decimal point in the sum directly below the other decimal points in the numbers that were added.

Examples

1. Add 5.67 + 6.7 + 13.458

```
   5.670
   6.700
+ 13.458
  25.828
```

Decimal:
from "deka" meaning ten.

2. Add .0067 + 230.01 + 34.0984

```
    .0067
 230.0100
+ 34.0984
 264.1151
```

Unit 2 – Number Operations

Topic 3.B – Subtracting Decimals

Subtracting Decimals

Decimal numbers are subtracted by lining up the smaller number under the larger number with the decimal points below each other. Use the rules for subtracting whole numbers to make exchanges and keep place values. Put the decimal point in the answer directly below the other decimal points.

Examples

1. Subtract 14.983 from 312.21

$$\begin{array}{r} 312.210 \\ -14.983 \\ \hline 297.227 \end{array}$$

2. Subtract 96.043 from 110

$$\begin{array}{r} 110.000 \\ -96.043 \\ \hline 13.957 \end{array}$$

Topic 3.C – Multiplying Decimals

Multiplying Decimals

To multiply decimals treat them the same way as whole numbers. The answer will have a decimal point to its right until you move it. Locate the decimal point in the answer with this rule:

Rule for Multiplying Decimals:

The number of decimal places in the product is **equal to the sum** of the decimal places in each factor. Move the decimal point in the product to the left this number of places.

Factor: a number that is multiplied by another number.

Notice that you do not have to line up the decimal points under each other when multiplying.

Examples

1. Multiply .00123 x 17.4

```
 0.00123   (5 decimal places)
  x 17.4   (1 decimal place)
 --------
     492
    861
   123
 --------
0.021402 (move 6 decimal places left = 5 + 1)
```

2. Multiply 764.34 x 100

```
  764.34   (2 decimal places)
    x100   (0 decimal places)
 --------
 76,434.00 (move 2 decimal places left = 2 + 0)
```

Tip:
$100 = 10^2$, $1000 = 10^3$

To multiply a decimal number by a power of 10, 10^n, n = the number of zeros, move the decimal point n places to the right in the decimal number.

$4.56x10 = 4.56x10^1 = 45.6$ (n = 1, move decimal point 1 place to the right)

$7.894x100 = 7.894x10^2 = 789.4$ (n = 2, move decimal point 2 places to the right)

$114.35x1000 = 114.35x10^3 = 114,350$ (n = 3, move 3 places to the right)

Topic 3.D – Dividing Decimals

Dividing Decimals

Divide decimals the same way as with whole numbers. Remember that every whole number has a decimal to its right. 5 = 5.0, 1213 = 1213.00. We can add zeros to the right of the decimal without changing the value of the number. Recall from section one that division of negative numbers gives a positive quotient, and that division with a negative and positive number gives a negative quotient.

Rule for division of decimal numbers

1. If the divisor is not a whole number, move the decimal point to the right of all the digits and count the number of places involved in the move.
2. Move the decimal point in the dividend this number of places (from step one) to the right.
3. Put the decimal point in the quotient (answer) directly above its location in step two for the dividend.
4. Divide the same way as with whole numbers.

recall: 100 = 10^2 , 1000 = 10^3
To divide a decimal number by a power of 10, 10n, (n=the number of zeros), move the decimal point n places to the **left**.

Examples

1. Divide 13,027 by 100.

   13,027 ÷ 100 = 13,027 ÷ 102 = 130.27 (move decimal in dividend 2 places left)

2. Divide 5.4 by 1000.

   5.4 ÷ 1000 = 5.4 ÷ 103 = .0054 (move decimal in dividend 3 places left)

3. Divide 197.442 by 0.4701.

   4701 $\overline{)1974420}$. (the decimal moved four places right: see rule step 1)

Now proceed as with long division to get 420. This answer can be checked by multiplying quotient times divisor (420 x 0.4701 = 197.4420)

4. Divide 5 by 16

```
        .3125
16  |5.0000        (no need to move decimal point because divisor is already
    -4.8            a whole number)
      .20
     -.16
       .40
      -.32
         80
         80
```

   check: 16 x .3125 = 5

Topic 3.D – Dividing Decimals

5. Divide -.37 by .32

   Begin by noticing that the answer (the quotient will be negative) . 32 goes into .37 once with a remainder of 5/32. This fraction cannot be reduced. Its decimal equivalent is .15625. the final answer is –1.15625.

Practice Questions

(Consult a math textbook for practice with operations on decimals)

Unit 3

Combining Operations and Numbers

Identify the largest or smallest number in decimal and fraction form

Any series of numbers can be ranked in order of increasing size, or in order of decreasing size. Once the series has been organized in one of these ways, the smallest or largest can be identified.

In order to compare number size we need to have each number being compared expressed as fractions with the same denominator or all expressed as decimals.

Unit 3 – Combining Operations and Numbers

Topic 1 – Identify the largest or smallest number in decimal and fraction form

Example

What is the largest number in the following list of decimal numbers?

32, 3.2, 3.032, 33.02, 3.12

These decimal numbers can be ordered by putting the decimal points directly under each other. Zeros can be added for clarity. This will reveal their relative size.

03.120 (three and 12 hundredths)

32.00 (thirty two)

03.200 (three and two tenths)

03.032 (three and thirty two thousandths)

33.020 (thirty three and two hundredths)

We can see that 33.02 is bigger than 32 by a difference of 1.02. 32 is bigger than 3.2 which is bigger than 3.12, which is bigger than 3.032. The smallest number is 3.032, the largest is 33.02.

A pattern emerges when we rank them in order of size with the largest at the top of a list:

33.020

32.000

03.200

03.120

03.032

Ordering a list of decimals requires that we compare the digits for each column starting with the largest column used in the list. In the above list the tens column is the largest that is used. Look to the left for the largest column.

Two numbers have entries in the tens column, 33.020 and 32.00. We can compare them. They each have 3 tens and we cannot tell which is larger from this alone. Move next to the ones column and compare them. 33 has one more unit than 32, therefore 33.020 is larger than 32.00. Now we go to the tenths column to the right of the decimal point and compare the remaining numbers in our list. 3.2 is larger than 3.12 because 3.2 has one more tenth than 3.12. Finally we have 3.032 as the smallest number in the list.

Unit 3 – Combining Operations and Numbers

Topic 1 – Identify the largest or smallest number in decimal and fraction form

Practice Example

Which is larger $3\frac{3}{5}$, or $2\frac{11}{16}$?

First change them into fractions over a common denominator and compare. Choose 80 as the LCM. (80 = 5x16)

$3\frac{3}{5} = \frac{288}{80}$

$2\frac{11}{16} = \frac{215}{80}$

We can see that $3\frac{3}{5}$ is larger by comparing the numerators, 288 is larger than 215.

Another approach is to see that the fraction parts of these mixed numbers are proper, i.e. less than one. This tells us that $3\frac{3}{5}$ is larger than $2\frac{11}{16}$ because $2\frac{11}{16}$ must be smaller than 3, and $3\frac{3}{5}$ is larger than 3. Therefore $3\frac{3}{5}$ is larger than $2\frac{11}{16}$

Practice Example

Which is the smallest fraction $\frac{11}{17}$, or $\frac{12}{16}$?

In order to compare these fractions we need either a common denominator or their decimal equivalents.

a) Common denominator method

Reduce $\frac{12}{16}$ to $\frac{3}{4}$

Now find a common denominator for $\frac{3}{4}$ and $\frac{11}{17}$. Try the product 17 x 4 = 68

$\frac{11}{17} = \frac{44}{68}$

$\frac{3}{4} = \frac{51}{68}$

Now we can see that $\frac{51}{68}$ is greater than 44/68.
Therefore $\frac{3}{4} = \frac{12}{16}$ is larger than $\frac{11}{17}$.

b) Decimal equivalent method

Change $\frac{11}{17}$ into its decimal equivalent = $17\overline{)11}$ = .647

Change 12/16 into its decimal equivalent = $16\overline{)12}$ = .75

.75 is greater than .647 because there are more tenths in .75

Unit 3 – Combining Operations and Numbers

Topic 2 – Converting Decimals to Fractions

Converting Decimals to Fractions

Decimals are aka fractions with a power of ten as the denominator. They are written to the right of a "decimal point". Every decimal number has a fraction number that it is equal to. See Unit 2, Topic 3 – Decimals for more background on decimals.

Examples

.2 = two **tenths**

2.2 = two and two **tenths** = 2 2/10 (a mixed number)

.02 = two **hundredths**

2.02 = two and two **hundredths**

.002 = two **thousandths**

2.002 = two and two **thousandths**

.002 (aka 2/1000) is ten times smaller than .02, (aka 2/100) and .02 is ten times smaller than .2, (aka 2/10). Two is the numerator and 1000, 100, 10 are the denominators of these increasingly smaller decimal numbers.

This means a decimal number is always some number (the numerator) "over" or divided by, either 10, 100 1000 or a larger power of ten. As these denominators grow larger, the decimal number gets smaller. 2/1000 is ten times smaller than 2/100, and 2/100 is ten times smaller than 2/10.

Tip:

The number of zeros minus one equals the number of zeros to put after the decimal point and before the numerator of the decimal number. The reverse is also true.

Example:
2/10,000 = .0002 "two ten thousandths"

4 zeros in the denominator-1 = 3 zeros in the decimal number

Example:
.00005 = 5/100,000 "five one hundred thousandths"

4 zeros in the decimal number + 1 = 5 zeros in the fraction number.

Remember: A decimal point is a "dot" put to the right of a number to create columns that get smaller by a power of ten each time we move another step to the right.

Decimal numbers can be read by saying "point" or "and" for the decimal point. For example, 39.4 could be read as "thirty nine point four", or as "thirty nine and four tenths."

Unit 3 – Combining Operations and Numbers

Topic 2 – Converting Decimals to Fractions

Place values get smaller to the right…

Each column is worth ten times less than its neighbor to the left.

…thousands,	hundreds,	tens,	units	.	tenths,	hundredths,	thousandths…
9x1000	**0**x100	**6**x10	**3**x1	.	**1**x1/10	**2**x1/100	**5**x1/1000

9,063.125 = 9 thousands, 0 hundreds, 6 tens, 3 units, 1 tenth, 2 hundreths, and 5 thousandths

"nine thousand sixty three and one hundred twenty five thousandths"

Examples

1. Convert .0067 to a fraction

Rule for converting decimals to fractions:

1. The denominator of the equivalent fraction will have 1 followed by as many zeros as there are decimal places in the decimal number.
2. The numerator of the fraction is the decimal number without the decimal point.
3. Simplify the answer if it is an improper fraction or a mixed number with an improper fraction part.

.0067 = 67/10,000 (.0067 has 4 decimal places and 10,000 has four zeros)

"sixty seven ten thousandths"

2. Convert 2.34 to a fraction

   2.34 = 234/100 (2.34 has two decimal places)

   "two hundred thirty four hundredths"

3. Convert 154.089 to a fraction

   154.089 = 154,089/1000 (154.089 has three decimal places)

   "one hundred fifty four thousand eighty nine thousandths"

4. Convert .0086 into a fraction and simplify the answer

   .0086 = 86/10,000 because 6 is in the ten thousandths place. Simplify with 2, the largest common divisor,

   86/10,000 = 43/5000

Unit 3 – Combining Operations and Numbers UNIT 3

Topic 3 – Converting Fractions to Decimals

Converting Fractions to Decimals

Recall that every fraction is also a division problem. Convert a fraction to a decimal by dividing the denominator into the numerator. You can see that every fraction has a decimal number it is equal to and vice versa.

Recall that fractions can include mixed numbers and improper fractions. The same rules will apply to all types of fraction numbers.

Examples

Divide bottom into top.

1. $^{1}/_{4} = 1 \div 4 =$ **.25** $= {}^{25}/_{100}$
2. $^{8}/_{5} = 5\overline{)8} =$ **1.60** $= {}^{160}/_{100}$
3. $35^{7}/_{8} = 35 + {}^{7}/_{8} = 35 + 8\overline{)7} = 35 + .875 =$ **35.875** = 35,875/1000
4. $^{5}/_{16} = 16\overline{)5} =$ **.3125** = 3,125/10,000
5. $^{2}/_{3} = 3\overline{)2.0} =$ **.66**… (the division could continue indefinitely, we round off to a desired place after the decimal and use "…" to show that division could continue for greater accuracy)
6. $^{3}/_{11} = 11\overline{)3.0} =$ **.272727…** (this division produces a repeating decimal. We can use a bar over the repeating number (27) to indicate the pattern, or use "…")
7. Sometimes a degree of accuracy is requested in the problem:

   "Write 7/11 as a decimal number to the nearest hundredth."

   $11\overline{)7.000} = 0.636 =$ **0.64** to the nearest hundredth. (We round up in the hundredths column because the value in the thousandths is greater than five, or more than half way to the next hundredth. If the value in a column = 5, we still round up unless otherwise instructed)

Topic 4 – The Order of Operations

The Order of Operations

(x , ÷) → (+, –)

STEP 1 STEP 2

Many problems have more than one step to the solution. **Always multiply and divide where indicated before adding and subtracting. Complete operations inside brackets (parentheses) first. Work from left to right. This order will give the right answer to problems that combine operations**.

An expression is a string of operations on numbers that solves a problem. On the exam you will be given questions that ask you to write expressions that solve problems. Decide which of the four operations are needed- and in what order- by breaking the problem into steps and writing an expression to solve each step.

Examples

What you need to know:

1. Do the operations inside brackets first.
2. Work from left to right.
3. Do multiplication and division before addition and subtraction.

1. Find (4 x 6) – (13 + 5)

   First work inside each bracket. 4 x 6 = 24 and 13 + 5 = 18, so we simplify the expression to:

   24 – 18 = 6 simplify means that we substitute the result of doing the operations inside each bracket for the original expression inside the bracket. This is an example of substituting one thing for another because they are equal. When there is only one value inside a bracket, the bracket is removed.

2. 98 – (6 x12 – 4) + (32 ÷ 4 + 3) = ?

   Work **inside the brackets first**, do **multiplication and division before addition and subtraction**, and go **left to right**.

   98 – (72 – 4) + (8 + 3) = ?

   98 – 68 + 11 = 41 (we can subtract 68 first from 98 or from the sum of 98 and 11)

Practice Questions

Question 1

Before taking the plane to work for two weeks, Robert buys himself 3 books to read for $5.75 each, a magazine for $3.50 and $15.00 of snacks for each week that he will be at work. What expression will give the total that he spent?

a) ($5.75 ÷ 3 + $15.00 x 2) + $3.50

b) $3.50 + $15.00 + ($5.75 x 3)

c) 3 x ($5.75 + 2 x $15.00)

d) (3 x $5.75) + (2 x $15.00) + $3.50

Answer: d

Explanation
This is a multi step problem. You can proceed by process of elimination or by writing an expression that tells how much Robert spent. **When you develop an expression, the order you choose may be equivalent to one of the choices given, but not in the identical order**. Remember that numbers can be switched when they are added or multiplied and still give the same answer.

With this in mind, first notice that we are looking for a total. This means that items will be added. Robert spent $3.50 on a magazine that will be part of the total he spent. Second, notice that some items are multiplied. A total of 3 books are bought, and each one costs $5.75. The total for books will be 3 x $5.75. Notice that in this question we do not need to calculate the total, only to build an expression for finding it. A total for snacks is needed over a two-week period, and we are told that Robert spent $15.00 for each week he is gone. 2 x $15.00 will give the snack total. With these three subtotals we can add them up. There are three items to combine:

Books = 3 x $5.75

Magazine = $3.50 (or 1 x $3.50)

Snacks = 2 x $15.00

The expression we want will bracket the items that have multiplication and then add the bracketed amounts.

(3 x $5.75) + $3.50 + (2 x $15.00)

This expression will work; now see if it, or an equivalent expression, is among the choices. Choice d, (3 x $5.75) + (2 x $15.00) + $3.50 is equivalent and is the correct answer.

Practice Questions

If you wanted to proceed by process of elimination, you could rule out the wrong answers this way:

Choice a is wrong because $5.75 ÷ 3 in the first bracket says to divide the cost of one book by three, when we need to multiply.

Choice b is wrong because $15.00 is not multiplied by two to find the snack total over a two-week period.

Choice c is wrong because the magazine item is missing and the number of books, 3, is not inside a bracket with $5.75.

Topic 5 – Using Percents

Using Percents

The next two topics cover what you need to know about the definition of percents and their relationship to fractions and decimals. Practical problems involving percentages, rates, unit rates, discount, and mark-up can be found in Unit 4, Topic 6 – Problems Involving Percentage, Topic 7 – Problems Involving and Topic 8 – Probles. These problems rely on the theory covered next.

Topic 5.A – Changing Fractions into Equivalent Percent Numbers

A percent is shorthand for a fraction with a denominator of 100. **The symbol "%" means "over a hundred".** 24% means 24/100, or .24. Percentage numbers select something that has been divided into one hundred equal parts. Review the sections 1b and 2b on fractions and apply everything to percents as well. 20% means something has been divided into 100 pieces, and we have selected twenty of them.

Recall that the denominator of any fraction describes how many parts something has been divided into, and the numerator selects or identifies how many of these parts we are considering. A percent is a number that tells how many parts out of 100 we are concerned with. A test score of 80% means that 80 points out of a possible 100 points were earned. A 5% rate of return on an investment means that $5.00 was earned for every $100 invested. Because percents are numbers over a power of ten, they are also decimal numbers and everything you learned in earlier unit about decimals in Unit 2, Topic 3 – Decimals as well as Unit 3, Topic 2 – Converting Decimals to Fractions and Topic 3 – Converting Fractions to Decimals, will also apply to percents.

> 25% aka 25/100 aka .25, aka $^1/_4$ aka "one fourth".

Examples

1. 50% = 50/100 = 5/10 = $^1/_2$

   Fifty percent means the same as one half of something that is divided into 100 parts. We can reduce (i.e. simplify) a percent to lowest terms the same way as any fraction number.

2. 30% = 30/100 = 3/10 = .30

3. 75% = 75/100 = .75 = $^3/_4$

   These are all expressions for the same amount, they are referred to as "equivalent expressions".

Topic 5.A – Changing Fractions into Equivalent Percent Numbers

Changing Fractions into Equivalent Percent Numbers

Example:
Change $\frac{3}{5}$ into a percent.

Step One:
Divide the denominator into the numerator to get a decimal number
3 ÷ 5 = .60.

Step Two:
Move the decimal point two places to the right by multiplying the decimal by 100. .60 becomes 60. You can also read this number as "sixty hundredths" which by definition is sixty percent.

Step Three:
Put the % sign after the number. 60% = .60 = $\frac{3}{5}$.

Notice that percentages can be added and subtracted because they are like fractions sharing 100 as their denominator.

30% + 60% = 90% is the same as 30/100 + 60/100 = 90/100 = 0.90 = 90%.

Remember that 100% = 1.00 = 100/100 and 1% = .01 = 1/100

Percentages can include improper fractions, for example, 300% = 3.00 = 300/100.

Rounding to hundredths gives approximate percentage numbers

1. Start at the rightmost column
2. If a number in a column is 5, 6, 7, 8, or 9 round it up and add one to the next column on the left, otherwise round it down and treat it as a zero.

Examples

a) .00894 rounds to .00890 then to .00900, then to .01

.00894 is approximately = 1/100 = 1% or exactly .894%

b) .1256 rounds to .1260, then to .1300

.1256 is approximately = 13/100 = 13% or exactly 12.60%

Topic 5.A – Changing Fractions into Equivalent Percent Numbers

Examples

1. Write 2/7 as a percent

   $7\overline{)2.000}$ = .2857… (use long division for decimals discussed earlier)

   This is an irrational number, and we can round to the nearest hundredth

   .2857 = .286 = .29 (rounded up to hundredths column)

   .29 = 29/100 = 29% (we could also have rounded to any other desired place, for example if we rounded to the nearest ten thousandth we would get 28.57%)[5].

2. Change 4/5 into a percent

   $5\overline{)4.000}$ = 0.80 = 80/100 = 80%

3. Write 7/8 as a percent

   $8\overline{)7.000}$ = 0.875 = 87.5%

4. Change $3\frac{3}{4}$ into a percent

   This is a mixed number. Recall that a mixed number is the sum of a whole number part and a fraction part. First change the whole number part into a percent number 3 = 300/100 = 300%. (multiply 3 by 100 and move two place to the right)

   Now change the fraction part, $\frac{3}{4}$, into a decimal and add it to the whole number and express as a percent.

   $\frac{3}{4} = 4\overline{)3.000}$ = 0.750 = 75/100 = 75%

   300% + 75% = 375% (this is the same as adding like fractions, 300 and 75 are both over a common denominator of 100).

5 See Unit 1, Topic 4 – Rational Numers and Signed Numbers for a review of irrational numbers, and Unit 2, Topic 3.D – Dividing Decimals for division with decimals.

Practice Questions

Question 1

30 apprentices took their exam and $^1/_3$ got failing grades.
What percent of the class got passing grades?

a) 33%

b) 69%

c) 66%

d) 50%

Answer: c

Explanation

This is a multi step problem. The question tells us that $^1/_3$ failed. Because we know that this is a class of thirty, we can find $^1/_3$ of 30 by multiplying. $^1/_3$ x 30 = 10. 10 people make up $^1/_3$ of the class, and that means 20 people, or the other $^2/_3$ passed. We are asked for the percentage equivalent of $^2/_3$. You may know that $^2/_3$ = 66%, or you may have to divide 3 into 2 to find the answer in hundredths. You will find a repeating decimal, .66…. You might want to round off to .67, which is a good approximation to the nearest hundredth. But the closest choice is c, and there is an algebraic demonstration that .66 is exactly equal to the rational number $^2/_3$. See the optional topic that follows for this demonstration.

Optional Topic: Rational Repeating Decimals and Fractions

Algebra helps us build an equation to prove that .66… = $^2/_3$

Let F = the repeating decimal you want to find an equivalent fraction for.

Here let F = .66…

This means that 10F = 6.66… (multiply both sides by 10 and they remain equal to each other)

Now imagine subtracting F from 10F to find what 9F equals. To do this you would have to write the infinitely repeating decimals under each other

6.6666…
-0 .66666…

After subtracting you will have 9F = 6, and solving for F by dividing both sides by 9, F = $^6/_9$ which reduces to $^2/_3$ exactly.

Topic 5.B – Changing Decimals into Equivalent Percent Numbers

Changing Decimals into Equivalent Percent Numbers

> Any numerator over a denominator of 100 can be expressed as the numerator followed by %. Remember that percent numbers can be added because they are like fractions with 100 as their common denominator.

To change a decimal number into a percent, multiply by 100. multiplying by 100 moves the decimal point two places to the right.

Example

1. Change .037, .37, and 3.37 into percents.

   Move the decimal two places to the right and add the percent sign

   .037 = 3.7%, .37 = 37%, and 3.37 = 337%

   A decimal number can be rounded to the nearest hundredth or another place value to give the percent number it is closest to.

2. Express .56 as a percent

   .56 = 56/100 = 56% (by definition, 56 hundredths = 56 percent)

3. Change .0087 to its closest percent value. We will round it off to the nearest percent

   .0087 = .87%, which rounds off to 1/100 = 1%

4. Change 2.34 to a percent

   a) This is a mixed number. First change the whole number part into its percent equivalent.

   2 = 200/100 = 200%

   b) Now change the decimal part into a fraction over 100 and express as a percent.

   .34 = 34/100 = 34%

   c) Add the whole number part to the fraction part using their common denominator of 100.

   200/100 + 34/100 = 234/100 = 234%

Topic 5.B – Changing Decimals into Equivalent Percent Numbers

5. Write .3478 as a percent

   Move the decimal two places right

   34.78%

   also note that

   .3478 = .348 = .35 (rounding up to the nearest hundredth)

   and .35 = 35/100 = 35%

To change a percent into a decimal divide by 100 and move the decimal point two places to the left.

6. Change 45% into a decimal

   45% = 45/100 by definition

   45/100 = .45

7. Change 3.5% to a decimal

   3.5% = 3.5/100 = 35/1000 = .035

Practice Questions

Question 1

Louis is a mine manager who lives in Gameti. He does some work from his office in Gameti and receives a fax alerting him to the fact that this month's mined ore has yielded .003 tons of concentrate for each ton of ore that was mined. What percentage of the ore is concentrate?

a) 30%

b) 3%

c) .3%

d) 3.3%

Answer: c

Explanation
We need to change the decimal number to its equivalent percent number. This will tell how many parts out of a hundred (in each ton) are concentrate. To change a decimal to a percent move the decimal point two places right and add the percent sign. .003 = .3% or "three tenths of one percent".

Question 2

What is 2.5% expressed as a decimal number?

a) .025

b) .25

c) 2.5

d) 25

Answer: a

Explanation
By definition, a percent is a number over 100. 2.5% = 2.5/100. Dividing by 100 means moving the decimal point two places to the left to give .025

Topic 5.C – Combining Percentages

Combining Percentages

Some situations require us to compare percentages using one of the four operations of addition, subtraction, multiplication, or division. Use your ability to convert percentages into decimals and fractions to find the answers. The problem will determine the operations you need to perform.

Examples

1. 30% of the apprentices in the Northwest Territories indicated on a survey that they preferred to study independently for the trades entrance exam. Of these, half said that their independent study was supported by some tutoring. What percentage of apprentices indicated that they preferred independent study and also had some tutoring support?

   The answer is 15% because we are told that one half of the 30% preferring independent study also had tutoring, and $\frac{1}{2}$ of 30% is 15%. You can also see this result by multiplying $\frac{1}{2}$ x 30/100 which equals 30/200, and this reduces to 3/20 = .15, and .15 = 15%. Recall that "of means times" to see that $\frac{1}{2}$ of 30% means $\frac{1}{2}$ times 30%.

2. A hydraulic pump ordered from a U.S. company costs $560 in Canadian funds. 7% GST and 12% import duty based on this cost are included in the invoice. What is the total invoice?

   We need to calculate 7% of $560 = $39.20, and 12% of $560 = $67.20 to find the amounts that will be added to the cost of the pump. The total invoice will be $560 + $39.20 + 67.20 = $666.40. This is a two step problem. First convert the percentages to decimals and multiply by the cost. Second, add the amounts to find the total invoice.

Unit 3 – Combining Operations and Numbers

Practice Questions

Question 1

Only 20% of all trade specific education takes place in high school according to a recent study. However, in one European country three times as much is covered during high school. What percentage of trades education is covered in this country during high school?

a) 23%

b) 90%

c) 60%

d) 66%

Answer: c

Explanation
Three times 20% equals 60%. These numbers allow us to compare rates of coverage in two kinds of high school.

Unit 3 – Combining Operations and Numbers

Topic 6 – Bases, Exponents and Square Roots

Bases, Exponents and Square Roots

Repeated multiplication of the same number by itself is called raising the number to a power. 2^3 means 2 x 2 x 2. In this example, 2 is the base and 3 is the exponent. Any number can be raised to any power.

Bases and Exponents

100^5 = 100 x 100 x 100 x 100 x 100 (100 is the base, 5 is the exponent)

$(.25)^3$ = .25 x .25 x .25 (.25 is the base, 3 is the exponent)

$(^1/_6)^4 = {^1/_6} \times {^1/_6} \times {^1/_6} \times {^1/_6}$ ($^1/_6$ is the base, 4 is the exponent)

A number raised to a power (an exponent) is called a base. Exponents are shorthand for repeated multiplication using the same base. The exponent tells how many times to multiply the base by itself.

Example: Calculating the square of a number

Multiply a number by itself to get it's square. **The square of a number is that number times itself.** 4 squared, written 4^2, is 4 x 4 = 16. The square of a number is the product of a number times itself one time. Integers, fractions, and decimals can be squared.

The square of a number is indicated by a small 2 written to the upper right of the number. This small number is also called an **exponent**, aka the **power** to which the number is being raised. The power can be greater than 2, for example if 3 is used, the number is being "cubed" or multiplied by itself three times. If the exponent is 4, the number will be multiplied by itself four times.

The terms "squared" and "cubed" come from geometry, where the area of a square is one side times itself, and the volume of a cube is one side (edge) times itself three times.

Powers of 0 and 1 are defined this way:

Any number with an exponent of 0 = 1 **$10^0 = 1$**

Any number with an exponent of 1 = itself **$10^1 = 10$**

Topic 6 – Bases, Exponents and Square Roots

Examples:

$3^2 = 3 \times 3 = 9$

$4^2 = 4 \times 4 = 16$

$5^2 = 5 \times 5 = 25$

$(1/4)^2 = 1/4 \times 1/4 = 1/16$

$(2/3)^2 = 2/3 \times 2/3 = 4/9$

$(^3/_4)^2 = 9/16$

$.5^2 = .5 \times .5 = .25$

$2.45^2 = 2.45 \times 2.45 = 6.0025$

$3\ 1/2^2 = 3 \times 3 = 9 + 1/2 \times 1/2 = 9 + {}^1/_4 = 9\ {}^1/_4$

Area of a square = s^2
where s = the length of a side

Volume of a cube = s^3
(see Math – Module 4 – Measurement)

Example: Calculating the Cube of a Number

The cube of a number is the result of multiplying the number by itself three times. The exponent 3 is used to express the cube of a number. A letter can be used to state the general case, or equation, for a cubed number.

Cubes

$a^3 = a \times a \times a$ "a cubed equals a times a times a, or a to the third power." The volume of a cube is found by multiplying the length of a side by itself three times.

$3^3 = 3 \times 3 \times 3 = 27$

Integers, fractions, and decimals can be cubed. Brackets can be used to enclose the number being cubed or raised to any power.

$(16)^3 = 16 \times 16 \times 16 = 4096$

Topic 6 – Bases, Exponents and Square Roots

Examples

1. Find the cube of 32.

   $32^3 = 32 \times 32 \times 32 = 32{,}768$

2. Find the cube of $^1/_4$.

   $(^1/_4)^3 = {^1/_4} \times {^1/_4} \times {^1/_4} = {^1/_{16}} \times {^1/_4} = {^1/_{64}}$

3. Calculate the cube of $3\ ^3/_4$

   Express the mixed number as an improper fraction and multiply:

   $(3\ ^3/_4)^3 = (15/4)^3 = \dfrac{15 \times 15 \times 15}{4 \times 4 \times 4} = \dfrac{3375}{64} = 52.73$ (rounded to nearest hundredth)

4. Find the cube of .7

   $(.7)^3 = .7 \times .7 \times .7 = .49 \times .7 = .343$

   or set $.7 = 7/10$, then $(.7)^3 = (7/10)^3 = \dfrac{7 \times 7 \times 7}{10 \times 10 \times 10} = \dfrac{343}{1000} = .343$

The Square Root of a Number

Finding the square root undoes what finding the square did. The cube root undoes what finding the cube root did. The square root of 16 is 4 because 4 x 4 = 16. -4 is also a square root of 16 because -4 x -4 = 16. The positive square root is called **the principal square root**. The square root of 9 is 3 because 3 x 3 = 9. The cube root of 27 is 3 because 3 x 3 x 3 = 27.

The square root of a given number is the number that can be multiplied by itself to produce this number. Here we will only discuss the principal (i.e. positive) square roots of positive whole numbers. The symbol for a square root is $\sqrt{\ }$. $3^2 = 9$ and $\sqrt{9} = 3$. A square root can also be written with an exponent of $^1/_2$.

$9^{1/2} = 3$.

Topic 6 – Bases, Exponents and Square Roots

Estimating Square Roots

To find the approximate square root of a number, find the nearest whole numbers that give products that are as close as possible above and below the number whose square root you seek.

Example
Estimate the square root of 6200.

First choose a whole number that can be squared to be as close to, but less than 62. That number is 7. 7 x 7 = 49.

Second, try the next largest whole number, 8. 8x8 = 64. We can conclude that the square root of 6200 is between 70 and 80. We can also see that the square root will be closer to 8 than to 7, because 6200 is closer to 6400 than to 4900. Next try the mid point, 75 x 75 = 5625. Or, you might skip this step because you already can see that 6200 is closer to 80 squared than to 70 squared. Continue homing in on the square root by trying the mid point of the boundaries you have discovered. We know 75 is too small and 80 is too big. Try 77 squared = 5929. Still too small, but closer to 6200. Try 78 squared = 6084. Try 79 squared = 6241. 6241 is closer to 6200 than 6084. 79 is the nearest whole number to the square root of 6200.

Practice Questions

Question 1

An environmental assessment of a northern lake shows that algae will increase at a rate of n^2 for the first hour. If n = the number of algae cells at the beginning of the hour = 50, what will be the number of algae cells after one hour?

a) 25,000

b) 250,000

c) 2500

d) 500

Answer: c

Explanation
To find the answer calculate 50 x 50 = 2500. n^2 is an expression that means the square of n, and we know that n = 50 in this problem.

Question 2

Find the nearest whole number to $\sqrt{8313}$

a) 100

b) 89

c) 91

d) 79

Answer: c

Explanation
Use the method of successive approximation described earlier. 90 squared is 8100 and 91 squared is 8281. 92 squared is 8464. 8313 is closer to 8281 than to 8464. The square root we seek is closer to 91.

Topic 6 – Bases, Exponents and Square Roots

Negative Exponents

A negative exponent, for example 2^{-3}, means one over the base,2, raised to the absolute value of the exponent, 3. In other words, the reciprocal of the base is raised to the positive power of the exponent.

Examples:

$2^{-3} = 1/2^3 = \frac{1}{8}$

$5^{-4} = 1/5^4 = 1/625$

Practice Questions

Question 1

Find $5^{-1/2}$

a) 2.5

b) 5/2

c) 12.5

d) $1/\sqrt{5}$

Answer: d

Explanation
The base is 5 and the exponent is negative ½. The fact that the exponent is negative tells us that the reciprocal of the base is being raised to the power of ½. The exponent of ½ tells us to find the square root of the base. Putting these facts together gives the answer of $1/\sqrt{5}$.

Topic 6.A – Simplifying Expressions with Exponents

Simplifying Expressions with Exponents

Some Background

Expressions contain numbers and variables. For example, 2h is an expression. It tells us to multiply a number by two. The value we choose for h can vary, or change, but any choice we make will be multiplied by two in order to satisfy the expression 2h. **A number multiplied by a variable is called a coefficient.** 2 is the coefficient of h in the expression 2h.

For example, We could set h = a person's height at age 4. Different children will have different heights at age four. The variable h will have a different value for each child. Evidence might be found to show that the expression 2h could equal a person's height at age 20. 2h = H is an equation stating this relationship if we set H = height at age 20. According to this equation, A person measuring 3 feet at age 4 would measure 6 feet at age twenty.

For Greater Clarity:

When an expression is equal to something else we have an equation. People create equations to solve problems and describe relationships that are useful. For example, distance travelled equals miles per hour (the speed) multiplied by time. This relationship can be described by the equation d = vt. Math – Module 3 – Variables and Equations explores this topic further.

An equation is a statement that one expression is equal to another. Only now, instead of using numbers on each side of the equal sign, we use letters to stand for any numbers we might choose. These letters are called **variables** because they can vary (change) according to the values we assign to them. For example, the expression k^2 becomes 9 when we set k = 3. The same expression becomes 25 when we set k = 5.

Expressions are Useful in Work and Business.

For example, the post office may have an expression of 2.5k for the charge of sending a parcel outside Canada, where k is the base rate for each ounce being mailed. 2.5 is the coefficient of k in this expression. When the base rate changes, the expression stays the same and allows the new cost to be easily calculated. Suppose that in 1999 k = $00.30, so 2.5K = $00.75. In 2002 k increased to 45 cents, now k = .45, so 2.5k became $1.125.

We could also change our expression from 2.5k to another coefficient for k. For example, 3.2k. Expressions allow us to calculate the impact of either kind of change.

Topic 6.A – Simplifying Expressions with Exponents

You Need to Know:

1. Two numbers written next to each other in brackets, or two letters written next to each other, are being multiplied by each other. We don't have to write the "x" symbol to show multiplication is involved. Ab means A x b, 35K means 35 x K and so on.
2. Equations are important, because they allow us to say in a general way what makes two expressions equal to each other. In an equation, a letter is called a "variable" and it stands for all of the numbers that make the equation produce true statements when they are used in place of the variable.[6]

Five Rules that Simplify Exponents

The following rules tell how to simplify numerical expressions with exponents in them.

When we simplify we do not need to calculate the answer. Simplifying puts an expression into a form with fewer operations to complete.

Let a = any whole number, and r = any whole number used as an exponent. The definitions used in earlier sections for exponents apply when a is not equal to zero:

Use the following rules to simplify expressions with exponents.

Definitions

1. $a^{-r} = 1/a^r = (1/a)^r$

   Example

   $3^{-3} = \frac{1}{3^3} = (\frac{1}{3})^3 = \frac{1}{27}$
2. $a^1 = a$ example: $3^1 = 3$
3. $a^0 = 1$ example: $3^0 = 1$

6 See Math – Module 3 – Variables and Equations for more on this topic

Topic 6.A – Simplifying Expressions with Exponents

Use the following rules to simplify expressions with exponents.

Rule One

Add the exponents when the base numbers being multiplied are the same. The answer is the base number with a new exponent equal to the sum of the two exponents. This is true for any number of terms being multiplied.

$a^r \times a^s = a^{r+s}$

Examples

$6^2 \times 6^3 = 6^5$

$2^3 \times 2^3 \times 2^4 \times 2^0 = 2^{10}$

Rule Two

A number raised to a power, and then raised again, is equal to the number raised to the product of the powers.

$(a^r)^s = a^{rs}$

Example

$(4^2)^3 = 4^6$ $(6 = 2 \times 3)$

Rule Three

The product of 2 base numbers raised to a power equals the product of the base numbers each raised to that power.

$(ab)^r = a^r \times b^r$

Examples

$(3 \times 6)^2 = 3^2 \times 6^2$

$(3 \times 3)^4 = 3^4 \times 3^4$

Topic 6.A – Simplifying Expressions with Exponents

Rule Four

The quotient of the same base numbers with exponents, is the base number raised to the power of the difference between the exponents.

$\frac{a^r}{a^s} = a^{r-s}$

Examples

$\frac{3^5}{3^2} = 3^{5-2} = 3^3 = 27$

$\frac{4^2}{4^3} = 4^{2-3} = 4^{-1} = \frac{1}{4}$ (see definition one)

$\frac{5^3}{5^5} = 5^{3-5} = 5^{-2} = 1/25$

Rule Five

A fraction raised to a power is equal to the numerator raised to that power divided by the denominator raised to that power.

$(a/b)^r = a^r/b^r$

Example

$(\frac{3}{4})^3 = 3^3 / 4^3$

Examples of Simplifying Expressions with Exponents

1. $(3^3)^4 = 3^{12}$ (rule two)
2. $(4/5)^5 = \frac{4^5}{5^5}$ (rule five)
3. $2^5 \times 2^3 \times 2^4 = 2^{12}$ (rule one)
4. $(6^2)^3 \times (6^4)^2 = 6^6 \times 6^8 = 6^{14}$ (rule two, rule one)
5. $(2 \times 4)^3 = 2^3 \times 4^3$ (rule three)
6. $\frac{(5 \times 2^3)^2}{2^4} = \frac{5^2 \times 2^6}{2^4}$ (rules 2 and 3) $= 25 \times 2^{6-4} =$ (rule four) $= 25 \times 2^2 = 100$

These examples could contain letters in place of numbers to give algebraic expressions based on the same rules. For example if we set 3 = a, then example seven would become $(a/2)^3 = a^3/2^3 = a^3/8$.

7. $(3/2)^3 = 3^3/2^3$ (rule five)

Simplifying Square Roots

The square root of many numbers can be simplified by using the fact that the square root of any product of whole numbers is equal to the product of the square roots of each of them.

Use this equation to simplify expressions with square roots

$\sqrt{ab} = \sqrt{a}\sqrt{b}$ **(the general case)**

Example:

Find the square root of 64

You may see that the answer is 8, but notice how a product of two perfect squares can also give us the answer we seek.

$\sqrt{64} = \sqrt{4 \text{x} 16} = \sqrt{4} \text{ x } \sqrt{16}$ **(an example of the general case)**

$\sqrt{4} = 2$ $(2 \text{ x } 2 = 4)$

$\sqrt{16} = 4$ $(4 \text{ x } 4 = 16)$

$\sqrt{64} = \sqrt{4}\sqrt{16} = (2 \text{ x } 4 = 8)$

The square root of 64 = 8

This fact is expressed by an equation: $\sqrt{ab} = \sqrt{a}\sqrt{b}$. In the next section you will see how the rules for working with exponents allow us to simplify equations with expressions that use scientific notation.

Examples

1. Find the square root of 256.

   Talk yourself through it this way:

   Step One: We need to find a number that will give us 256 when it is multiplied by itself. Let's call the number we are looking for "a". We know it exists, but we can't give it its name until we find it.

   However, we do know this much: $a^2 = 256$ and $\sqrt{256} = a$.

   Step Two: Try using trial and error. We know that 8 x 8 = 64, 10 x 10 = 100 etc. How close can we get to 256? Answer: 16. It turns out that 16 x 16 is exactly 256. By letting a = 16 we find the answer, and solve the equation for a.

Topic 6.A – Simplifying Expressions with Exponents

2. Find the square root of 512.

   Step One: Estimate (i.e. get close to) the answer by dividing with numbers that are perfect squares. This will allow you to rewrite the problem as a product of square roots. Try using 4, a number whose square root we know is 2.

   $512/4 = 128$

   Step Two: Rewrite the square root (also called a radical) as a product of square roots using the rule given above.

   $\sqrt{512} = \sqrt{4}\sqrt{128} = 2\sqrt{128}$

   This tells us that the product of 2 and $\sqrt{128} = 512$

   Step Three: Repeat this process to simplify further. Try using 16 as a factor of 128. You will see that 8 x 16 = 128. Now rewrite the square root reaching in step two again:

   $2\sqrt{128} = 2\sqrt{16}\sqrt{8} = 2 \times 4\sqrt{8} = 8\sqrt{8}$

   Once more we can use 4 to factor 8 into 4x2 to get:

   $8\sqrt{8} = 8\sqrt{4}\sqrt{2} = 8 \times 2\sqrt{2} = 16\sqrt{2}$. We have found that the square root of 512 is $16\sqrt{2}$.

   Step Four: If no factors allow us to express 2 as the product of a perfect square and another number, we are done. The value of $\sqrt{2}$ will be somewhere between 1.4 and 1.5, but we don't need to find it in this section because we are dealing only with the squares of whole numbers. $\sqrt{2}$ is an irrational number, meaning that it cannot be expressed as a fraction with whole numbers in the numerator and denominator.

Practice Questions

Question 1

What is the square root of 1100?

a) 32

b) 34

c) $10\sqrt{11}$

d) 110

Answer: c

Explanation
This question could be estimated as 33. (33.166247.. is the calculated answer.) However choice c uses the product rule for square roots to simplify the problem $\sqrt{1100} = \sqrt{100}\sqrt{11}$, and $\sqrt{100} = 10$.

Question 2

What is the base in the expression 5^n ?

a) 5 x n

b) n x 5

c) 5

d) n

Answer: c

Explanation
The base is the number that is being multiplied by itself the number of times indicated by the exponent. In this example the exponent is n. n is a variable that can be set to any value we chose for the expression. If we are doing carpentry and want to know the area of a floor that is 5 feet on a side, we will set n = 2 and find 52 = 25 square feet. If we want to know the volume of an ice cube that is 2 inches on a side, we will set n = 3 and find 23 = 8 cubic inches. Area and volume are discussed further in Module 4.

Practice Questions

Question 3

Evaluate this expression $3^3 + 4^2 - 1$

a) 24

b) 35

c) 31

d) 42

Answer: d

Explanation
Complete the multiplications described by the exponents first.
33 = 3 x 3 x 3 = 27, and 42 = 16. Now do the addition and subtraction from left to right. 27 + 16 – 1 = 42

Question 4

Simplify $(4^3)^3$

a) 4^6

b) 4^9

c) 4

d) 64

Answer: b

Explanation
(use rule two)

Question 5

Simplify $(2/3)^4$

a) $\frac{2^4}{3^4}$

b) $(2/3)^4$

c) $(3/2)^4$

d) (2/3) x 4

Answer: a

Explanation
(use rule five)

Topic 6.B – Using Scientific Notation

Using Scientific Notation

Recall that every whole number can be expressed as a number times a power of ten. For example, 16 = 1 x 10 plus 6 x 1, or 1.6 x 10, and 500 = 5 x 100 plus 0 times ten plus 0 times one. Secondly, recall that every column in a whole number is a power of ten. We can use exponents to describe the powers of ten in each column. For example 100 = 10^2, 1000 = 10^3 etc.

650,000,000 (650 million) becomes 6.5 x 10^8 in scientific notation. **Scientific notation uses exponents as a shorthand way to write very small or very large numbers using powers of ten.** Scientists and technicians work with these kinds of numbers. Scientific notation keeps track of powers of ten so that we don't have to write out all of the zeros in very large or in very small numbers.

Any decimal number can be written in scientific notation using a power of ten. For example the number 120 becomes 1.2 x10^2 in scientific notation. This is read, "1.2 times ten squared." 45 x 10^8 is read, "forty five times ten to the eighth power", and this is equal to 4.5 x 10^9 in scientific notation. 80 million can be written as 8.0 x 10^6, instead of as 8,000,000.

A number in scientific notation is always a decimal number expressed as a digit from 1 to 9 followed by the decimal point, then any remaining digits, and then the power of ten that will expand the number correctly if desired. For example, 54,034 = 5.4034 x 10^4, .05034 = 5.034 x 10^{-2}.

The power to which ten is raised is also called the exponent. Review Unit 1, Topic 3 – Decimals and make sure you understand columns, place value, and the powers of ten that represent each place value.

Review

100 = 10^2, 1000 = 10^3

To multiply a decimal number by a power of 10, 10^n, n = the number of zeros, move the decimal point n places **to the right** in the answer.

4.56 x 10 = 4.56 x 10^1 = 45.6 (1 decimal place to the right)

7.894 x 100 = 7.894 x 10^2 = 789.4 (2 decimal places to the right)

114.35 x 1000 = 114.35 x 10^3 = 114,350 (3 decimal places to the right)

To divide a decimal number by a power of 10, 10^n, (n = the number of zeros), move the decimal point n places to the **left**.

Topic 6.B – Using Scientific Notation

Converting Large Numbers into Scientific Notation

Recall that every whole number can be written with a decimal to its right.
Example: 6 = 6. = 6.0 = 6.00.., 78 = 78.00…, 134 = 134.000…

Step One:
Move the decimal point in the number being converted to the left until you reach the last digit to the left. This will be a whole number with the decimal point to its right. Count the number of moves required to reach this place.

Step Two:
Multiply the number created in step one by 10 raised to a power that is equal to the number of decimal moves that were made.

Examples

1. Express 93,000,000 miles, the distance from the earth to the sun, in scientific notation.

   Step One: 9.3000000 (We moved 7 decimal places to the left)

   Step Two: 9.3×10^7 (Scientific notation uses the power of ten that equals the places we moved the decimal, and saves us from writing all of the zeros.)

   Check: expand 9.3×10^7 using the rule for multiplying decimals.

   $9.3 \times 10^7 = 93{,}000{,}000$ = 93 million

2. Put 59,000.25 into scientific notation

   Step One: 5.900025 (we moved 4 decimal places to the left)

   Step Two: 5.900025×10^4

   Check: expand 5.900025×10^4

   $5.900025 \times 10^4 = 59{,}000.25$ (move four places right to multiply by 10^4)

Topic 6.B – Using Scientific Notation

Converting Large Numbers in Scientific Notation to Decimal Numbers

These are multiplication problems using the rule for multiplying by powers of ten. Recall that we move the decimal point the same number of places to the right as the exponent (power of ten) in the scientific notation.

Examples

1. Convert (expand) 56.2×10^5 to a decimal number

   Move five places to the right

   5,620,000

2. Convert 5.62×10^6 to a decimal number

   Move six places to the right

   5,620,000

3, Convert 78.003×10^4 to a decimal number

   Move four places to the right

   780,030

Topic 6.B – Using Scientific Notation

Converting Small Numbers into Scientific Notation

Very small numbers are also decimal numbers and all of the rules for multiplying and dividing decimals will apply. We multiply by negative powers of ten to express decimal numbers that are less than one.

> **Negative powers of ten**
>
> **In general: $10^{-n} = 1/10^n$**
>
> $10^{-1} = 1/10$
>
> $10^{-2} = 1/100$
>
> $10^{-3} = 1/1000$

Examples

.2 = two **tenths**

.2 = 2.0×10^{-1}

.02 = two **hundredths**

2.0×10^{-2}

.002 = two **thousandths**

2.0×10^{-3}

2.002 = two and two **thousandths**

$2 + 2 \times 10^{-3}$

or, 2002×10^{-3}

.005007 = 5.007×10^{-3} = 5007.0×10^{-6} = five thousand and seven ten millionths.

> To expand small numbers written in scientific notation move the decimal place to the left the number of places that equals the negative power of 10.

Examples

56×10^{-4} = .0056 (move the decimal point in 56.0 four place left)

4.67×10^{-6} = .00000467 (move the decimal point in 4.67 six places left).

Practice Questions

Question 1

Which number is equal to 3.45×10^5?

a) 345

b) 345,000

c) 34.500

d) 34,500

Answer: b

Explanation
The exponent is positive five, so move the decimal point five places to the right.

Question 2

What is .0000185 in scientific notation?

a) 1/185,000

b) 1.85×10^{-5}

c) 185×10^{-3}

d) 18.5×10^{-2}

Answer: b

Explanation
The number must start with 1.85 to be in scientific form, and this required that the decimal point move five places right. The exponent of ten will therefore be -5.

Practice Questions

Question 3

Express 45 ten thousandths in scientific notation.

a) 45 x 10,000

b) 45×10^{4}

c) 4.5×10^{-4}

d) 4.5×10^{-3}

Answer: c

Explanation
First write the number as a decimal, .00045. Next move the decimal to the right four places to get 4.5×10^{-4} This is a small number and has a negative exponent to show that 4.5 is over (or divided by) 10,000. Notice that this number could also be written 4.5 x 1/10,000

Question 4

What is $(2.3 \times 10^{3}) – (1.2 \times 10^{3})$?

a) 1100

b) 120

c) 1200

d) 1000

Answer: a

Explanation
Remember the order of operations, do multiplication before adding and subtracting. Expand each scientific number to get 2300 (move decimal three places right) and 1200 (move decimal three places right). Now do the subtraction to get 1100.

Topic 6.B – Using Scientific Notation

Use the exponent laws to multiply and divide numbers in scientific notation

The rules for simplifying exponents given in the last topic can be used on numbers in scientific notation to make calculations with them easier.

1. **To multiply numbers in scientific notation,** multiply the first numbers, aka factors, and then multiply the powers of ten by adding the exponents.

   Examples

   $(4.5 \times 10^{6}) \times (3.2 \times 10^{5}) = (4.5 \times 3.2 \times 10^{6+5}) = 14.4 \times 10^{11}$

   $(2.0 \times 10^{-2}) \times (4.2 \times 10^{4}) = (8.4 \times 10^{-2+4}) = 8.4 \times 10^{2}$

   $(3.1 \times 10^{4}) \times (5.0 \times 10^{3}) = 15.5 \times 10^{4+3} = 15.5 \times 10^{7} = 1.55 \times 10^{8}$

   In the last example 15.1×10^{7} is converted into 1.51×10^{8} because scientific notation requires that the first number be between 1 and 9.

2. To divide numbers in scientific notation, divide the first numbers, aka factors, and then divide the exponents of ten by using the rule

   $\mathbf{\dfrac{a^r}{a^s} = a^{r-s}}$

   Examples

   1. $3 \times 10^{5} \div 2 \times 10^{2} = \dfrac{3 \times 10^{5}}{2 \times 10^{2}} = 3/2 \times 10^{5-2} = 1.5 \times 10^{3}$

   2. $5 \times 10^{2} \div 2 \times 10^{4} = \dfrac{5 \times 10^{2}}{2 \times 10^{4}} = 5/2 \times 10^{2-4} = 2.5 \times 10^{-2}$

   3. $30 \times 10^{4} \div 2 \times 10^{2} = 30/2 \times 10^{2} = 15 \times 10^{2} = 1.5 \times 10^{3}$

Practice Questions

Question 1

A machinist discovers an error of 2.6×10^{-4} inches on each part that he has milled. If he has milled 13 of these parts and they are joined together, what is the total error?

a) 3.38×10^{-3}
b) 23×10^{-4}
c) 2.38×10^{-5}
d) 2.38×10^{3}

Answer: a

Explanation
We can solve this problem by adding 2.6×10^{-4} 13 times, by multiplying 2.6×10^{-4} by 13, or by changing the scientific number into an equivalent decimal or fraction and then multiplying by 13. The simplest way is to leave it in scientific form and the answer choices are in this form. $13 \times (2.6 \times 10^{-4}) = 33.8 \times 10^{-4}$. This answer is not in scientific form until it is changed into the equivalent form of 3.38×10^{-3}.

Question 2

If 4.5×10^{4} tonnes of concentrate will have 10% pure gold, how much will the pure gold weigh?

a) $.45 \times 10^{4}$
b) 4.5×10^{3}
c) 4.5×10^{-4}
d) 4.5×10^{2}

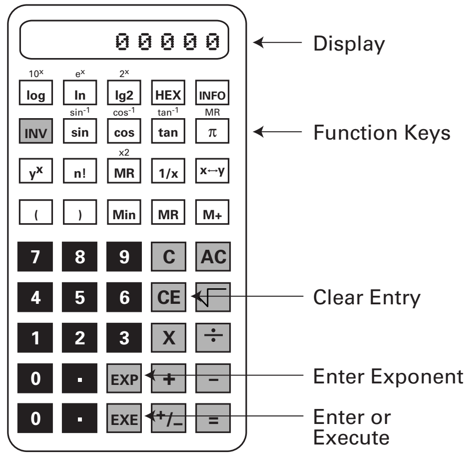
Answer: b

Explanation
To find 10%, multiply by .1. This moves the decimal place one place to the left. You can also see that ten percent of 45,000 (4.5×10^{4}) is 4500, and $4500 = 4.5 \times 10^{3}$.

Unit 3 – Combining Operations and Numbers UNIT 3

Topic 7 – Using A Scientific Calculator

A scientific calculator can be used to solve problems with rational numbers. Scientific calculators can operate on numbers in scientific notation and use algebraic logic. This means the calculator will follow the order of operations reviewed in Unit 3, Topic 4 – The Order of Operations.



Problems involving exponents and operations on signed numbers can be done as well. Calculators also have a CE (clear entry key) that removes the last keystroke's input. You must press the = key when you want the calculator to complete an operation. Some calculators have an EXE key meaning "execute" to perform the operation that has been entered.

Calculators can have different keyboard layouts. Some have = and come have EXE to complete an operation. See the booklet of instructions that comes with a calculatr to learn what each key means and the order of keystrokes for various operations.

Signed Numbers

On some calculators, when a negative number is entered after a multiplication sign, minus sign, addition or division sign, the +/- key is used to enter the number correctly. This key changes the sign of the last number entered. For example 6 – (-5) is entered as

6 **–** **5** **+/–** **=**

and the display shows 11. When a negative number is entered for a problem, for example -2, first press **2**, then **+/-**. The display will show 2-. The minus sign appearing after the number means that it is negative.

On other calculators, 6 – (-5) is entered this way:

6 **—** **(-)** **5** **EXE**

to get the answer of 11. A negative number is entered 2 (-) to get a display of -2.

Topic 7 – Using A Scientific Calculator

Scientific Notation

A number in scientific notation is entered in two steps. First enter the decimal number part of the scientific notation and then press the EE key (enter exponent EE key, sometimes marked EE. Now enter the exponent part of the scientific number. Once this is done, you can operate on the number using the + , -, x, and ÷ keys and the answer will be displayed in scientific form, with a number followed by the correct exponent of ten. Remember that when you want to enter a negative exponent you must use the +/- key after the exponent is entered on some calculators.

Example

Enter

1 . 2 EXP 6 +/–

or on some calculators

1 . 2 EXP (-) 6 EXE

The display will show 1.2 6 - to mean 1.2 x 10-6

If possible, do the following examples on a calculator as you read. These examples refer to a calculatr with a +/- key.

Examples

1. **Find 9 ÷ 5/6**

   Method one: change fraction into a decimal and divide into 9. Keystrokes:

   5 ÷ 6 = **.83333**

   9 ÷ .83333 =

   Method two: change into a multiplication problem, invert and multiply

   9 x 6 ÷ 5 = 10.8

2. **-1/5 divided by 2/7**

   1 +/– ÷ 5 X 7 ÷ 2 = **.7 – which means -.7**

   Here 2/7 was inverted to change the division into a multiplication problem.

   An alternative sequence of keystrokes can use the = sign to find the result of each step:

   1 +/– ÷ 5 (display shows 0.2-)

Topic 7 – Using A Scientific Calculator

Continue by operating on 0.2- as follows:

X **7** **÷** **2** (invert and multiply by 2/7) The calculator will display the final answer .7- which means -.7.

3. **Find the square root of 163**

163 **$\sqrt{}$** (the display will show 12.767145).

Some calculators reverse this order and

$\sqrt{}$ **163** will give the answer.

This is an irrational number and can be rounded off to any desired place value. The nearest hundredth, $\sqrt{163}$ = 12.78. Some calculators will reverse this order and you will enter the square root sign first.

4. **Find the reciprocal of 273**

273 **1/x** or use **273** **shift** **x^{-1}** depending on your calculator.

(the display will show the decimal answer = .003663) Notice that the x in the keystroke is a variable. Any number entered before you press 1/x will be divided into one to give the reciprocal as a decimal number.)

Unit 3 – Combining Operations and Numbers

Practice Questions

Question 1
Use a scientific calculator to evaluate the following expression: $(3.4 \times 10^5) \div 48$

a) 10, 480
b) 7083.33
c) 480,000
d) 708.33

Answer: b

Explanation
The keystrokes are:

$\boxed{3}$ $\boxed{.}$ $\boxed{4}$ $\boxed{X}$ $\boxed{10}$ $\boxed{EXP}$ $\boxed{5}$ $\boxed{\div}$ $\boxed{48}$ $\boxed{=}$ or $\boxed{EXE}$

This will return 7083.33. You can check your answer by multiplying 48 x 7083.33. The calculator will return 339,999.99.. This rounds off to 3.4 x 105

Question 2
What would be the result of the following keystrokes on a scientific calculator?

$\boxed{50}$ $\boxed{+/-}$ $\boxed{\div}$ $\boxed{10}$ $\boxed{+}$ $\boxed{5}$

a) -5
b) 5
c) 0
d) 10

Answer: c

Explanation
50 becomes -50 after the $\boxed{+/-}$ keystroke. Do division before addition. $-50 \div 10 = -5$, and $-5 + 5 = 0$.

Question 3
The amount of material needed to be scraped off an underground kimberlite deposit is estimated at 2.56×10^7 tons. If this amount is to be moved in four equal phases, how much material will be removed in the first phase?

a) 256,000 tons
b) 2.56×10^2 tons
c) 6.4×10^6 tons
d) 6.4×10^8

Answer: c

Explanation
The amount to be removed will be divided into four equal phases. We need to divide 2.56×10^7 tons by 4. First enter the total: 2.56 ex 7. Now press ÷ 4 = and the answer will be returned in scientific notation as 6.4 6, which equals 6.4×10^6.

Unit 3 – Combining Operations and Numbers

Topic 8 – Ratios, Rates and Proportions

Ratio

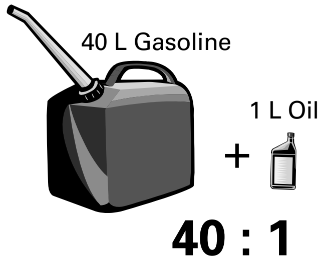
A ratio is a fraction. Any fraction can be looked at as a relationship between two numbers. When we call a fraction a ratio, we are referring to the comparison that can be made between the numerator and the denominator. When we say , "He is twice as tall she is, we can express the ratio of his height to hers as 2/1. This means that for every foot of her height there are two feet of his height. Everything you have learned about working with fractions will also apply to working with ratios. You may want to review earlier sections on fractions for this topic. See Unit 1, Topic 2 – Fractions and Unit 2, Topic 2 – Fractions for more on fractions.

Examples

1) $^{2}/_{3}$ is a ratio that compares two to three. The comparison can be expressed as a relationship this way: for every group of three we have two of something. For example, for every two days of sunshine we get three days of rain, or for every two wins our team has three losses. "for every three hours he sleeps, I sleep two hours”. These statements express ratios.
2. $^{1}/_{2}$ is a ratio that compares one to two. For every two of something, we are thinking of one of something. " for every dollar that I make, he makes two", or equivalently, " for every two dollars he makes, I make one". The relationship between dollars I make and dollars he makes is one to two.
3. $^{2}/_{1}$ is the same relationship viewed from the opposite perspective. "He makes twice what I make, for every \$1.00 I make, he gets \$2.00." His rate of pay compared to mine is in the ratio of $^{2}/_{1}$, or two to one, while my rate of pay compared to his is $^{1}/_{2}$.

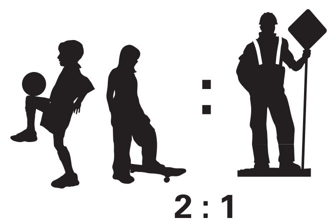
Two Examples for Greater Clarity

1. In my chainsaw I put in 40 parts of gas for one part of oil – this can be described by the ratio 40/1 (also written 40:1)

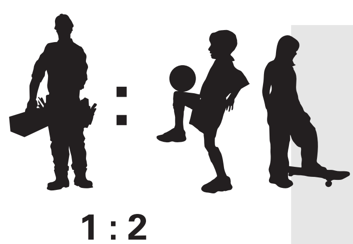


OR, In my chainsaw I put 1 part oil for every 40 parts of gas – this can be described by the ratio 1/40 (also written 1:40) . It's the same relationship between the oil and the gas, and its the same saw, **but we must be clear about what the numerator and the denominator represent**. 1/40 is oil to gas, and 40/1 is gas to oil. Serious errors have been made by confusing this ratio in filling up saws. 40 will always refer to the gas, never the oil part of the relationship. As you will see in the section on measurement, it is also important that the parts are the same size. 1/40 is the same relationship when we compare ounces to ounces or gallons to gallons – but we cannot compare ounces to gallons using this ratio.

2. The ratio of **young people under the age of 12 to adults** in some communities is two to one. This relationship can be expressed by the ratio 2/1. There are two young people under 12 for every (one or single) adult.



OR, The ratio of **adults to young people** under the age of twelve in some communities is one for every two, or $\frac{1}{2}$. There is one adult for every two young people under 12.



In both cases it is the same relationship being described.

Topic 8 – Ratios, Rates and Proportions

The Unit Rate (Ratio)

A ratio with a denominator of 1 is called a unit ratio. For example, if you earn $25.00 per hour, this can be expressed as the ratio 25/1. This is the rate of pay for each hourly unit of time. Because you are paid at the rate of $25.00 per hour, you know you will earn $100.00 in four hours by multiplying the base rate times 4. If you are told that someone earns $100 in four hours, you can see that the hourly rate of pay is $25.00 by dividing $100 into four parts.

Example

12 dozen eggs cost $11.29. What is the unit rate?

Here we are told that 12 dozen eggs cost $11.29. The ratio of dozens of eggs to their cost is 12/$11.29. The cost for one dozen eggs will be the unit rate in this example. Dozens of eggs are the units being discussed in this problem. You could calculate the cost per egg as well, and get the unit rate per egg. However, the problem gives us a dozen eggs as the basic unit on which the cost is based, and therefore we calculate the rate per dozen to find the answer. $11.29/12 = $.94, or 94 cents per dozen. 94 cents is the unit rate.

The unit rate is useful for budgeting and purchasing because it can be used to calculate the cost of any number of items at that rate. For example, if a camp cook wants to order 560 dozen eggs, she can multiply 560 x .94 to get the total cost because she knows the unit cost is $.94 (in this case the units are dozens of eggs).

Some Unit Rates Used in Trades:

The price per gallon of gas (transportation, machinery)

The cost per board foot of lumber (construction)

The cost per litre of shampoo (hairdressing)

The weight per brick (bricklaying)

"per" means "of one unit"

Ratios Can Be Expressed As Percents

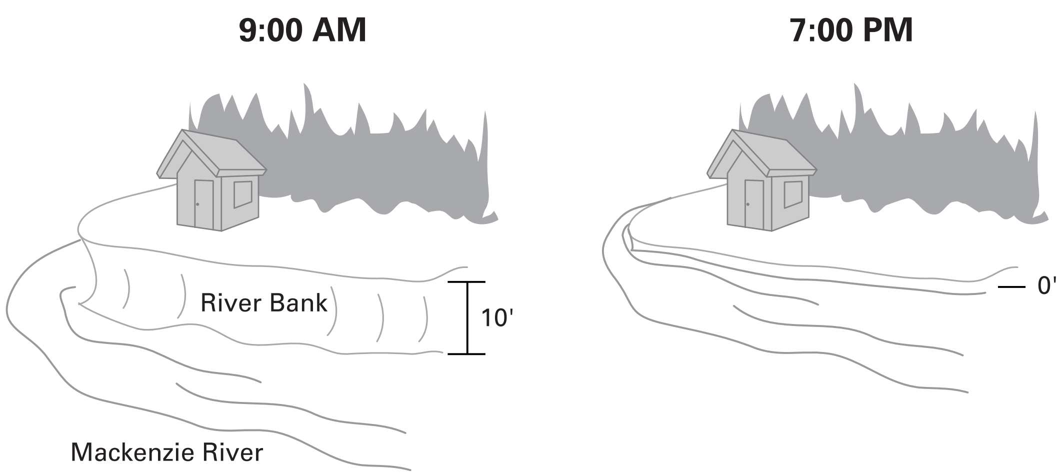
For example, if a manager knows that 12 out of every 16 shipments are being received without any damage, he can express this as the ratio $^{12}/_{16}$ or as $^{3}/_{4}$ or as .75/1. Three of every four arrive undamaged, which also implies that one out of four is damaged. As you know from previous topics, these are equivalent numbers. A percent number can also be used to express a ratio .75 = 75%. The manager can say that 75%, or three quarters, of his shipments are arriving undamaged.

Topic 8 – Ratios, Rates and Proportions

The related idea of a rate uses the same information to describe how fast something happens. For example, if we know that three out of every four shipments arrive in undamaged condition, we can conclude that **at that rate** we will have four shipments of damaged goods after 12 shipments are received.

More On Rates

Rates are ratios that tell us how fast something happens or how much of something changes over a period of time. For example, if water is rising at the rate of one foot per hour on the Mackenzie river, The unit or base rate is 1 foot/1hour, or " one foot per hour". If the bank is 10 feet above the water level, the shore will begin to flood after ten hours at this rate (of one foot per hour). Remember that a ratio can be reversed and still describe the same relationship as long as we are clear about which term in the relationship we are putting first. In this example we can switch perspectives and also say that one foot of the bank is being covered by water each hour.



A second example: if snow accumulates at the rate of 1 foot per hour we can predict, at that rate, that it will be 3 feet deep after three hours. A rate can be expressed as a percent that describes how fast something happens or how much of something happens. For example, if taxpayers are taxed at the rate of 2 cents for every dollar that their property is assessed for, then the tax rate is 2% because the ratio of 2 cents to 100 cents is 2/100 = .02 = 2%.

Practice Questions

Question 1

Bill earns one third more than Sally. What is the ratio of Bill's earnings compared to Sally's?

a) $^3/_4$
b) $^4/_3$
c) $^2/_3$
d) $^1/_3$

Answer: b

Explanation

We are told that Bill earns one third more than Sally. This means that for every dollar that Sally earns Bill will earn $l.33 because that is one third more than what she earns: 1 + $^1/_3$ of one = 1$^1/_3$ = 1.33. The unit rate comparing Bill's salary to Sally's is 1.33/1. Choice b expresses 1.33 as the equivalent fraction of $^4/_3$. It means the same thing to say that Bill earns four dollars for every three that Sally earns, and to say that Bill earns $1.33 (one and one third dollars) for every dollar that Sally earns (this is the unit rate).

Going back to our basic picture of what a fraction means (see Unit 1, Topic 2 – Fractions), $^4/_3$ tells us that we have divided Sally's earnings into three equal parts and then selected four of these parts to assign to Bill. This ratio can also be expressed as a percent. Bill earns 133% of Sally's earnings. All three expressions refer to the same relationship: 133%, $^4/_3$, and $1.33/$1.00.

Question 2

Bill receives a notice that his hydro rates will increase 5% next month because smaller communities will no longer be subsidized by the GNWT. He pays $00.12 for each kilowatt-hour. How much will he pay for each kilowatt-hour after the increase?

a) $00.126
b) $00.125
c) $00.17
d) $00.52

Answer: a

Explanation

The rate is increasing by 5%. To find the amount of the increase, multiply .05 x .12 to find $.006. This is the amount of the increase. Next, add this increase to the old (base) rate to find the new rate of $00.12 + $00.006 = $00.126 per kilowatt-hour.

Practice Questions

Question 3

A truck is being loaded at the rate of 100 lbs. per minute. At this rate, how long will it take to finish loading a truck that can carry 3000 lbs?

a) 3 minutes
b) 30 minutes
c) 15 minutes
d) 20 minutes

Answer: b

Explanation

The unit in this question is the minute. The unit rate is 100 pounds per minute. It will take thirty minutes because 30 x 100 = 3000. To find how long it takes we divided the base rate (unit ratio) of 100/1 into 3000. An equation could be used to set up the problem: loading time required = total weight of 3000/unit rate of 100.

Question 4

An outboard motor requires mixed fuel, with 1 part of oil for every 50 parts of gas. What is the ratio of gas to oil?

a) 1 to fifty
b) fifty to one
c) one to one
d) fifty to five

Answer: b

Explanation

The relationship is one part oil to 50 parts gas, but the question asks us to express this relationship in reverse: with parts of gas compared to parts of oil. We will mix fifty parts of gas for every part of oil. For example, for every fifty ounces of gas, we will add one ounce of oil. Put this way, the ratio of gas to oil is 50 to one.

Practice Questions

Question 5

What is the base rate charged by a guide outfitter if a 10 day trip cost $1500, a 20 day trip cost $3000, and a 15 day trip cost $2250?

a) $200 per day
b) $300 per day
c) $150 per day
d) $225 per day

Answer: c

Explanation

The base rate (or unit rate) can be found by reducing the ratio of days divided by cost. We have three ratios to choose from: 20/1500, 15/3000, and 15/2250. These are equivalent fractions that are based on the same daily rate. They all reduce to the unit ratio of 1/150. The daily rate is the unit rate in this problem and is equal to $150.00

Topic 8 – Ratios, Rates and Proportions

Proportion

A proportion compares two ratios that are equal. $^1/_2$ and $^5/_{10}$ form a proportion because they are equal. $^1/_2 = {}^5/_{10}$. A proportion is also an equation that says one fraction equals another. $^1/_2 = 50/100$ is a proportion equal to $^5/_{10}$. A proportion can have more than two terms, for example $^5/_{25} = {}^{10}/_{50} = {}^1/_5$.

Normally we would reduce the ratios being compared in a proportion to lowest terms, but the value of using proportions is that we can find any one of the four terms involved in two term proportions if we know the other three.

> This means that equivalent fractions are proportions
>
> $^3/_{12} = {}^1/_4$
>
> $^5/_6 = {}^{10}/_{12}$

Examples

$^3/_4 = {}^{75}/_{100}$ is a true statement, and it tells us that taking three parts for every four that a whole is divided into, is the same amount as is taking 75 parts for every 100 parts that a whole is divided into. Spending three fourths of your income, is the same as spending 75 cents out of every dollar, the ratios are equal. $^1/_2 = {}^{75}/_{100}$ is a proportion.

When two ratios are equal to each other, we only need to know three of the terms in order to find the fourth one.

A proportion has two ratios that are equal

There are four terms in two ratios

2 numerators, and 2 denominators

$$\frac{N_1}{D_1} = \frac{N_2}{D_2}$$

To find the missing term in a proportion

1. Set up the proportion with a? for the missing term you need
2. Cross multiply (each numerator times opposite denominator)

   $N_1 \times D_2 = N_2 \times D_1$

3. Divide both sides by the number next to the ? (the coefficient of the number you seek)

Unit 3 – Combining Operations and Numbers

Topic 8 – Ratios, Rates and Proportions

Example

Suppose someone offered you $75.00 out of every hundred dollars you bring in for a business, and says that it is equivalent to what someone else is getting when they keep 3 parts of every $4.00 that they earn. Would this claim be true?

You could solve the problem using the procedure for finding the missing term in a proportion.

1. $3/4 \stackrel{?}{=} 75/100$ Set up the problem

   Test the claim that 3 has a relation to four that is the same relationship that 75 has to 100. You can put a "?" over the equal sign to remind yourself that you are not sure the two ratios are actually equal to each other.

2. **Cross multiply**

   3 x 100 = 75 x 4

   300 = 300

The ratios are indeed equal because their cross products are equal.

The offer made in this problem means you would get $75.00 out of $100. You would keep the same amount of money, dollar for dollar, as would the other person who keeps $3.00 out of every $4.00 that they earn. The offer is a fair comparison of rates of pay. You both work for the same base rate of $3/4$ or equivalently, 75%.

Examples

1. Solve this proportion problem

   5/? = 15/24

   5 x 24 = 15x? (cross multiply)

   $\frac{5 \times 24}{15} = \frac{15x?}{15}$ (divide both sides by 15)

   8 = ? (8 is the answer)

   Check: 5/8 = 15/24 (common divisor of 3 reduces 15/24 to 5/8), also the cross products are equal, 100 = 100. You can also see that 15/24 reduces to 5/8.

Topic 8 – Ratios, Rates and Proportions

Alternate Method Based on Equivalent Fractions

Because 5/? = 15/24, we know that a common multiple can be used to multiply 5 to get fifteen, and also to multiply our unknown (?) to get 24. We can see that three does the job for the numerators, therefore three times some number will equal 24 in the denominator. This means 3 x ? = 24. Divide both sides by 3 and the answer is 8.

1. Find the missing term

   14/6 = 7/?

   14 x ? = 7 x 6 (cross multiply)

   14 x ? = 42

   ? = 42/14 (divide both sides by 14)

   ? = 21/7 = 3

   Check: 14/6 = 7/3

2. ?/6 = 11.5/23

   ? x 23 = 11.5 x 6

   ? x 23 = 69

   ? = 3

   Check: 3/6 = 11.5/23

3. 6/2.1 = ?/7

   6x7 = ? x 2.1

   42 = ? x 2.1

   20 = ?

   Check: 6/2.1 = 20/7

4. A cook knows that 15lbs of flour needs to be combined with 2 pounds of butter in a bread recipe. If the kitchen has 65 lbs. of flour to make bread, how much butter will be required?

   a) 12 pounds
   b) 8.66 pounds
   c) 5 pounds
   d) 10 pounds

   Answer: b

   This problem can be solved by setting up a proportion where three of the numbers are known. We are given the ratio of 2 lbs. butter for every 15 pounds of flour. This can be set equal to an unknown amount of butter for 65 pounds of flour. The proportion can express butter to flour, or flour to butter. Here the numerators are pounds of flour, and the denominators pounds of butter.

   15/2 = 65/?

   15 x ? = 2 x 65 (cross multiply)

   ? = 130/15 = 8.66 pounds of butter (divide both sides by 15)

5. What is the unit rate of butter to flour for this bread recipe?

   a) 2/15
   b) 1/15
   c) 1/7.5
   d) 1/65

   Answer: c

   The ratio given in the problem is 2 pounds of butter for 15 pounds of flour. We need to know how many pounds of flour are required for one pound of butter to find the unit ratio. 2/15 can be reduced to 1/7.5 by dividing top and bottom by 2.

Unit 4

Practice Exam Questions Involving Basic Operations

The sample questions in this unit apply the competencies reviewed in Units 1 to 3 on basic operations. You may want to use this section as a pre- test to see which topics you need to study in Units 1 to 3. The explanations of the correct answers make use of the material covered in these earlier units. You may find that these explanations are sufficient for your purposes, or you may find that you need to study the earlier units.

General Guidelines for Working on Exam Questions

Many problems require more than one operation and more than one-step. Diagrams and pictures can help organize the order in which you answer the parts of a question. First describe in words to yourself what it is you want to find out. Then list the facts you have to work with. Write down any formulas, exchange factors, or definitions that you will use.

Use these questions to guide your thinking:

1. Is something being put together, combined, added, made into a sum or total? Consider adding
2. Is something being taken away, removed, subtracted, or put somewhere else? Consider subtraction
3. Is something being added over and over again? Is something being expressed as a rate related to something else? Examples: 50% **of** something, $^2/_3$ **of** something, .7 **of** something? Consider multiplying.
4. Is a proportion involved that compares two ratios or fraction quantities? Cross multiply and divide.
5. Is something being divided, broken into sections or parts? Use division.
6. If a series of events is described, put them in the order in which they happened. Write down the facts that you know for sure. Describe any important relationships between the facts using numbers and operations.

Unit 4 – Practice Exam Questions Involving Basic Operations

Two Principles Will Be Useful:

1. Integers, decimals, and fractions can be changed into each other.
2. Before adding, have all of the numbers in the same form. Problems involving more than one kind of number require first changing all the numbers into the same kind so we can add fractions to fractions, integers to integers, and decimals to decimals.

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 1 – Adding numbers containing integers, decimals and fractions

Unless the problem asks for the answer as a fraction, integer, or decimal, you can choose the form of your answer. Generally speaking, the "convert to decimal method" is easier, but each situation can be different. You may want to review the earlier topic on adding decimals.

Guidelines

1. Know what form the answer is required in and use the method that produces that form.
2. Converting to decimals relies on division and does not require searching for common denominators

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 1 – Adding numbers containing integers, decimals and fractions

Question 1

Add $\frac{5}{12}$ and 3.45

a) 6.45
b) 3.87
c) $\frac{7}{12}$
d) $3\frac{7}{12}$

Answer: b

Explanation
We can change the fraction into a decimal, or the decimal into a fraction. Which form will be easier to add?

a) Convert to fractions method

3.45 = 345/100 = 69/20

We still have to find a common denominator for 69/20 and 5/12 before we can add them. Try 120

69/20 = 414/120

5/12 = 50/120

Now add these like fractions to get 464/120

Simplify: $120\overline{)464}$ = 3.866

This answer is in a decimal form. It can be rounded up to the nearest hundredth: 3.87. This is given as choice b). If a fraction form had been requested we would divide $120\overline{)464}$ and get 3 R104, or 3 104/120 which simplifies to 3 52/60. This would be the same answer (i.e. 3.866…) expressed in its equivalent fraction form. This is not an option given in the list of answer choices.

b) Convert to decimal method

1. 5/12 = $12\overline{)5.0}$ = .4166.. We have a choice about what to round up to. Since 3.45 stops at the hundredths column, we can round up the hundredths place for this addition problem.

   Now add the decimals

$$\begin{array}{r} 0.4166 \\ +\ 3.4500 \\ \hline 3.8666 \end{array}$$

3.8666 = 3.87 (rounded to nearest hundredth)

This is the answer in decimal form. If a fraction form had been requested, we would have to change this decimal into its fraction equivalent as in a method above, or express it as 387/100. Notice that 387/100 = 3 52/60

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 1 – Adding numbers containing integers, decimals and fractions

Question 2

Add 516, $8^{2}/_{3}$ and .67

a) 525.34
b) $527^{2}/_{3}$
c) 526.8
d) 528

Answer: a

Explanation
Convert to decimals method:

First add the whole number and decimal number. 516 is a whole number = 516.00 that can be added to .67 by lining up the decimal points.

$$\begin{array}{r} 516.00 \\ +\ .67 \\ \hline 516.67 \end{array}$$

Now change $8^{2}/_{3}$ into its decimal equivalent

$8^{2}/_{3}$ = 26/3 = $3\overline{)26}$ = 8.6666…

Since we are adding to numbers that only go to the hundredths column, we can round this decimal up to the hundredths place = 8.67

Now add 8.67 + 516.67 = 525.34, which is given as choice a.

If the answer had been requested in fraction form, we would convert this number to its equivalent mixed number.

535.34 = 535 34/100 = 535 17/50

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 2 – Subtracting numbers containing integers, decimals and fractions

The same guidelines apply as for addition, but you may have to decide which number is larger in order to subtract correctly. See Unit 3, Topic 1 – Identify the Largest or Smallest Number in Decimal and Fraction Form on identifying the largest and smallest numbers from a list of fractions or decimals. You may also want to review the earlier section on subtracting decimals.

Question 1

Find the difference between 13 5/8 and 13.625

a) $^5/_8$
b $^1/_4$
c) $^1/_3$
d) 0

Answer: d

Explanation
We need to know which number is larger before we can subtract. Both numbers have 13 units. The question becomes: is 5/8 larger or smaller than .625? The answer can be found by changing 5/8 into its decimal equivalent and comparing this with .625.

$5/8 = 8\overline{)5} = .625$

We see that the decimal equivalent of 13 5/8 = 13.625

The answer to our problem is 0. There is no difference between 13 5/8 and 13.625

Question 2

Subtract 39.6 from $112^3/_4$

a) 72.75
b) 73.15
c) 69.65
d) 73.25

Answer: b

Explanation
Change 112 $^3/_4$ into its decimal equivalent

$112^3/_4 = 112.75$

Subtract

$$\begin{array}{l} 112.75 \\ -39.6 \\ \hline 73.15 \end{array}$$

Topic 2 – Subtracting numbers containing integers, decimals and fractions

Question 3

Subtract $17^{2}/_{3}$ from $25^{1}/_{2}$ and express the answer as a decimal number.

a) $7\ {}^{7}/_{8}$
b) 7.42
c) 7.83
d) $7\ {}^{5}/_{12}$

Answer: c

Explanation
Change into decimal equivalents

$17^{2}/_{3}$= 17.66 (round up to hundredths place = 17.67)

$25^{1}/_{2}$= 25.50

Subtract the equivalent decimal numbers

$$\begin{array}{r} 25.50 \\ -17.67 \\ \hline 7.83 \end{array}$$

Question 4

Subtract 14.23 from 16 1/5 and express the answer as a fraction.

a) $1^{97}/_{100}$
b) $2^{1}/_{5}$
c) $2^{1}/_{4}$
d) $1^{3}/_{4}$

Answer: a

Explanation
Change $16^{1}/_{5}$ into its decimal equivalent

$16^{1}/_{5}$ = 16 + 1/5 = 16 + .2 = 16.20

Subtract

$$\begin{array}{r} 16.20 \\ -14.23 \\ \hline 1.97 \end{array}$$

Change 1.97 to the fraction it equals

1.97 = 1 97/100

Is 97/100 in lowest terms? Yes, there is no whole number to go evenly into both 97 and 100.

The answer in fraction form is 1 97/100.

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 3 – Multiply numbers containing integers, decimals and fractions

The same guidelines apply as for addition and subtraction of decimals. For this section review the earlier topics on multiplying decimal numbers.

Question 1

Find the product of 4.5, $^2/_3$ and 16 to the nearest hundredth

a) 48
b) 48.24
c) 48. 52
d) 48.25

Answer: a

Explanation
Change into decimal equivalents

$^2/_3$ = .66.. = .67 rounded up to the nearest hundredth

Multiply the integer and decimal numbers

$4.5 \times {}^2/_3 \times 16 = {}^9/_2 \times {}^2/_3 \times 16 = 3 \times 16 = 48$

Question 2

Multiply 30.015 x 16 3/5

a) 502.10
b) 498.25
c) 480
d) 499.25

Answer: b

Explanation
Change $16^3/_5$ into its decimal equivalent

$16^3/_5 = 16.60$

Multiply

$16.60 \times 30.015 = 498.25$

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 3 – Multiply numbers containing integers, decimals and fractions

Question 3

Multiply 12.63, $^3/_8$, and $1^1/_2$

a) 7.10
b) $24^1/_8$
c) 14.10
d) 8.10

Answer: a

Explanation
Change the factors into decimal numbers

$^3/_8$ = .375

$1^1/_2$ = 1.5

Multiply in any order

12.63 x 1.5 = 18.945 x .375 = 7.10 (rounded to the nearest hundredth)

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 4 – Dividing numbers containing integers, decimals and fractions

The same guidelines apply for division. You may want to review the earlier topic on dividing with decimals.

Question 1

6 5/12 ÷ 13

a) .57
b) 89/144
c) 77/156
d) .67

Answer: c

Explanation
Convert to fractions method

$6\frac{5}{12} = \frac{77}{12}$

13 = 13/1

Identify the divisor

The divisor is $\frac{13}{1}$

We are now dividing one fraction by another. Invert the divisor and multiply

$$\frac{1}{13} \times \frac{77}{12} = 77/156 = .49 \text{ (rounded to nearest hundredth)}$$

Either answer is correct unless an answer in fraction form or decimal form is specified.

The "convert to decimals" method would also work and produce the same answers. Try this for yourself to see.

Question 2

What is 133.45 divided by $6\frac{3}{5}$?

a) 25.11
b) $27\frac{1}{5}$
c) 20.22
d) 22.20

Answer: c

Explanation
Change $6\ \frac{3}{5}$ into its decimal equivalent

$6\ \frac{3}{5} =$ 6.60

Divide
$6.60 \overline{)133.45}$ = 20.22 (rounded to nearest hundredth)

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 4 – Dividing numbers containing integers, decimals and fractions

Question 3

Express the quotient as a fraction: $5.5 \div 63\ ^1\!/_2$

a) 433/5000
b) 43/500
c) 3/5
d) 3/8

Answer: a

Explanation
Convert the divisor, $63\ ^1\!/_2$ to a decimal = 63.50

Divide $63.50\overline{)5.5}$ = .0866

Change this decimal number into its fraction equivalent

.0866 = 866/10,000 = 433/5000 (divide numerator and denominator by 2)

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 5 – Problems involving money. Add, Subtract, Multiply, Divide (payroll, unit price)[7]

The exchange factor for money is 1 dollar = 100 cents. Money uses the decimal system. All of the guidelines for working with decimals will apply to problems involving money. Adding and subtracting, multiplying and dividing money is the same as with other decimal numbers. Organize problems with money by seeing what operations with decimals will give you the answer.

Question 1

Bill spent $54.69 on some fishing tackle and bought a snack for $3.45. How much did he spend?

a) $58.34
b) $68.14
c) $58.14
d) $62.24

Answer: c

Explanation
Two items are being combined. Add $54.69 and $3.45

  $54.69
+ $3.45
―――――
  $58.14

Question 2

Sally was overcharged $3.09 at the store. She went back with her receipt to claim a refund from the $15.35 she had paid. What is the correct amount she should have paid?

a) $13.05
b) $18.44
c) $12.34
d) $12.26

Answer: d

Explanation
Look at the sequence of events. First she overpaid by giving $15.35, then she discovered the overcharge. Subtract $3.09 from $15.35

$15.35
 -$3.09
―――――
$12.26

Sally should have paid $12.26

7 You may want to review converting units to sub units in Math – Module 4 – Measuring Time, Shapes and Spaces. See also ratios and unit rates in Unit 3, Topic 8 – Ratios, Rates and Propportion.

Topic 5 – Problems involving money. Add, Subtract, Multiply, Divide (payroll, unit price)

Question 3

Wanda Norwegian and Jean Billet work for the environmental review board for $25.00 per hour. Wanda worked 22 hours this week, and Jean worked 30 hours. What is the total earned by both workers?

a) $1700.00
b) $1300.00
c) $1250.00
d) $1400.00

Answer: b

Explanation
Wanda earned 22 x $25.00 = $550.00

Jean earned 30 x $25.00 = $750.00

The sum is $1300.00

Question 4

The fish plant on Great Slave Lake produces 600 pounds of fish per hour. Each pound is worth $2.50. What is the value of the fish produced in 8 hours?

a) $4800.00
b) $12,000.00
c) $7800.00
d) $6500.00

Answer: b

Explanation
We need to know the total weight of the fish produced in twelve hours before we can calculate the value of the total. We could add 600 pounds eight times to get the total number of pounds, but Multiplying is quicker.
Multiply 8 hours times 600 lbs. = 4800 pounds of fish.

Now we can think about what 4800 pounds is worth.
One pound = $2.50, so 4800 pounds x $2.50 = $12,000.00

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 5 – Problems involving money. Add, Subtract, Multiply, Divide (payroll, unit price)

Question 5

Ed's pay cheque has a deduction of 17% for income tax. If Ed earns $1540 bi-weekly, what will be deducted from each pay cheque for income tax?

a) $130.80

b) $340.00

c) $261.80

d) $154.80

Answer: c

Explanation
We know that Ed is paid $1540 every two weeks. We also know that he has a 17% deduction from his pay cheque, therefore we need to find what 17% of $1540 is. Recall that "of means times", and that 17% = .17.

We find the Answer by multiplying .17 x $1540 = $261.80

Question 6

Ethel is paid $2300 each month from her job at the band office. She receives a pay cheque with several deductions:

Unemployment insurance $45.00
Income tax $460.00
Canada Pension Plan $35.00

What is Ethel's take home pay each month?

a) $635.00

b) $1760.00

c) $2135.00

d) $1835.00

Answer: b

Explanation
We need to add the deductions and then subtract the total from $2300.

The total of the three deductions is $540. $2300 – $540 = $1760.00

Topic 5 – Problems involving money. Add, Subtract, Multiply, Divide (payroll, unit price)

Question 7

The Wekweti coop orders 50 cases of canned milk for $150.00. Each case has 20 cans. What is the cost per can of milk for the coop?

a) $.15
b) $.30
c) $.12
d) $.20

Answer: a

Explanation
We need to know the total number of cans in order to figure out the price of each can. The price per can is the unit price we seek. If 50 cases each have 20 cans, then 50 x 20 = 1000 cans in total.

We know that the cost of 1000 cans = $150.00. By dividing 1000 into $150.00 we will get the cost per can. $150/1000 = .15 . Each can costs 15 cents.

Question 8

Pete makes $40.00 per hour of regular time worked, and time and a half for overtime. In January Pete worked 100 regular hours and 15 overtime hours. How much did he earn in January?

a) $2500.00
b) $4900.00
c) $6500.00
d) $3300.00

Answer: b

Explanation
We need to add the pay from regular hours to the pay from overtime hours.

Pay from regular hours = $40.00 x 100 hours = $4000.00

Pay from overtime is $1\frac{1}{2}$ times as much as regular pay. This means overtime is 1.5 x $40.00 = $60.00 per hour.

The overtime pay = $60.00 x 15 = $900.00

Now combine the overtime pay with the regular pay to get the answer

$900.00 + $4000.00 = $4900.00

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 6 – Problems Involving Percentages

Percentages are used to compare quantities in different situations. Percentage numbers also express rates. Unit 3, Topic 5 – Using Percents for a review of percents, rates, and ratios.

Question 1

Someone guesses that 30% of the people in the Northwest Territories have never been outside of the Territory. If the population is 55,000 people, how many people have left the Northwest Territories at least once according to this guess?

a) 38,500
b) 16,500
c) 22,500
d) 12,500

Answer: a

Explanation
If 30% have never been out of the Territory, then 70% have been out at least one to make up the total of 100%. We know that 100% = 55,000 people. Recall that "of means times". To find how many have been out of the Territory find 70% of 55,000. To do this, multiply .7 x 55,000 = 38,500 people.

Question 2

550 people bought fishing licenses in Yellowknife last week. Of these, 16% were women. If half of the women who bought licenses were under age 50, how many were over 50 years old?

a) 88
b) 44
c) 25
d) can't tell from this information

Answer: b

Explanation
First find 16% of 550 = 88. This is the number of people who bought licenses and are women. We are told that half of these are under age fifty, which means the other half of them are over fifty. Divide 88 by 2 to find that 44 of the women were over fifty and 44 under fifty years old.

Topic 6 – Problems Involving Percentages

Question 3

If 20% of a wood supply is used to heat a building for one week, how much of the total wood supply will be left after three weeks?

a) 60%
b) 30%
c) 50%
d) 40%

Answer: d

Explanation
We know that 20%,or 1/5 of the total supply is used each week. Three weeks will use 3 times 20% or 60%. This leaves a remaining supply of 100% – 60% = 40%.

Question 4

A carpenter estimates that 15% of his cost for materials will be waste. If he spends $15,000 on materials, how much will be spent on wasted material?

a) $225
b) $1550
c) $2250
d) $1250

Answer: c

Explanation
We need to find what 15% of $15,000 is equal to . "of means times", and so we multiply .15 x $15,000 to find $2250 will be spent on wasted material.

Question 5

Twenty percent of the applicants for a pilot's license fail on their first test flight. Of those who fail, five percent decide not to try again. What percent of those who apply for a license decide not to try again?

a) 5%
b) 2.5%
c) 1%
d) 10%

Answer: c

Explanation
This problem asks us to find a percent of a percent. We need to know what five percent of twenty percent is since this is the number of applicants who both fail and decide not to try again. Use multiplication to find the answer by converting to decimals: .05 x .2 = .01 = 1%.

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 7 – Problems involving discount (x% off)

A sign announcing "30% off" is describing a discount. Discount is a rate applied to a price. The rate is expressed as a percent. The percent is multiplied times a price to give the amount of the discount. The answer is subtracted from the price to arrive at a discounted, or lowered price. This price is often called a sale price. Discount rates are expressed as percents. You may want to review the topic on percentage numbers and ratios and proportions for this topic.

Example

A pair of insulated coveralls is on sale for $42.50 at the coop. Here is how the sale price was calculated:

1) Original price (list price, undiscounted price) $50.00
2) Rate of discount 15%
3) Discount amount = $7.50 (.15 x $50.00)
4) Sale price (discounted price) = $42.50 ($50.00 – $7.50)

In a discount problem we are trying to find one or more of these four items. **Solve discount problems by identifying which of the four items are given in the problem. Then use multiplication, division, and subtraction to find the remaining items in a discount problem.**

Formulas based on the relationships in the example will relate each item in the list to the other items.

Question 1

A new car is offered at 15% off the listed price of $23,000.00 what is the discounted price?

a) $22,500

b) $19,500

c) $21,500

d) $18,500

Answer: b

Explanation
First find the discount amount. We need to find 15% of $23,000. "of means times"- we multiply 15% x $23,000.

15% = .15 (definition of percent)

.15 x $23,000 = $3450 (the discount amount)

Subtract the discount amount from the list price to find the sale price

$23,000 – $3450 = $19,550

$19,550 is 15% less than $23,000.

Topic 7 – Problems involving discount (x% off)

In this problem we were given the discount rate and the list price. We had to first find the discount amount. This allowed us to find the discounted, or sale price. Now we know all four items.

Question 2

A fur trapper buys a snow machine for his trap line on sale at a cost of $13,000. Later he learns that this price was a sale price that was based on a discount of 25%. He bought the machine at 25% below list price. What was the list price?

a) $17,333.33
b) $19,000
c) $15,000
d) $18,333.33

Answer: a

Explanation
We are given the sale price (the discounted price) and the discount rate. We need to find the list price (the price before the discount was applied). The discounted price equals the list price minus the discount.

We know that the discount = .25 of the list price. (i.e. 25% off)

Let P = the list price we want to find. An equation will show the relationship this way: $13,000 = P – .25P

In words: $13,000 is equal to the list price minus 25% of the list price . We need to solve the equation and find the value of P that makes it a true statement.

The distributive property of multiplication is helpful.

$r (a + b) = ra + rb$ and $r (a – b) = ra – rb$

This is true for any values of r, a, and b. r distributes over a and b. Multiplication distributes over addition.

Topic 7 – Problems involving discount (x% off)

Therefore \$13,000 = P – .25P = P(1 – .25) = P(.75) (distributive rule, simplifying)

\$13,000 = P.75 (\$13,000 is equal to 75%, or $^3/_4$ of the price)

Dividing both sides by .75 gives the value of P

P = \$13,000/.75 = \$17,333.33 which is the list price before the discount was applied to it.

Question 3

How much was the discount worth to the trapper in dollars saved?

a) \$5000.00
b) \$2533.33
c) \$4333.33
d) \$3330.00

Answer: c

Explanation
The dollar value of the discount to the trapper is the difference between what he paid and the list price.

\$17,333.33 – \$13,000 = \$4333.33

check: .25 x \$17,333.33 = \$4333.33, and \$17,333.33- \$4333.33 = \$13,000

Alternate method: set up a proportion

We could also set up a proportion and find the missing term by cross multiplying and dividing with the factor next to the missing term.[8]

We know this relationship exists:

\$13,000/P = .75/1

In words: \$13,000 stands in the same relationship to the list price as $^3/_4$ths does to 1. (we can say this because we know that \$13,000 is three fourths of the full cost after a 25% discount was applied.)

Cross Multiply

\$13,000 = .75P

Divide both sides by .75

\$13,000/.75 = P

P = \$17,333.33

8 See Unit 3 topic 8.

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 7 – Problems involving discount (x% off)

Question 4

What is the discount rate on Joan's parka if she paid \$650 for it when it was on sale marked down from a list price of \$800?

a) 15%
b) $16\frac{1}{4}\%$
c) $18\frac{3}{4}\%$
d) 20%

Answer: c

Explanation
We are given the list price and the discounted price. We have to find the rate of discount, a percent number.

We know that the difference between \$800 and \$650 = \$150. This difference is the value of the discount because \$150 was subtracted from the list price. Now we need to know what percent of \$800 is represented by \$150. Recall that a percent is a fraction with a denominator of 100. We need the numerator of that fraction such that

?/100 x \$800 = \$150

?x\$800 = \$150 (Multiply)

100

? X \$800 = \$15000 (simplify by multiplying both sides by 100)

? = \$15000/\$800 (simplify by dividing both sides by \$800)

? = 18.75%

The discount rate was $18\frac{1}{2}$ percent.

Check: .1875 x \$800 = \$150

Unit 4 – Practice Exam Questions Involving Basic Operations

Topic 8 – Problems involving markup (x% above cost)

Merchants mark up the wholesale price that they pay for their goods in order to make a profit. **The rate that determines how much above cost something will be sold for is called the markup.** In banking it is called the spread between what the bank pays for borrowing money and what it charges its customers for borrowing money.

The difference between a wholesale price and a retail price is referred to as a markup. You may want to review the sections on percent, ratio and proportion, and problems involving discount for this topic.

Example

A used parka retails for $57.50. Here is how the retail price was calculated:

Wholesale price (cost before markup,) $50.00

Rate of markup 15%

Markup amount = $7.50 (.15x$50.00)

Retail price (price after markup is added) = $57.50 ($50.00 + $7.50)

In a mark-up problem we are trying to find one or more of these four items. Solve markup problems by identifying which of the four items are given in the problem. Then use multiplication, division, and subtraction to find the remaining numbers.

Formulas based on the relationships in the example will relate each item in the list to the other items.

Question 1

Tuktu Expediting Company buys flour in 100 lb bags wholesale for $35.00 per bag. The company repackages flour in 20 lb bags for fly-in fishing camps. They sell the rebagged flour for $10.00 per bag. What is the markup per pound of flour?

a) 10 cents
b) 12 cents
c) 15 cents
d) 20 cents

Answer: c

Explanation
We need to know the cost per pound wholesale and the cost per pound retail and then find the difference to get the mark-up amount per pound.

Wholesale cost per pound = $35.00/100 = .35 = 35 cents per pound

Retail price per pound = $10.00/20 = .50 = 50 cents per pound

The difference is .50 – .35 = .15. The markup is .15 or 15 cents per pound.

Topic 8 – Problems involving markup (x% above cost)

Question 2

What is the rate of this mark-up?

a) 15%
b) 23%
c) 43%
d) 53%

Answer: c

Explanation
We know that the flour has been marked up $.15 per pound. We need to know the relationship between this amount and what the wholesale price per pound is. A proportion allows us to compare the relationship between the mark-up and the wholesale price and express the result as a percentage.

.15/.35 = ?/10 = 42.85/100 = 43% (rounded off). The profit margin for this expediter will be what is left after subtracting the costs of bagging into 20 lb amounts for retail.

Question 3

Shirley buys the work of Northern artists and marks it up 25% for sale through ads on her website. Calculate the following:

a) If Shirley paid $3500 for a carving, how much will she retail it for on her website?
a) $4000
b) $4375
c) $4573
d) $4250

Answer: b

Explanation
We know she marks up 25%, so she will calculate 25% of $3500 and add that amount to $3500.

.25 x $3500 = $875.00

$875.00 + $3500 = $4375

She will offer the carving for sale at a price of $4375.00

Topic 8 – Problems involving markup (x% above cost)

Question 4

Shirley wants to increase her markup so that she can list the carving that she paid \$3500 for a new price of \$5000.00. What will her new markup be?

a) 28%
b) 39%
c) 45%
d) 43%

Answer: d

Explanation
We know the wholesale price and the retail price. We need to express the difference between them as a percent.

\$5000 – \$3500 = \$1500

In order to express the difference as a percent we need to know what part of \$3500 (her cost) \$1500 (the markup amount) equals. Divide \$1500 by \$3500.

\$1500/\$3500 = x%

.4285 = x/100

42.85% = x

Shirley will have to markup 43% (rounded to the nearest percent)

Check: 43% of \$3500 = \$1505, and \$3500 + \$1505 = \$5005. (\$5.00 is due to rounding up from 42.85%)

Question 5

If Shirley sells a carving for \$4200 that she has marked up 30% from what she paid for it. What did she pay for it?

a) \$3500.77
b) \$3750.77
c) \$3230.77
d) \$3467.50

Answer: c

Explanation
We know the marked up price and the rate (percent) of the markup. To find what the wholesale price is we solve the following equation:

Retail price = 30% of the wholesale price + the wholesale price

Retail price = \$4200

Set W = the wholesale price we are looking for and substitute:

\$4200 = .3W + W

Topic 8 – Problems involving markup (x% above cost)

In words: we are looking for a number, such that when we add 30% of it to itself we get $4200.

$4200 = W (.3 + 1) (use the distributive rule, factor out W)

$4200 = W x 1.3 (simplify .3 + 1)

W = $3230.77 (divide both sides by 1.3)

Shirley paid $3230.77 for the carving.

Check: 30% of $3230.77 = $969.23

The retail price = $3230.77 (her cost) + $969.23 (the markup amount) = $4200.00.

Unit 5

Practice Exam Questions for Math – Module 1 – Foundations

There are four modules in the common core required for all trades entrance math exams. Each module includes a set of practice exam questions. Each topic in the table of contents for the Foundation section has five questions that will test your preparation for the trades entrance exam.

You should aim for 100% and study the sections of the curriculum for any topics that you do not have mastery over. The core curriculum is based on "need to know" competencies that are important in all trades. You may want to use the following sample exam questions both as a way of assessing what you need to learn before you work on the curriculum, and as a test of what you know after you have completed your preparation for the exam.

The practice questions begin with topic two and follow the table of contents for the Foundations curriculum. Turn to the appropriate section of the curriculum whenever you need help.

Practice Exam Questions

Number Operations

Integers

1) Adding Integers
2) Subtracting Integers
3) Signed Numbers
4) Multiplying Integers
5) Dividing Integers

Question 1

What is the sum of 89, 567, and 418?

a) 897

b) 912

c) 1011

d) 1074

Question 2

What is the sum of the integers in the following list?

3/2, 89, 4.67, 2003, 11/16

a) 62

b) 9/2

c) 2092

d) 2007.67

Question 3

What is the result of adding 5671 and 379?

a) 6050

b) 6150

c) 5991

d) 6040

Question 4

What is the answer when you subtract 876 from 1034?

a) 234

b) 158

c) 234

d) 168

Number Operations

Question 5
Multiply 32 times 64

a) 1048

b) 1348

c) 2048

d) 3048

Question 6
What is 345 divided by 5?

a) 79

b) 95

c) 69

d) 59

Question 7
Subtract 45 from 516

a) 441

b) 461

c) 473

d) 471

Question 8
What is the product of 456 and 39?

a) 18,684

b) 17,784

c) 19, 684

d) 18,784

Question 9
Divide 36 into 181

a) 5 R10

b) 5 R15

c) 4 R1

d) 5 R1

Question 10

Subtract 5003 from 10,113

a) 6110

b) 5110

c) 6113

d) 6115

Answers:

1) d
2) c
3) a
4) b
5) c
6) c
7) d
8) b
9) d
10) b

Practice Exam Questions

Number Operations – Fractions

Fractions

1) Adding Fractions
2) Subtracting Fractions
3) Multiplying Fractions
4) Dividing Fractions

Question 1

What is 5/12 multiplied by 6?

a) 3/2

b) 30/2

c) 2 1/2

d) 30/12

Question 2

Divide 5/6 by 12

a) 5/72

b) 10

c) 65

d) 14 1/2

Question 3

Add 12 + 2/3 + 1/4

a) 15/4

b) 12 11/12

c) 15/12

d) 14 1/4

Question 4

Subtract 11/16 from 4/5

a) 7/11

b) 1/5

c) 9/80

d) 44/50

Practice Exam Questions

Number Operations – Fractions

Question 5

Add 1/3 and 3/5

a) 14/15

b) 2/3

c) 4/5

d) 4/15

Question 6

Multiply 3/16 x 2/3

a) 1/4

b) 12/10

c) 1/8

d) 6/16

Question 7

Subtract the smaller fraction from the larger one, 8/14 and 7/8

a) 1/8

b) 6/7

c) 17/112

d) 17/56

Question 8

Divide 3/16 by 1/8

a) 1 1/2

b) 2 1/2

c) 1 1/4

d) 2/3

Question 9

Add 1/5 and 7/8

a) 8/13

b) 1 2/5

c) 1 3/40

d) 1 2/3

Practice Exam Questions

Number Operations – Fractions

Question 10

Subtract 9/16 from 4/5

a) 5/16

b) 19/80

c) 45/64

d) 36/45

Answers

1) c
2) a
3) b
4) c
5) a
6) c
7) d
8) a
9) c
10) b

Practice Exam Questions

Number Operations – Decimals

Decimals

1) Adding decimals
2) Subtracting decimals
3) Multiplying decimals
4) Dividing decimals

Question 1

Add 3.56 + 4000.32

a) 4103.88

b) 4003.88

c) 4030.88

d) 4300.88

Question 2

Subtract .007 from 109

a) 93

b) 109.993

c) 108.993

d) 102.9

Question 3

Multiply 4.57 x 34.21

a) 156.3397

b) 165.3

c) 125.35

d) 155.21

Question 4

Divide 67 by 2.5

a) 33.52

b) 124.35

c) 34.25

d) 26.8

Practice Exam Questions

Number Operations – Decimals

Question 5
Multiply 2000 x 67.9

a) 135,800

b) 114.9

c) 1240.90

d) 267,009

Question 6
Subtract 45.67 from 90.007

a) 44.337

b) 55.003

c) 45.003

d) 54.900

Question 7
Divide 56 by 10,000

a) 560

b) .0056

c) .056

d) .0566

Question 8
Multiply 83 by 1000

a) 8300

b) 830,000

c) 83,000

d) 83.000

Question 9
Add 56.2 + 78.45 + 78

a) 112.65

b) 212.65

c) 214.75

d) 212.76

Number Operations – Decimals

Question 10
Multiply 56.78 x .003

a) 0.17034

b) .80829

c) 17.034

d) 80.829

Answers

1) b

2) c

3) a

4) d

5) a

6) a

7) b

8) c

9) b

10) a

Practice Exam Questions

Combining Operations and Numbers – Identify the largest or smallest number in decimal and fraction form

Question 1
Identify the smallest of .0345, .03366, 3.0300, and .04000

a) .0345

b) .03366

c) 3.0300

d) .04000

Question 2
Which is the largest fraction, 2/3, 5/8, 15/16, 7/8

a) 2/3

b) 5/8

c) 15/16

d) 7/8

Question 3
Which is the smallest number, 2.345, 2.043, 1.999, 2.012?

a) 2.345

b) 2.043

c) 1.999

d) 2.012

Question 4
Which is the largest number, 5/12, .77, 3/5, 1.7

a) 5/12

b) .77

c) 3/5

d) 1.7

Question 5
Which is the smallest number, 5/16, 3/8, 13/27, 7/13

a) 5/16

b) 3/8

c) 13/27

d) 7/13

Practice Exam Questions

Combining Operations and Numbers – Identify the largest or smallest number in decimal and fraction form

Question 6
What is the correct order of 3/8, 4/5, 1/2. And 1/4 from largest on the left to smallest on the right?

a) 3/8, 4/5, 1/2, 1/4

b) 4/5, 1/2, 3/8, 1/4

c) 1/2, 4/5, 3/8, 1/4

d) 3/8, 4/5, 1/4, 1/2

Question 7
Which of the following numbers is greater than 1/2?

a) 7/14

b) 5/4

c) 1/3

d) 75/250

Answers

1) b

2) c

3) c

4) d

5) a

6) b

7) b

Practice Exam Questions

Combining Operations and Numbers – Converting Decimals to Fractions

Question 1
Express .004 as a fraction

a) 4/10

b) 4/1

c) 4/1000

d) 4/100

Question 2
Convert .56 into a fraction

a) 14/25

b) 56/25

c) 56/1000

d) 56/25

Question 3
Change 234.2 into a fraction

a) 234,200

b) 234/200

c) 234 1/5

d) 234 1/100

Question 4
Change .0076 into a fraction

a) 100/76

b) 7.6

c) 76/100

d) 19/2500

Question 5
Convert 45.089 into a fraction

a) 45,089/1000

b) 450/890

c) 45,089/10,000

d) 45/1089

Practice Exam Questions

Combining Operations and Numbers – Converting Decimals to Fractions

Answers

1) c
2) a
3) c
4) d
5) a

Practice Exam Questions

Combining Operations and Numbers – Converting Fractions to Decimals

Question 1
Change 7/8 into a decimal number

a) .075

b) .870

c) 1.875

d) 0.875

Question 2
Express 4 3/4 as a decimal

a) 44.3

b) 16.3

c) 4.75

d) 4.45

Question 3
Convert 11/23 into a decimal number

a) .478

b) .2311

c) 22. 23

d) 4.780

Question 4
Express 9/3 as a decimal

a) 39.33

b) 93

c) 9 1/3

d) 3.0

Question 5
Convert 5/16 into a decimal number

a) 3.1

b) .51

c) 0.3125

d) .2575

Practice Exam Questions

Combining Operations and Numbers – Converting Fractions to Decimals

Question 6

What is the decimal equivalent to 3/4?

a) .34

b) three fourths

c) 1.34

d) .75

Question 7

What is the decimal number for 7/12 to the nearest hundredth?

a) .59

b) .58

c) .583

d) .59

Answers

1) d

2) c

3) a

4) d

5) c

6) d

7) b

Practice Exam Questions

Combining Operations and Numbers – The Order of Operations

Question 1
What operation must be carried out first in the expression 6 x 5 + 8?

a) 6 + 8

b) 5 + 8

c) 6 x 5

d) 6 x 8

Question 2
Find 14 – 3 x 2

a) 1/2

b) 10

c) 8

d) 21

Question 3
What is the answer for 24 ÷ (6 x 2)?

a) 3

b) 2

c) 8

d) 4

Question 4
Calculate (5 x 3) + (10 ÷ 2) – 3.

a) 32

b) 21

c) 17

d) 23

Question 5
When brackets appear in an expression, what rule should you follow?

a) do the operation inside the brackets last

b) do the operation inside the brackets first

c) do the operation inside the left bracket first

d) Ignore the brackets if you can add them

Practice Exam Questions

Combining Operations and Numbers – The Order of Operations

Answers

1) c
2) c
3) b
4) c
5) b

Practice Exam Questions

Combining Operations and Numbers – Using Percents

Using Percents

a) Changing fractions into equivalent percent numbers

b) Changing decimals into equivalent percent numbers

Question 1

What is 3.45 expressed as a percent?

a) 345

b) 34.5%

c) 34.05%

d) 345%

Question 2

Express 1/5 as a percent

a) 50%

b) 20%

c) 25%

d) 15%

Question 3

What is .1568 in percentage terms

a) 156%

b) 157%

c) 16%

d) 15%

Question 4

Express 8/5 as a percent number

a) 160%

b) 16%

c) 58%

d) 85%

Combining Operations and Numbers – Using Percents

Question 5

What is 7 3/5 expressed as a percent?

a) 760%

b) 735%

c) 75.5%

d) 76%

Answers

1) d

2) b

3) c

4) a

5) a

Practice Exam Questions

Combining Operations and Numbers – Bases, Exponents and Square Roots

Simplifying expressions with exponents

Question 1

What is the square of 32?

a) 64

b) 96

c) 1024

d) 16

Question 2

Calculate 7.5 squared

a) 49.15

b) 4905

c) 490.25

d) 56.25

Question 3

What is $(2/3)^2$?

a) 4/9

b) 6/9

c) .667

d) 2/9

Question 4

Calculate 5.67 squared

a) 25.7

b) 32.15

c) 55.67

d) 33.125

Combining Operations and Numbers – Bases, Exponents and Square Roots

Question 5

Calculate 1 7/16 squared

a) 1 14/16

b) 17 1/16

c) 1 49/160

d) 2.07

Answers

1) c

2) d

3) a

4) b

5) d

Practice Exam Questions

Adding, Subtracting, Multiplying and Dividing – Adding Numbers Containing Integers, Decimals and Fractions

Sample Exam Questions involving Basic Operations

a) Adding numbers containing integers, decimals and fractions

b) Subtracting numbers containing integers, decimals and fractions

c) Multiplying numbers containing integers, decimals and fractions

d) Dividing numbers containing integers, decimals and fractions

Question 1
What is the sum of 13 7/8 and 42.90?

a) 156.7

b) 55.77

c) 58.777

d) 56.775

Question 2
Divide 405 3/8 by 3/8

a) 135.375

b) 1081

c) 1375

d) 135.475

Question 3
Express the product of 7/12 and 6.78 as a fraction

a) 3/25

b) 3 7/12

c) 3 11/12

d) 3 24/25

Question 4
Subtract 78.09 from 89.75

a) 12.66

b) 10.56

c) 12 2/3

d) 11.66

Practice Exam Questions

Adding, Subtracting, Multiplying and Dividing – Adding Numbers Containing Integers, Decimals and Fractions

Question 5
Multiply 6 1/8, 32, and .77

a) 180 3/8

b) 180.66

c) 150.92

d) 151.9

Question 6
Add 133.04 and 16/3

a) 138.37

b) 149.33

c) 139.33

d) 138 5/8

Question 7
Divide 56.3 by 2.25

a) 23.25

b) 25.25

c) 25.02

d) 23.02

Question 8
Subtract .89 from 2 1/2

a) 1.18

b) 1.21

c) 1.55

d) 1.61

Question 9
Find the sum of 18 1/4, .15, and 38

a) 56.4

b) 56.25

c) 56. 15

d) 156.15

Practice Exam Questions

Adding, Subtracting, Multiplying and Dividing – Adding Numbers Containing Integers, Decimals and Fractions

Question 10

Multiply 2/3, 4/5, and .16

a) 1 12/15

b) .085

c) 16 8/15

d) 1.085

Answers

1) d

2) b

3) d

4) d

5) c

6) a

7) c

8) d

9) a

10) b

Practice Exam Questions

Combining Operations and Numbers – Calculate the Square Root of a Number or of an Expression

Question 1
What is the square root of 625?

a) 62.5

b) 55

c) 25

d) 25.25

Question 2
Find the square root of 69

a) 16.9

b) $8\sqrt{5}$

c) $5\sqrt{8}$

d) 8.5

Question 3
Calculate $\sqrt{176}$

a) 17.5

b) $4\sqrt{11}$

c) $11\sqrt{16}$

d) $41\sqrt{14}$

Question 4
Find the square root of 36

a) 9

b) 18

c) 13

d) 6

Question 5

Calculate $\sqrt{16}\sqrt{144}$

a) 2304

b) 4.12

c) 812

d) 48

Practice Exam Questions

Combining Operations and Numbers – Calculate the Square Root of a Number or of an Expression

Answers

1) c
2) b
3) b
4) d
5) d

Practice Exam Questions

Combining Operations and Numbers – Calculate the Cube of a Number

Question 1
Calculate the cube of 75

a) 775

b) 7075

c) 421,875

d) 707,750

Question 2
Find the cube of 3/8

a) 81/64

b) 81/16

c) 27/64

d) 27/512

Question 3
Find the cube of 2.16

a) 110.08

b) 11.08

c) 10.08

d) 8.32

Question 4
What is the cube of 2 2/3?

a) 10 1/3

b) 8 8/3

c) 19.96

d) 18 26/27

Question 5
Calculate the cube of 16/2

a) 922

b) 412

c) 512

d) 812

Practice Exam Questions

Combining Operations and Numbers – Calculate the Cube of a Number

Answers

1) c
2) d
3) c
4) d
5) c

Practice Exam Questions

Combining Operations and Numbers – Simplify Expressions Involving Exponents

Question 1
Simplify $(2/3)^4$

a) 80/81

b) 16/27

c) 8/81

d) 16/81

Question 2
Simplify $3^{20}/3^{15}$

a) 3^{35}

b) 1^{35}

c) 3^{15}

d) 3^{5}

Question 3
Simplify $(4^2/4^5)^2$

a) 4^{-6}

b) 4^{6}

c) 4^{4}

d) 4^{10}

Question 4
Simplify $(3 \times 4)^2 \times (3 \times 4)^3$

a) 12^{6}

b) 144^{3}

c) 144^{2}

d) 12^{5}

Question 5
Simplify $(-3 \times 5)^4$

a) $(5/3)^4$

b) $(15)^{-4}$

c) $(-3)^4 \times (5)^4$

d) $(1/15)^4$

Practice Exam Questions

Combining Operations and Numbers – Simplify Expressions Involving Exponents

Answers

1) d
2) d
3) a
4) d
5) c

Practice Exam Questions

Combining Operations and Numbers – Bases, Exponents and Square Roots

Using Scientific notation

Question 1

Rewrite 56,000,000 in scientific notation

a) 56.0×10^{6}

b) 5.6×10^{7}

c) 5.6×10^{-7}

d) 560×10^{6}

Question 2

Expand 8.4×10^{-4}

a) .00084

b) 84,000

c) .084084

d) .0084

Question 3

Put ,00098 into scientific notation

a) 8.0×10^{-5}

b) 8001×10^{-4}

c) 9.8×10^{-5}

d) 9.8×10^{-4}

Question 4

Convert 7.01×10^{5}

a) 701,000

b) 70,001

c) 710,000

d) 770,000

Practice Exam Questions

Combining Operations and Numbers – Bases, Exponents and Square Roots

Question 5

Rewrite 897,030,000 in scientific notation

a) 89.73×10^6

b) 8.9703×10^8

c) 8.97×10^7

d) 8.97×10^8

Answers

1) b
2) a
3) d
4) a
5) b

Practice Exam Questions

Combining Operations and Numbers – Ratios, Rates, and Proportions

Question 1
What is the ratio of men to women working at a Diamond mine if there are 3 men working there for every woman who works there?

a) 2/3

b) 3/3

c) 3/1

d) 3/2

Question 2
Express the proportion between 2/3 and 8/12

a) 2/3 = 12/8

b) 2/3 = 8/12

c) 12/8 = 3/2

d) 3/2 = 8/12

Question 3
What is the proportion between 3/4 and ?/20?

a) 3/4 = 2/20

b) 3/4 = 12/16

c) 3/4 = 15/20

d) 3/20 =4/3

Question 4
What is the missing term in the proportion between ?/18 and 3.5/9

a) 7

b) 2/9

c) 3

d) 6

Practice Exam Questions

Combining Operations and Numbers – Ratios, Rates, and Proportions

Question 5

Which ratio makes a proportion with 2/3?

a) 1/3

b) 3/2

c) 4/6

d) 12/6

Answers

1) c
2) b
3) c
4) a
5) c

Practice Exam Questions

Problems Involving Discount (x% off)

Question 1

The coop store has marked all canned goods down 20% for a special clear out sale. How much will Bruce pay for 10 cans of beans that normally sell for $1.39 per can?

a) $13.00

b) $10.50

c) $11.12

d) can't tell from this information

Question 2

What discount rate would have to be applied to a boat listed at $14,000 to bring it down to a sale price of $12,000?

a) 12%

b) 13%

c) 20%

d) 14.3%

Question 3

Jim bought a rifle on sale for $130.00. It had been marked down 10%. What was the original list price?

a) $150.00

b) $145.00

c) $144.44

d) $154.00

Question 4

The theatre is giving away a pass that entitles the holder to a 25% discount on tickets. The Saturday show costs $15.00 for a ticket. What will someone pay who has a pass?

a) $12.25

b) $11.25

c) $10.00

d) $12.00

Practice Exam Questions

Problems Involving Discount (x% off)

Question 5
How much would Sally have to discount her asking price for her car if she gets an offer that equals half of her asking price?

a) 30%

b) 25%

c) 40%

d) 50%

Answers

1) c

2) d

3) c

4) b

5) d

Practice Exam Questions

Problems Involving Markup (x% above cost)

Question 1
The food in the supermarket is marked up 18% above cost. If the supermarket pays $150 for an item what will it be sold in the store for?

a) $27.00

b) $168.00

c) $177.00

d) $200.00

Question 2
A craft coop marks up the carvings it sells by 35%. If a carving is sold for $2500, what did the coop pay for it?

a) $875

b) $3375

c) $1851.85

d) $1535.00

Question 3
A car cost a dealer $17,000. How much should this car be sold for so that it sells for 50% more than it cost?

a) $23,000

b) $34,000

c) $18,000

d) $25,500

Question 4
Bill paid $675 for a canoe after it had been marked up $15%. What did the canoe cost before the markup?

a) $825

b) $600

c) $586.96

d) $576.98

Practice Exam Questions

Problems Involving Markup (x% above cost)

Question 5
Bill sold his house for 45% above his cost. He paid $60,000 for his house. How much did he sell it for?

a) $107,000

b) $87,000

c) $70,000

d) $645,000

Answers

1) c

2) c

3) d

4) c

5) b

Practice Exam Questions

Involving Basic Operations

Question 1

John was paid $450.00 the first week, $344.00 the second week, $236.70 the third week, and $884.75 the fourth week. How much did he make at the end of these four weeks?

a) $1915.45

b) $1030.70

c) $1925.45

d) $1130.70

Question 2

What is the unit cost of a can of sardines if a case of 50 cans costs $62.50?

a) $2.50

b) $00.75

c) $1.25

d) $1.50

Question 3

Bill has payroll deductions of $15.50 and $54.00 from his monthly salary of $1475.00. What is his monthly pay after deductions?

a) $1425.50

b) $1405.50

c) $1305.50

d) $1545.50

Question 4

Ellen earns 1 1/2 times as much when she works overtime. If she normally earns $12.00 per hour, what does she earn per hour when she works overtime?

a) $12.50

b) $16.50

c) $24.00

d) $18.00

Practice Exam Questions
Involving Basic Operations

Question 5
$57.00 is to be shared equally between 3 people. How much will each person receive?

a) $18.50

b) $22.50

c) $5.70

d) $19.00

Answers

1) a

2) c

3) b

4) d

5) d

Practice Exam Questions

Problems Involving Money. Add, Subtract, Multiply, Divide (payroll, unit price)

Question 1

Steve and Sally work on a drilling rig near Inuvik. They are both paid at the same rate. If Steve makes $1700 for 25 hours of work, how much will Sallly make for 20 hours of work?

a) can't tell from the information given

b) $1360

c) $1700

d) $1200

Answer: b

Question 2

What is the base rate of pay for Sally?

a) can't tell from this information

b) $55.00 per hour

c) $1500 per week

d) $68.00 per hour

Answer: d

Question 3

One truckload of gravel costs $110.00. How much will 39 loads cost?

a) $4290

b) $3910

c) $2580

d) $7650

Answer: a

Practice Exam Questions

Problems Involving Money. Add, Subtract, Multiply, Divide (payroll, unit price)

Question 4

Every paycheque that Margaret receives includes a deduction of 17% from her gross earnings. If Margaret earns $1250 before this deduction, how much will she take home?

a) $1150.50

b) $1450

c) $1037.50

d) $988.75

Answer: c

Question 5

1800 pounds of flour cost $780. What is the cost per pound?

a) Can't tell from this information

b) $23.00

c) $00.78

d) $00.43

Answer: d

Resources

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Appendix A

The following topics from the Alberta Entrance Level Competencies are covered in Section I, titled ***Foundations: Number Concepts and Operations*** in this curriculum.

Section One: Number Concepts and Operations

A. Decimals And Integers Including Whole Numbers

Outcome: Demonstrate a number sense for decimals and integers, including whole numbers. (1, 2, 3, 4, 5)

1. Define and use power, base and exponent to represent repeated multiplication.
2. Write a whole number as an expanded numeral; using powers of 10, scientific notation, and vice versa.
3. Use divisibility rules to determine if a number is divisible by 2, 3, 4, 5, 6, 9, and 10.
4. Read and write numbers to any number of decimal places.
5. Demonstrate and describe equivalent mixed numbers and improper fractions concretely, pictorially and symbolically.
6. Compare and/or order improper fractions, mixed numbers and decimals to thousandths.
7. Recognize and illustrate that all fractions and mixed numbers can be represented in decimal form.
8. Convert from terminating decimals to fractions.
9. Convert from single-digit repeater decimal numbers to fractions, using patterns.
10. Demonstrate, concretely and pictorially, that the sum of opposite integers is zero.
11. Represent integers in a variety of concrete, pictorial and symbolic ways.
12. Compare and order integers.

B. Rational Numbers, Common Fractions, Integers And Whole Numbers

Outcome: Demonstrate a number sense for rational numbers, including common fractions, integers and whole numbers. (1, 2, 3, 4, 5)

1. Demonstrate and explain the meaning of a negative exponent, using patterns (limit to base 10).
2. Represent any number in scientific notation.
3. Define, compare and order any rational numbers.
4. Demonstrate concretely, pictorially and symbolically that the product of reciprocals is equal to 1.
5. Express 3-term ratios in equivalent forms.
6. Represent and apply fractional percent, and percent greater than 100, in fraction or decimal form, and vice versa.
7. Represent square roots concretely, pictorially and symbolically.
8. Distinguish between a square root and its decimal approximation as it appears on a calculator.

C. Structure And Interrelationship Of Rational Numbers

Outcome: Explain and illustrate the structure and the interrelationship of the sets of numbers within the rational number system. (1, 2, 3, 4, 5)

1. Give examples of numbers that satisfy the conditions of natural, whole, integral and rational numbers, and show that these numbers comprise the rational number system.
2. Describe, orally and in writing, whether or not a number is rational.
3. Give examples of situations where answers would involve the positive (principal) square root, or both positive and negative square roots of a number.

D. Exponents And Rational Bases

Outcome: Develop a number sense of powers with integral exponents and rational bases. (1, 2, 3, 4, 5)

1. Illustrate power, base, coefficient and exponent, using rational numbers or variables as bases or coefficients.
2. Explain and apply the exponent laws for powers with integral exponents.
3. Determine the value of powers with integral exponents, using the exponent laws.

Appendix A

Section Two: Number Operations

A. Arithmetic Operations Using Decimals And Integers

Outcome: Apply arithmetic operations on decimals and integers, and illustrate their use in solving problems. (1, 2, 3, 4, 5)

1. Use patterns, manipulatives and diagrams to demonstrate the concepts of multiplication and division by a decimal.
2. Use estimation strategies to justify or assess the reasonableness of calculations.
3. Add, subtract, Multiply and divide decimals (for more than 2-digit divisors or multipliers; the use of technology is expected).
4. Add, subtract, Multiply and divide integers concretely, pictorially and symbolically.
5. Illustrate and explain the order of operations, using paper and pencil or a calculator.

B. Problem Solving Using Rates, Ratios, Percentages And Decimals

Outcome: Illustrate the use of rates, ratios, percentages and decimals in solving problems. (1, 2, 3, 4, 5)

1. Estimate and calculate percentages.
2. Distinguish between rate and ratio, and use them to solve problems.
3. Explain, demonstrate and use proportion in solving problems.
4. Solve problems by mentally converting, among fractions, decimals and percent.

C. Problem Solving Using Whole Numbers And Decimals

Outcome: Apply arithmetic operations on whole numbers and decimals in solving problems. (1, 2, 3, 4, 5)

1. Add, subtract, Multiply and divide fractions concretely, pictorially and symbolically.
2. Estimate, compute and verify the sum, difference, product and quotient of rational numbers, using only decimal representations of negative rational numbers.
3. Estimate, compute (using a calculator) and verify approximate square roots of whole numbers and of decimals.

D. Problem Solving In Meaningful Context

Outcome: Apply the concepts of rate, ratio, percentage and proportion to solve problems in meaningful contexts. (1, 2, 3, 4, 5)

1. Use concepts of rate, ratio, proportion and percent to solve problems in meaningful contexts.
2. Calculate combined percentages in a variety of meaningful contexts.
3. Derive and apply unit rates.
4. Express rates and ratios in equivalent forms.

E. Using A Calculator

Outcome: Use a scientific calculator or a computer to solve problems involving rational numbers. (1, 2, 3, 4, 5)

1. Document and explain the calculator keying sequences used to perform calculations involving rational numbers.
2. Solve problems, using rational numbers in meaningful contexts.

F. Using Exponents

Outcome: Explain how exponents can be used to bring meaning to large and small numbers, and use calculators or computers to perform calculations involving these numbers. (1, 2, 3, 4, 5)

1. Understand and use the exponent laws to simplify expressions with variable bases, and evaluate expressions with numerical bases.
2. Use a calculator to perform calculations involving scientific notation and exponent laws."